TIØ 1: Financial Engineering in Energy Markets

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COURSE OUTLINE

- ★ Introduction (Chs 1–2)
- ★ Mathematical Background (Chs 3–4)
- ★ Investment and Operational Timing (Chs 5–6)
- ★ Entry, Exit, Lay-Up, and Scrapping (Ch 7)
- ★ Recent Theoretical Work I: Capacity Sizing
- ★ Recent Theoretical Work II: Risk Aversion and Multiple Risk Factors
- ★ Applications to the Energy Sector I: Capacity Sizing, Timing, and Operational Flexibility
- ★ Applications to the Energy Sector II: Modularity and Technology Choice

LECTURE OUTLINE

- Entry and exit strategies
- Lay-up, re-activation, and scrapping

ENTRY AND EXIT STRATEGIES: Setup

- Costless suspension and resumption of operations is not always realistic
  - Suspension may lead to dissipation of human capital or deterioration of equipment
  - In order to resume operations from a suspended state, the entire investment cost must be borne again
- Suppose output price, P, follows a GBM and variable operation cost is C
- Investment cost is I, whereas abandonment cost is E (may be negative as long as $I + E > 0$)
  - Intuitively, invest (abandon) when price reaches upper (lower) threshold P_H (P_L)
  - Once invested (abandoned), maintain *status quo* until lower (upper) threshold is reached
  - Note that the options are compound: part of the value of an active firm, $V_1(P)$, is the option to abandon
  - Similarly for the value of an idle firm, $V_0(P)$

ENTRY AND EXIT STRATEGIES: Solution

- Obtain an ODE for $V_0(P)$ following the contingent claims approach
  - $\frac{1}{2}\sigma^2 P^2 V_0''(P) + (r-\delta)PV_0'(P) - rV_0(P) = 0$
  - General solution is $V_0(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2}$
  - Boundary condition for an idle firm: $\lim_{P \to 0} V_0(P) = 0 \Rightarrow A_2 = 0$
  - Thus, the value of an idle firm is $V_0(P) = A_1 P^{\beta_1}$ for $P \in (0, P_H)$

- Since an active firm pays instantaneous cash flow $(P - C)dt$, the ODE for $V_1(P)$ is $\frac{1}{2}\sigma^2 P^2 V_1''(P) + (r - \delta)PV_1'(P) - rV_1(P) + P - C = 0$
  - General solution is $V_1(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r}$
  - Since the last two terms are the expected NPV of cash flows, the first two terms must be the option value of abandonment
  - Boundary condition: abandonment option is nearly worthless at high prices, i.e., $\lim_{P \to \infty} V_1(P) = \frac{P}{\delta} - \frac{C}{r}$
  - Therefore, $B_1 = 0$ and $V_1(P) = B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r}$ for (P_L, ∞)

ENTRY AND EXIT STRATEGIES: Thresholds

- At P_H, the VM and SP conditions involve exchanging an idle project for an active one
  - $V_0(P_H) = V_1(P_H) - I$ and $V_0'(P_H) = V_1'(P_H)$
- Similarly, at P_L, the VM and SP conditions involve exchanging an active project for an idle one
  - $V_1(P_L) = V_0(P_L) - E$ and $V_1'(P_L) = V_0'(P_L)$
- Inserting equations for $V_0'(P)$ and $V_1'(P)$ gives the following system of non-linear equations:
  - $A_1 P_H^{\beta_1} - B_2 P_H^{\beta_2} - \frac{P_H}{\delta} + \frac{C}{r} = -I$
  - $\beta_1 A_1 P_H^{\beta_1 - 1} - \beta_2 B_2 P_H^{\beta_2 - 1} - \frac{1}{\delta} = 0$
  - $-A_1 P_L^{\beta_1} + B_2 P_L^{\beta_2} + \frac{P_L}{\delta} - \frac{C}{r} = -E$
  - $-\beta_1 A_1 P_L^{\beta_1 - 1} + \beta_2 B_2 P_L^{\beta_2 - 1} + \frac{1}{\delta} = 0$
  - Solve numerically for four unknowns

ENTRY AND EXIT STRATEGIES: Comparison with Myopic Decisions

- Marshallian theory concludes that investment should occur when $P = C + rI$
  - Similarly, abandon if the price drops below the operating cost minus the amortised abandonment cost, $C - rE$
  - However, these conclusions are based on the assumption that the given price will prevail forever
- In order to compare P_H and P_L with $C + rI$ and $C - rE$, respectively, define $G(P) = V_1(P) - V_0(P) = B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r} - A_1 P^{\beta_1}$
  - For (P_L, P_H), $G(P)$ is the incremental value of becoming active
  - For low P, $G(P) \to B_2 P^{\beta_2}$, i.e., decreasing and convex in P
  - For high P, $G(P) \to -A_1 P^{\beta_1}$, i.e., decreasing and concave in P
  - VM and SP conditions become: (i) $G(P_H) = I$, (ii) $G'(P_H) = 0$, (iii) $G(P_L) = -E$, and (iv) $G'(P_L) = 0$
  - Figure 7.1 indicates the S shape over (P_L, P_H)

ENTRY AND EXIT STRATEGIES: Comparison with Myopic Decisions

★ At P_H, subtract ODE for $V_0(P)$ from that of $V_1(P)$ to obtain $\frac{1}{2}\sigma^2 G''(P) + (r-\delta)G'(P) - rG(P) + P - C = 0$

- Use VM and SP at P_H to obtain $\frac{1}{2}\sigma^2 G''(P_H) - rI + P_H - C = 0 \Rightarrow P_H - C - rI = -\frac{1}{2}\sigma^2 G''(P_H) > 0$
- Therefore, $P_H > C + rI$, i.e., firms are more reluctant to invest compared to the Marshallian case

