TIØ 1: Financial Engineering in Energy Markets

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COURSE OUTLINE

- ★ Introduction (Chs 1–2)
- ★ Mathematical Background (Chs 3–4)
- ★ Investment and Operational Timing (Chs 5–6)
- ★ Entry, Exit, Lay-Up, and Scrapping (Ch 7)
- ★ Recent Theoretical Work I: Capacity Sizing
- ★ Recent Theoretical Work II: Risk Aversion and Multiple Risk Factors
- ★ Applications to the Energy Sector I: Capacity Sizing, Timing, and Operational Flexibility
- ★ Applications to the Energy Sector II: Modularity and Technology Choice

LECTURE OUTLINE

★ Continuous scaling

★ Discrete scaling

CONTINUOUS CAPACITY SCALING: Dangl (1999)

- Often, size of the capacity is an endogeneous variable
- Dangl (1999) addresses this within the context of investment and sizing under uncertainty
  - Assume that the firm with a perpetual option to invest faces a downward-sloping inverse demand curve
  - Marginal investment costs are non-increasing in the capacity size
  - Tradeoff between smaller capacity (higher price, but higher per unit investment cost) and larger capacity (lower price, but lower per unit investment cost)
- Approach becomes analytically intractable unless one of the conditions is relaxed, i.e., either firm is a price taker or has constant marginal investment cost

CONTINUOUS CAPACITY SCALING: Price Taker without Operational Flexibility

- Suppose $\theta_t = P_t - c_t$, where $d\theta_t = \alpha t + \sigma dz_t$ and $\theta_0 = \theta$
- Investment cost is $I(Q) = \eta Q^\gamma$, $\gamma \geq 1$
- Instantaneous profit flow is $\pi(\theta_t, Q) = \theta_t Q$
- Assume exogeneous discount rate $\rho > \alpha \geq 0$
- Formally, the problem is $F(\theta) \equiv \max\left\{e^{-\rho dt}\mathcal{E}_\theta\left[F(\theta + d\theta)\right], \max_Q\left[V(\theta, Q) - I(Q)\right]\right\}$
- Expected project PV is $V(\theta, Q) = \mathcal{E}_\theta\left[\int_0^\infty e^{-\rho t}\pi(\theta_t, Q)dt\right]$
  - $V(\theta, Q) = \int_0^\infty Qe^{-\rho t}\mathcal{E}_\theta[\theta_t Q]dt$
  - $\Rightarrow V(\theta, Q) = \int_0^\infty Qe^{-\rho t}\left(\theta + \alpha t\right)dt = \frac{Q}{\rho^2}[\theta\rho + \alpha]$