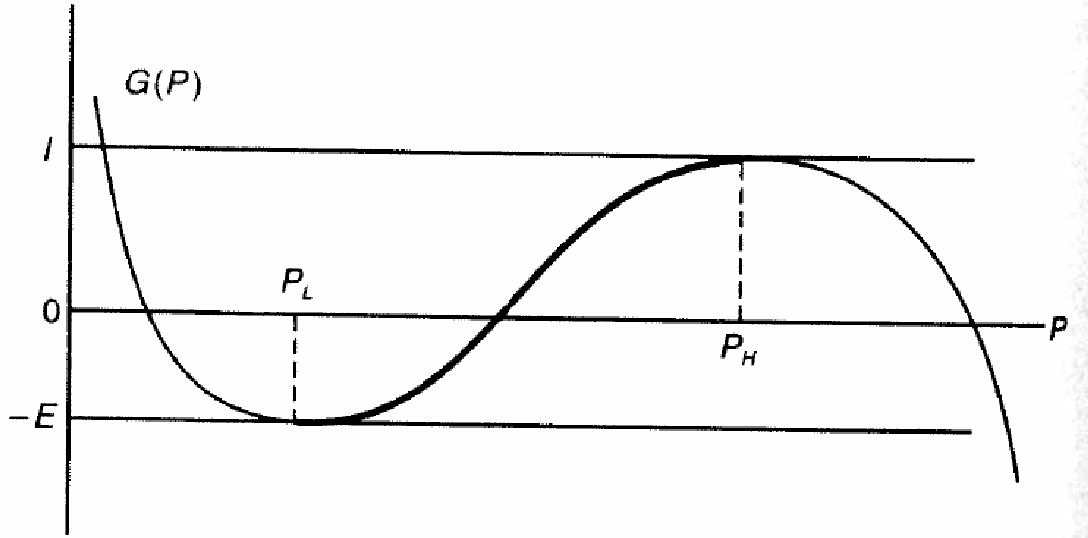

Figure 7.1. Determination of the Thresholds P_L and P_H

ENTRY AND EXIT STRATEGIES: Comparative Statics

- ★ How do P_H and P_L respond to changes in parameters?
  - Not straightforward with r, δ, and σ
  - Analytics are easier with I, E, and C
- ★ Express G as a function of P, A_1, and B_2
  - VM and SP conditions: (i) $G(P_H, A_1, B_2) = I$, (ii) $G(P_L, A_1, B_2) = -E$, (iii) $G_P(P_H, A_1, B_2) = 0$, (iv) $G_P(P_L, A_1, B_2) = 0$
  - Total differentiation of (i) and (ii) yields $G_A(H)dA_1 + G_B(H)dB_2 = dI$ and $G_A(L)dA_1 + G_B(L)dB_2 = 0$
  - Since $G_A(H) = -P_H^{\beta_1}$ and $G_B(L) = P_L^{\beta_2}$, the solutions are $dA_1 = -\frac{P_L^{\beta_2}dI}{\Delta}$ and $dB_2 = -\frac{P_L^{\beta_1}dI}{\Delta}$, where $\Delta = P_H^{\beta_1}P_L^{\beta_2} - P_H^{\beta_2}P_L^{\beta_1} > 0$
  - Differentiate the SP condition at P_H: $G_{PP}(H)dP_H + G_{PA}(H)dA_1 + G_{PB}(H)dB_2 = 0$
  - Thus, $G_{PP}(H)dP_H = -\frac{\left[\beta_1 P_H^{\beta_1-1}P_L^{\beta_2} - \beta_2 P_H^{\beta_2-1}P_L^{\beta_1}\right]dI}{\Delta} \Rightarrow \frac{dP_H}{dI} > 0$ because $G(P)$ is concave at P_H
  - Similarly, find that $\frac{dP_L}{dE} < 0$, $\frac{dP_L}{dI} < 0$, and $\frac{dP_H}{dE} > 0$

ENTRY AND EXIT STRATEGIES: Numerical Example

- An example from the copper industry
  - Mine produces 10 million pounds of copper per year forever
  - $I = 20$ and $E = 2$ (in million \$), while $C = \$0.80$ per pound
  - $\mu = 0.06$, $\delta = \mu - \alpha = 0.04$, $r = 0.04$ (all real rates per annum), and $\sigma = 0.20$ (allow for a range of estimates)
- Solve system of four VM and SP equations numerically using initial guesses for P_H, P_L, A_1, and B_2
  - Marshallian: $NPV_1 = -20 + \frac{10P}{0.04} - \frac{8}{0.04}$ (in million \$), which implies $P_H^0 = 0.88$, and $NPV_0 = -2 - \frac{10P}{0.04} + \frac{8}{0.04} \Rightarrow P_L^0 = 0.79$
  - Figure 7.2 indicates how uncertainty drives these thresholds further apart from each other as uncertainty increases
  - As C increases, investment becomes more difficult, while abandonment is easier (Figure 7.3)
  - Neither threshold is very sensitive to E although the zone of inaction widens as it becomes more difficult to abandon (Figure 7.4)
  - The value curves are parallel at P_H and differ by I (and differ by E at P_L) in Figure 7.5
  - Function $G(P)$ has the characteristic S shape (Figure 7.6)

ENTRY AND EXIT STRATEGIES: Figure 7.2

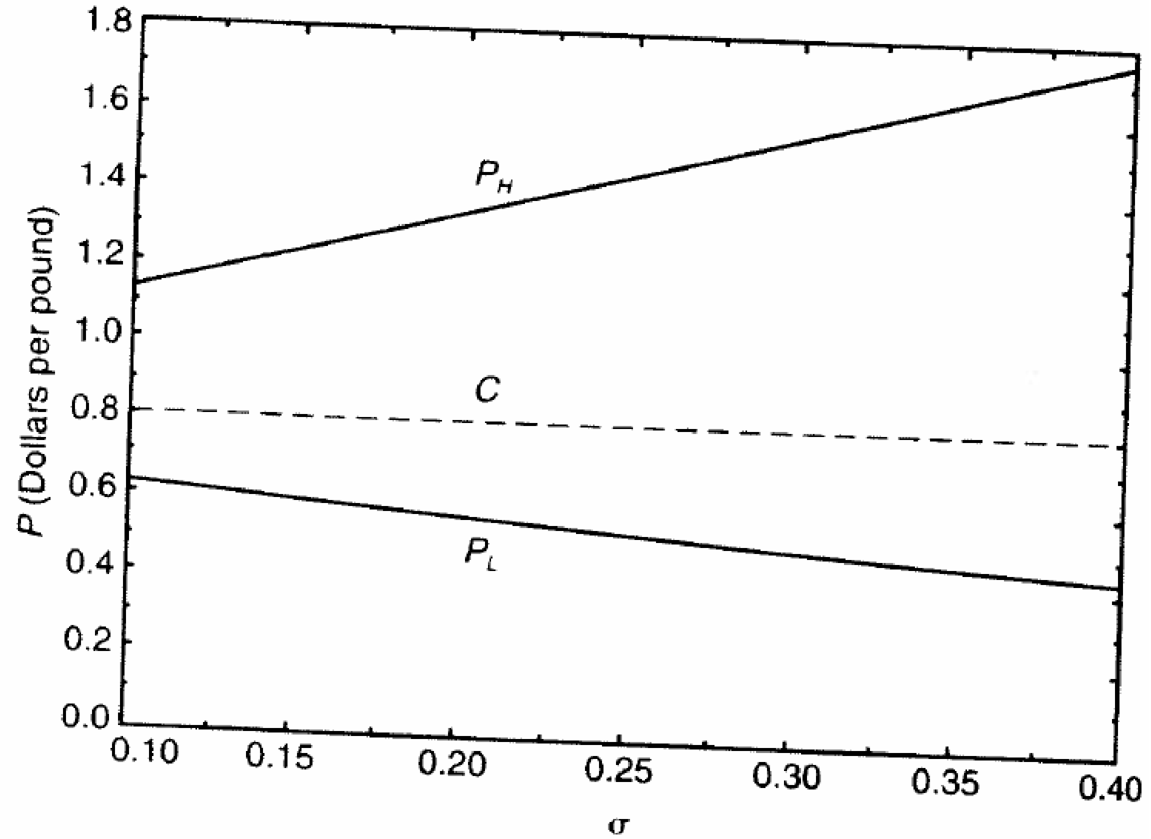

Figure 7.2. Copper: Entry and Exit Thresholds as Functions of σ

ENTRY AND EXIT STRATEGIES: Figure 7.3

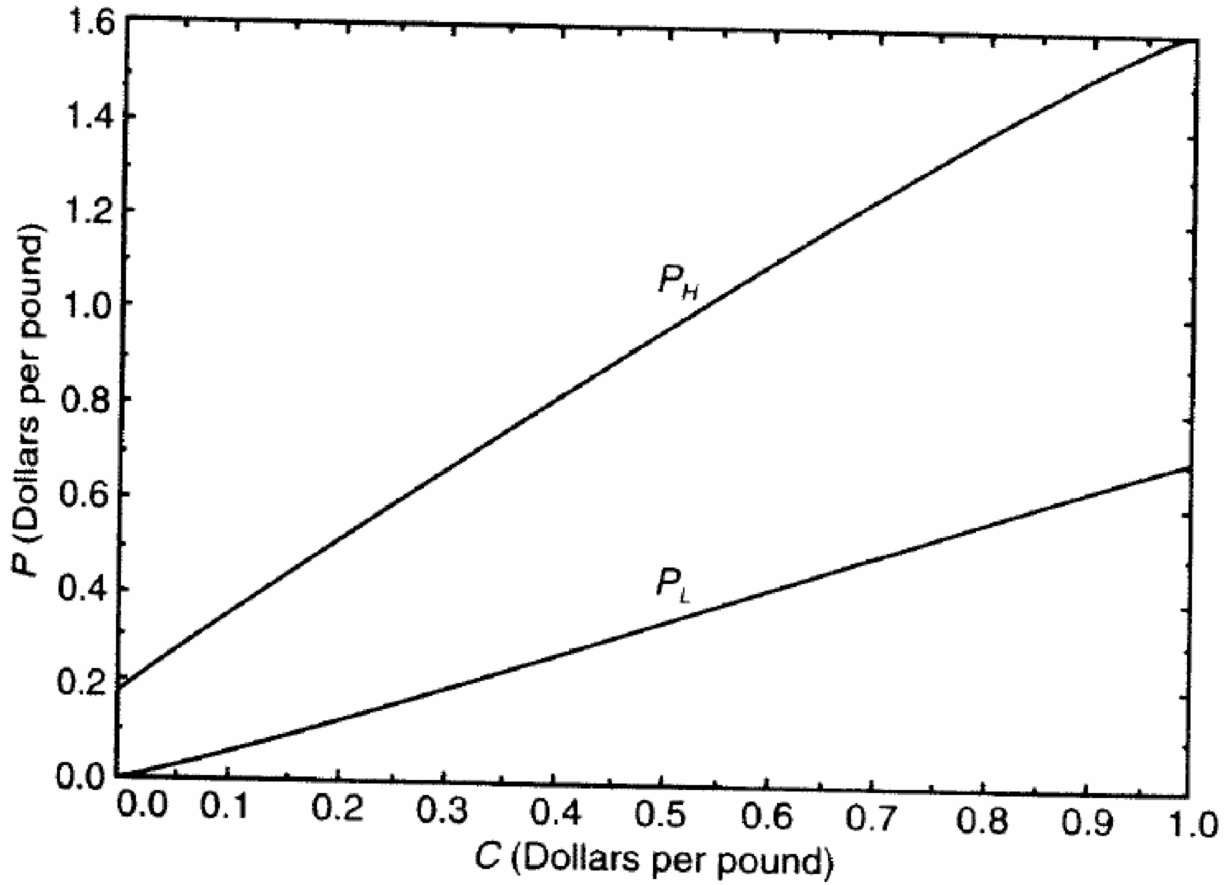

Figure 7.3. *Entry and Exit Thresholds as Functions of Operating Cost C, for $\sigma = 0.2$*

ENTRY AND EXIT STRATEGIES: Figure 7.4

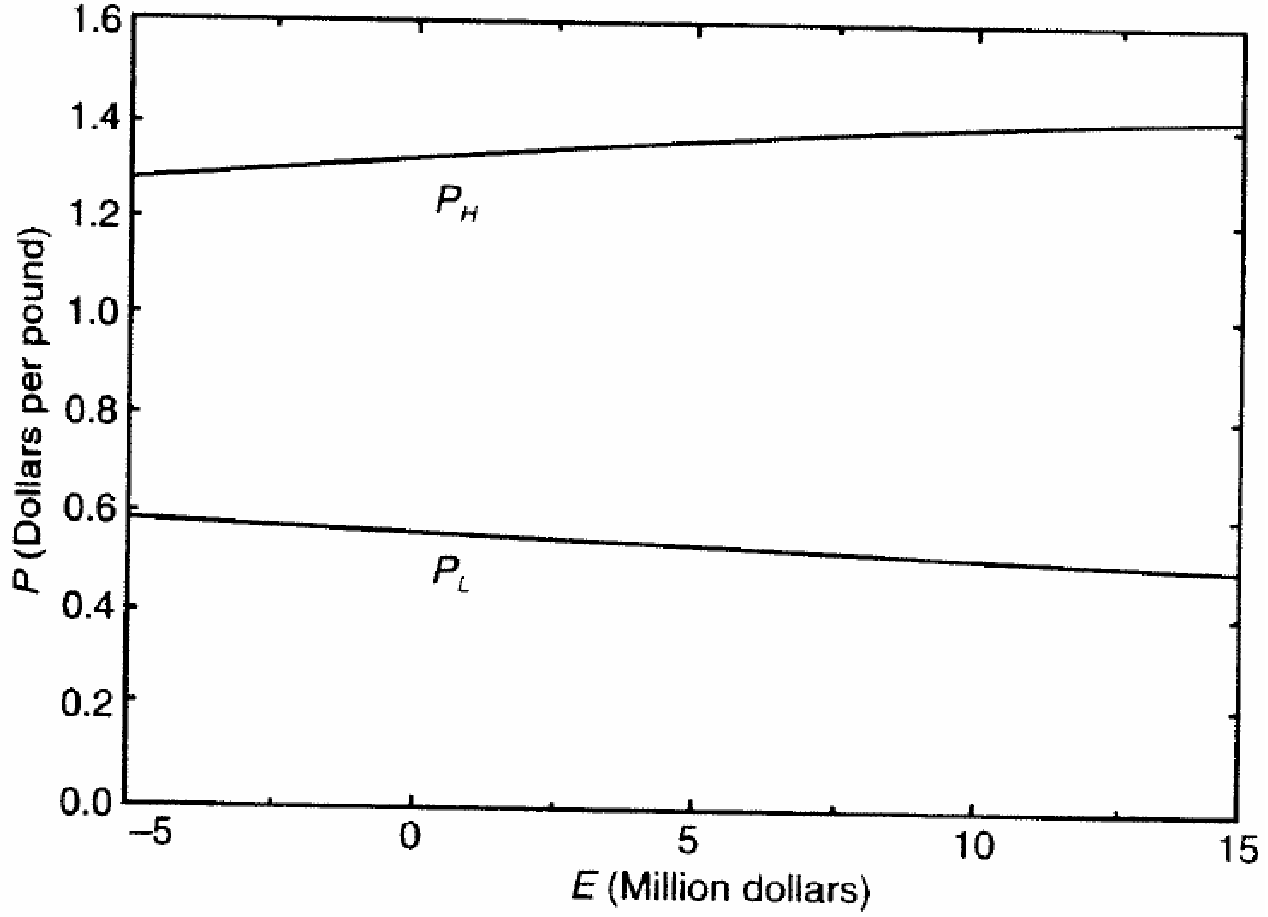

Figure 7.4 Entry and Exit Thresholds

ENTRY AND EXIT STRATEGIES: Figure 7.5

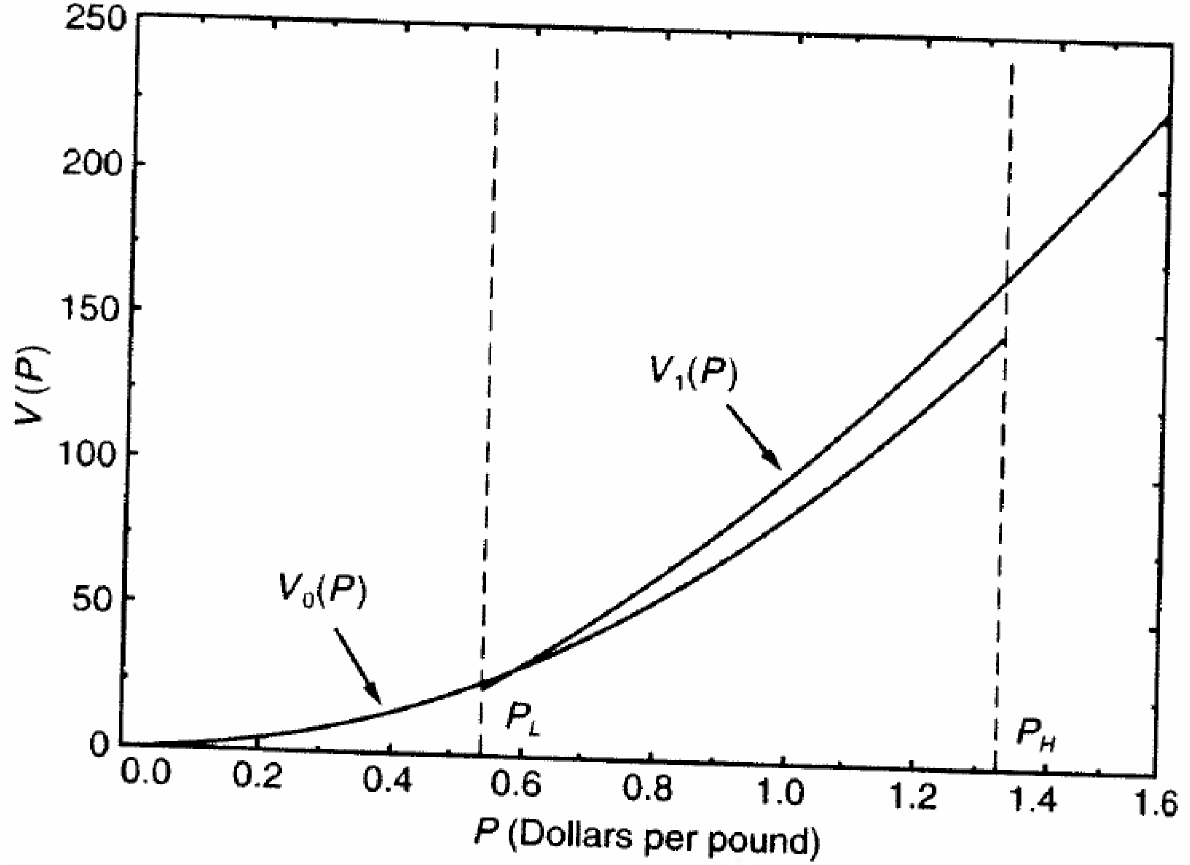

Figure 7.5. $V_0(P)$ *and* $V_1(P)$ *as Functions of* P

ENTRY AND EXIT STRATEGIES: Figure 7.6

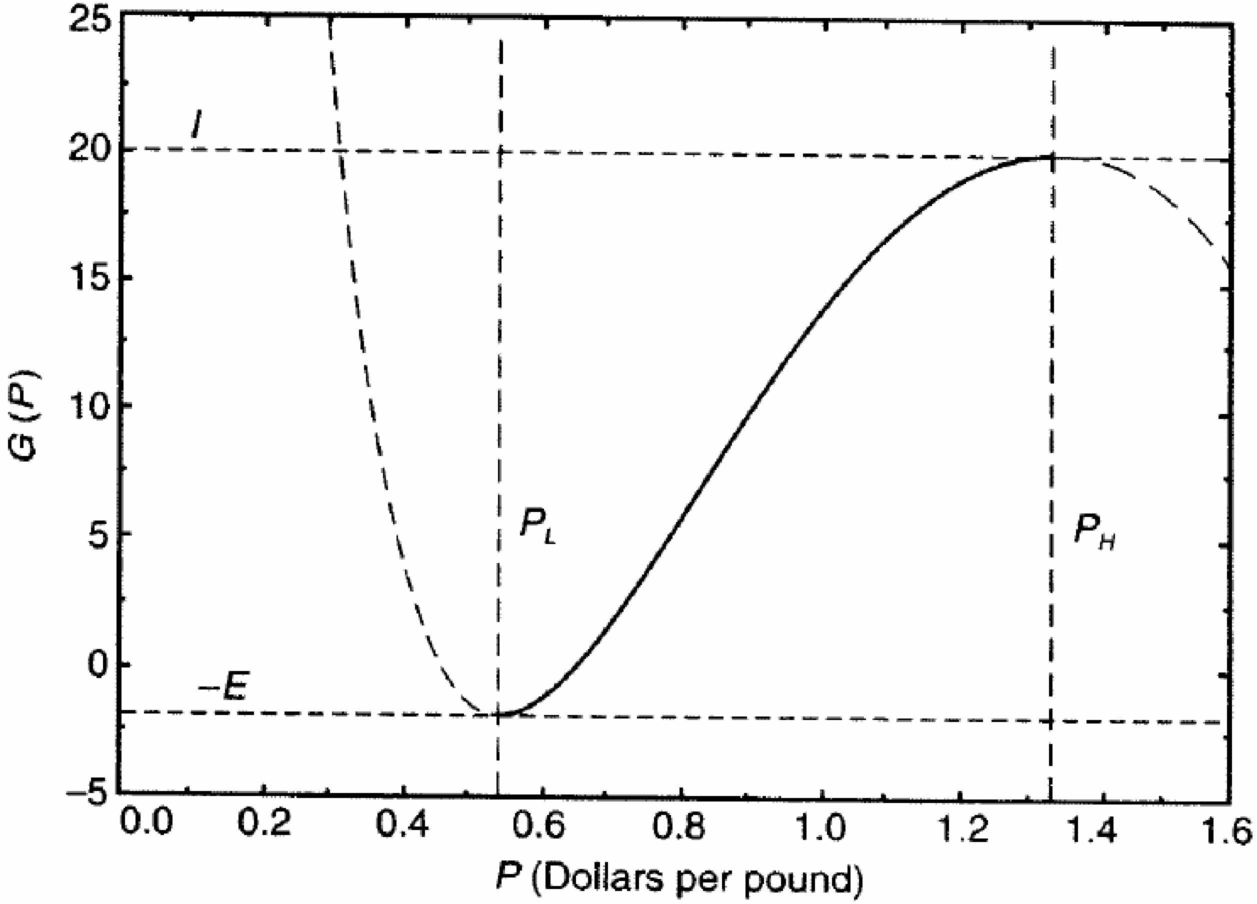

Figure 7.6. $G(P) = V_1(P) - V_0(P)$

LAY-UP, RE-ACTIVATION, AND SCRAPPING: States and Value Functions

- In addition to investment and abandonment, a producer may have other intermediate options, such as mothballing
  - Mothballing requires sunk cost, E_M, and ongoing maintenance cost, $M < C$
  - From a mothballed state, either re-activate operations by paying a sunk cost, $R < I$, or abandon as before at cost E_S such that $E = E_S + E_M$
  - Thus, there will be four price thresholds: P_H, P_M, P_R, and P_S

- There are three possible states
  - Idle: $V_0(P) = A_1 P^{\beta_1}$ for $(0, P_H)$
  - Active: $V_1(P) = B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r}$ for (P_M, ∞)
  - Mothballed: $V_m(P) = D_1 P^{\beta_1} + D_2 P^{\beta_2} - \frac{M}{r}$ for (P_S, P_R)
  - Since two transitions are possible from the mothballed state, there are a total of four possible transitions

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Optimal Switching

- Write four VM and four SP conditions to solve for eight unknowns
  - Idle to active: $V_0(P_H) = V_1(P_H) - I$ and $V_0'(P_H) = V_1'(P_H)$
  - Active to mothballed: $V_1(P_M) = V_m(P_M) - E_M$ and $V_1'(P_M) = V_m'(P_M)$
  - Mothballed to active: $V_m(P_R) = V_1(P_R) - R$ and $V_m'(P_R) = V_1'(P_R)$
  - Mothballed to idle: $V_m(P_S) = V_0(P_S) - E_S$ and $V_m'(P_S) = V_0'(P_S)$
- Examine the set of equations relating mothballing to an active state
  - $-D_1 P_R^{\beta_1} + (B_2 - D_2)P_R^{\beta_2} + \frac{P_R}{\delta} - \frac{(C-M)}{r} = R$
  - $-\beta_1 D_1 P_R^{\beta_1 - 1} + \beta_2(B_2 - D_2)P_R^{\beta_2 - 1} + \frac{1}{\delta} = 0$
  - $-D_1 P_M^{\beta_1} + (B_2 - D_2)P_M^{\beta_2} + \frac{P_M}{\delta} - \frac{(C-M)}{r} = -E_M$
  - $-\beta_1 D_1 P_M^{\beta_1 - 1} + \beta_2(B_2 - D_2)P_M^{\beta_2 - 1} + \frac{1}{\delta} = 0$
  - Solve for four unknowns P_M, P_R, $B_2 - D_2$, and D_1 as before

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Optimal Switching

- Now consider the set of equations relating idle/active and mothballing/idle states
  - $-A_1 P_H^{\beta_1} + B_2 P_H^{\beta_2} + \frac{P_H}{\delta} - \frac{C}{r} = I$
  - $-\beta_1 A_1 P_H^{\beta_1 - 1} + \beta_2 B_2 P_H^{\beta_2 - 1} + \frac{1}{\delta} = 0$
  - $(D_1 - A_1) P_S^{\beta_1} + D_2 P_S^{\beta_2} - \frac{M}{r} = -E_S$
  - $-\beta_1 (D_1 - A_1) P_S^{\beta_1 - 1} + \beta_2 D_2 P_S^{\beta_2 - 1} = 0$
  - Solve for six unknowns P_H, P_S, A_1, B_2, $D_1 - A_1$, and D_2 using solutions for D_1 and $B_2 - D_2$
- Comparative statics
  - If M and R are zero, then we have costless suspension/resumption
  - Increasing R while holding M constant: P_R increases, P_M decreases; also, both P_H and P_S increase
  - May reach a point with R high enough that mothballing is not used, i.e., proceed directly to abandon
  - Similar story for increasing M while holding R constant: since saving from mothballing is reduced, both P_R and P_M decrease, while P_H and P_S both increase

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Numerical Example

- Illustrate intuition with example from the VLCC industry
  - Assume $I = 40$, $E_M = 0.20$, $E_S = -3.4$ and $R = 0.79$, while the annual maintenance cost is $M = 0.515$ (all in million \$)
  - Annual operating cost is $C = 4.4$, and the revenue, P, follows a GBM with $P_0 = 7.3$, $\alpha = 0$, and $\sigma = 0.15$, and have $r = 0.05$ and $\delta = 0.05$
- Comparative statics
  - As R increases, P_H, P_S, and P_R increase, while P_M decreases (Figure 7.8)
  - Increasing E_M has a similar impact (Figure 7.9)
  - Higher M increases P_H, decreases P_M, decreases P_R, and increases P_S (Figure 7.10)
  - Increasing C increases P_H, increases P_M, increases P_R, and increases P_S (Figure 7.11)
  - Increasing σ increases P_H, decreases P_M, increases P_R, and decreases P_S (Figure 7.12)

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Figure 7.8

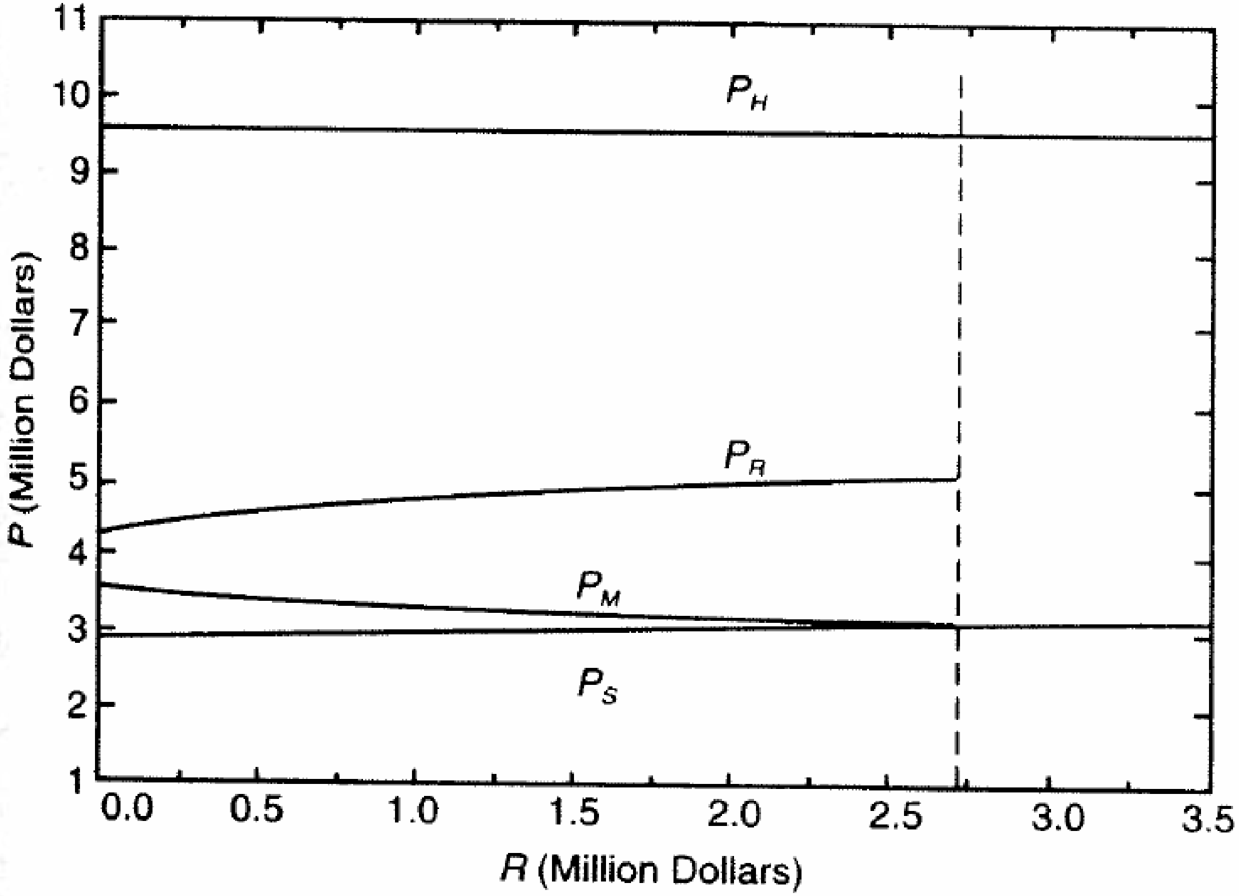

Figure 7.8. *Critical Thresholds as Functions of Reactivation Cost R*

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Figure 7.9

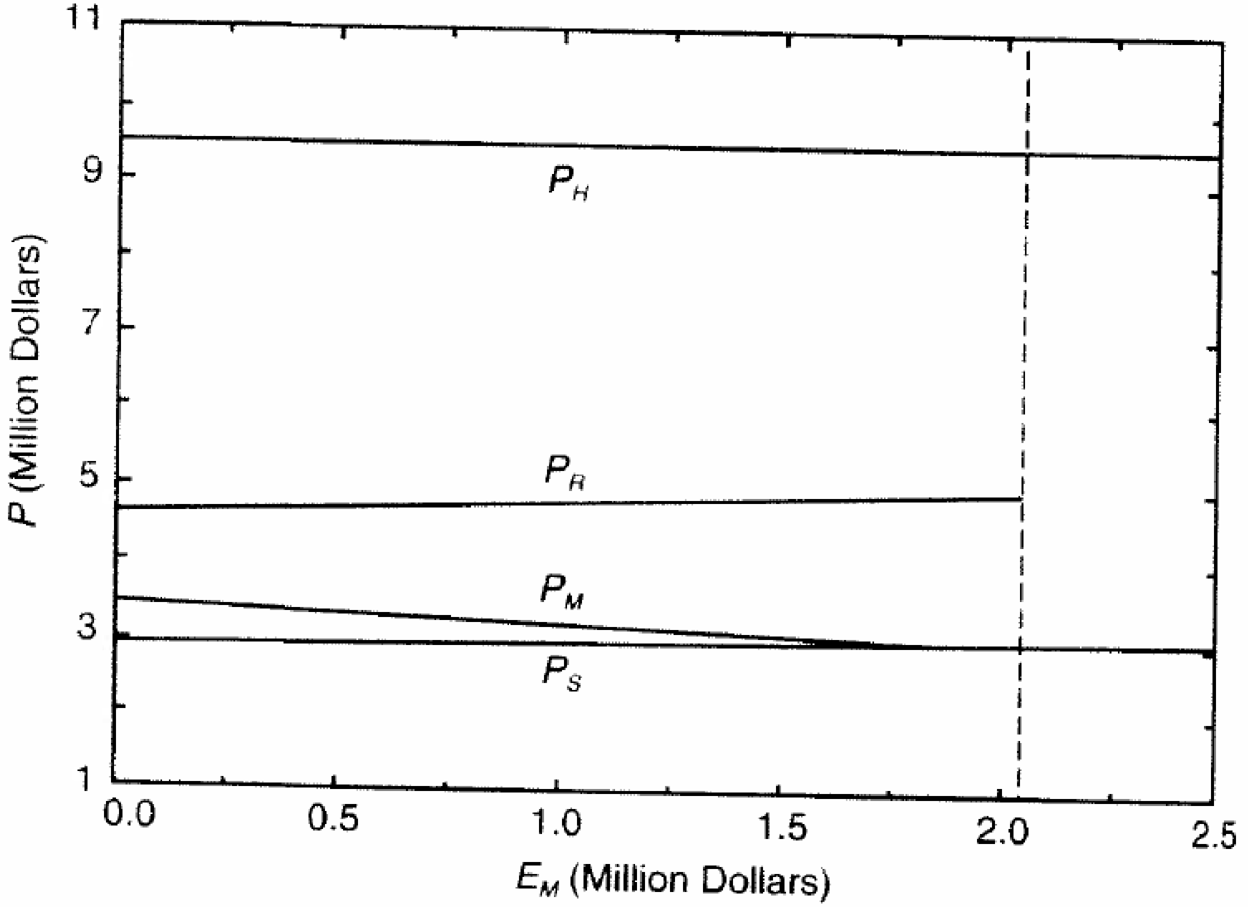

Figure 7.9. Critical Thresholds as Functions of One-Time Cost of Mothballing E_M

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Figure 7.10

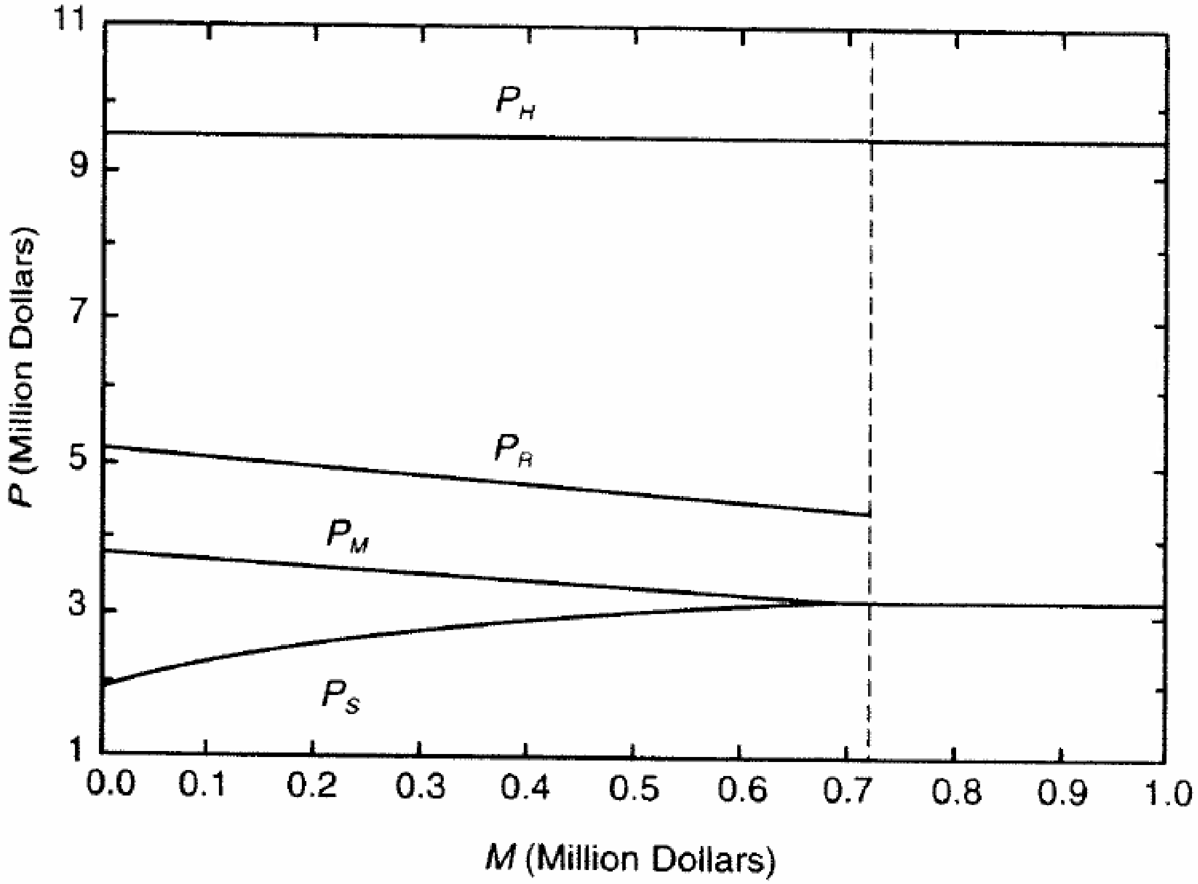

Figure 7.10. *Critical Thresholds as Functions of Annual Maintenance Cost M*

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Figure 7.11

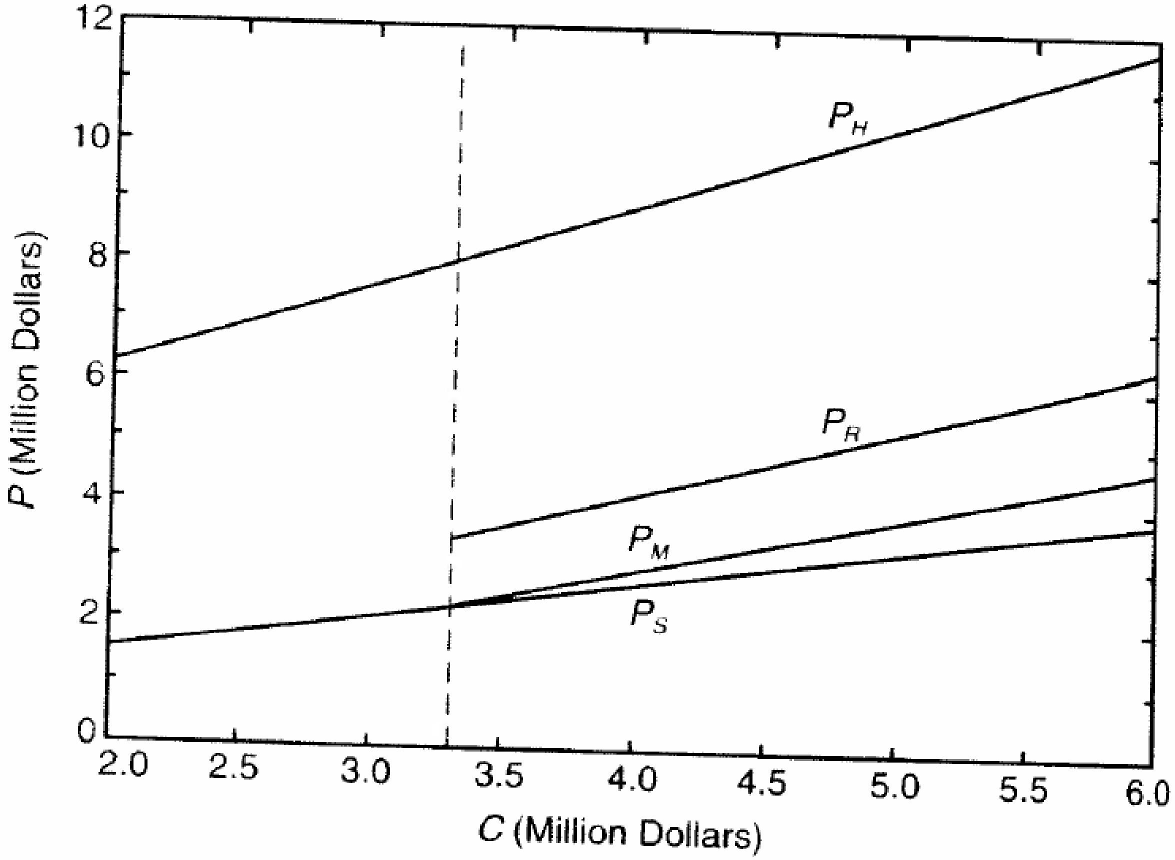

Figure 7.11. Critical Thresholds as Functions of Annual Cost of Operation C

LAY-UP, RE-ACTIVATION, AND SCRAPPING: Figure 7.12

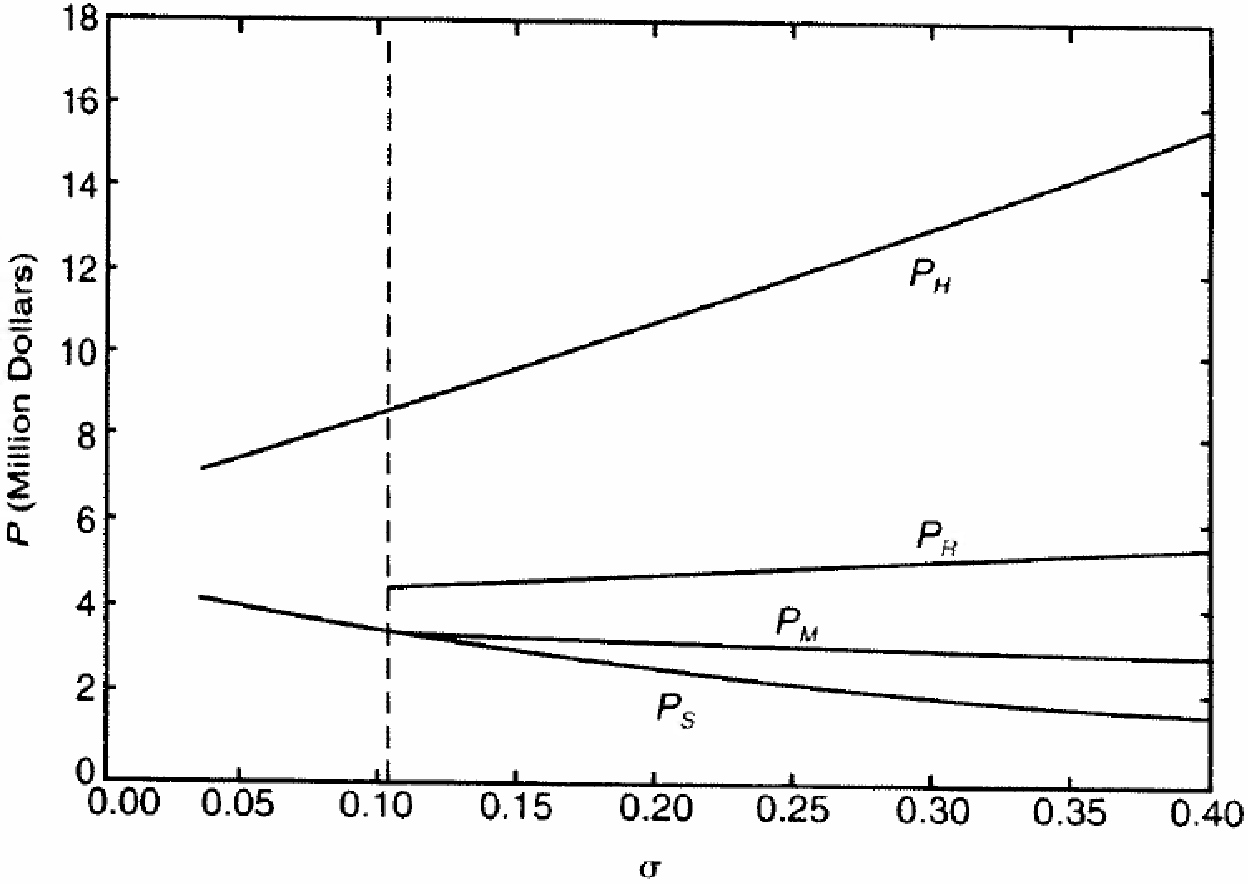

Figure 7.12. *Critical Thresholds as Functions of Volatility σ*

QUESTIONS