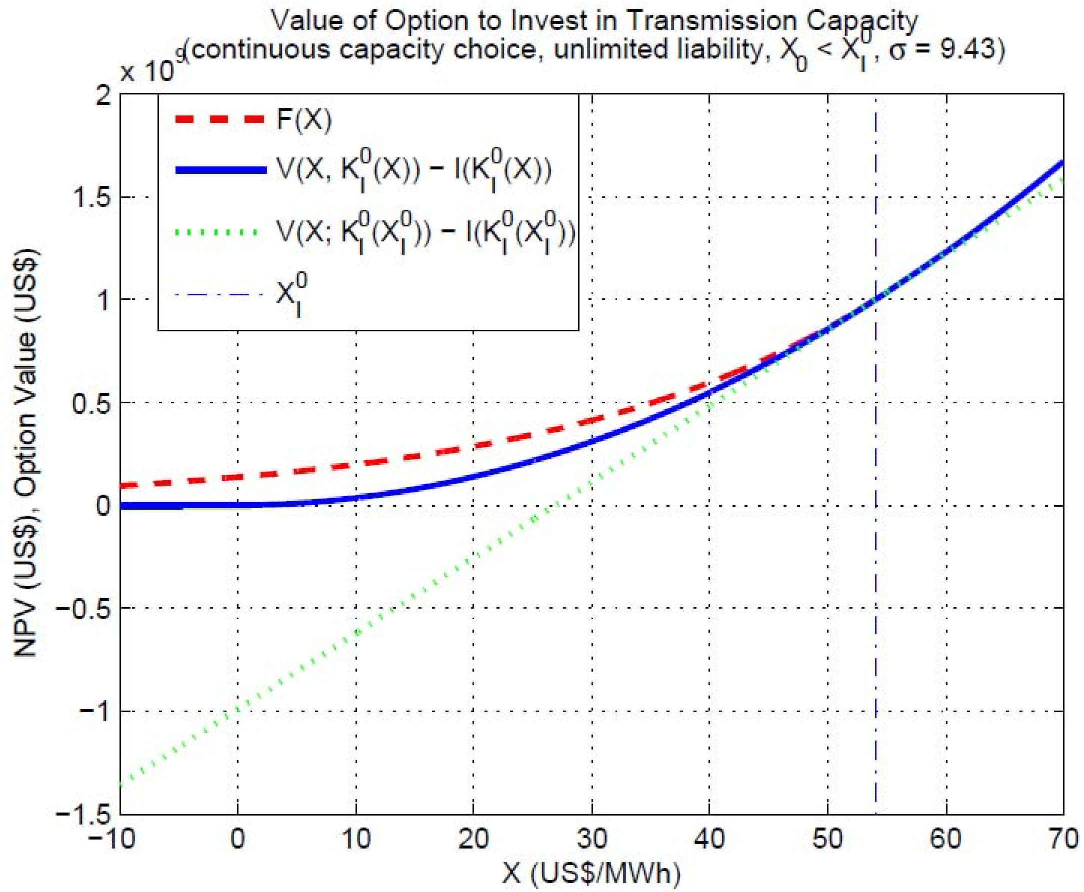
CONTINUOUS CAPACITY SCALING: Price Taker without Operational Flexibility

- Solution to the inner extremum yields $Q^*(\theta) = \left[\frac{\theta\rho+\alpha}{\gamma\eta\rho^2}\right]^{\frac{1}{\gamma-1}}$
  - The maximised expected project NPV is $V(\theta, Q^*(\theta)) - I(Q^*(\theta)) = \left[\frac{\theta\rho+\alpha}{\gamma\eta\rho^2}\right]^{\frac{1}{\gamma-1}} \left[\frac{\theta\rho+\alpha}{\rho^2}\right] \left(\frac{\gamma-1}{\gamma}\right)$
- Option value is $F(\theta) = a_1 e^{\beta_1\theta}$, where β_1 is the positive root of $\frac{1}{2}\beta^2\sigma^2 + \alpha\beta - \rho = 0$
  - VM: $a_1 e^{\beta_1\theta^*} = \left[\frac{\theta^*\rho+\alpha}{\gamma\eta\rho^2}\right]^{\frac{1}{\gamma-1}} \left[\frac{\theta^*\rho+\alpha}{\rho^2}\right] \left(\frac{\gamma-1}{\gamma}\right)$
  - SP: $\beta_1 a_1 e^{\beta_1\theta^*} = \left[\frac{\theta^*\rho+\alpha}{\gamma\eta\rho^2}\right]^{\frac{1}{\gamma-1}} \left(\frac{1}{\rho}\right)$
  - Thus, $\theta^* = \frac{\gamma}{\beta_1(\gamma-1)} - \frac{\alpha}{\rho}$, $a_1 = \left[\frac{\theta^*\rho+\alpha}{\eta\gamma\rho^2}\right]^{\frac{1}{\gamma-1}} \frac{e^{-\beta_1\theta^*}}{\rho\beta_1}$, and $Q^*(\theta^*) = \left[\frac{1}{\beta_1\eta\rho(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$
- The distinction between $V(\theta, Q^*(\theta)) - I(Q^*(\theta))$ and $V(\theta, Q^*(\theta^*)) - I(Q^*(\theta^*))$ is that the latter is a linear function of θ

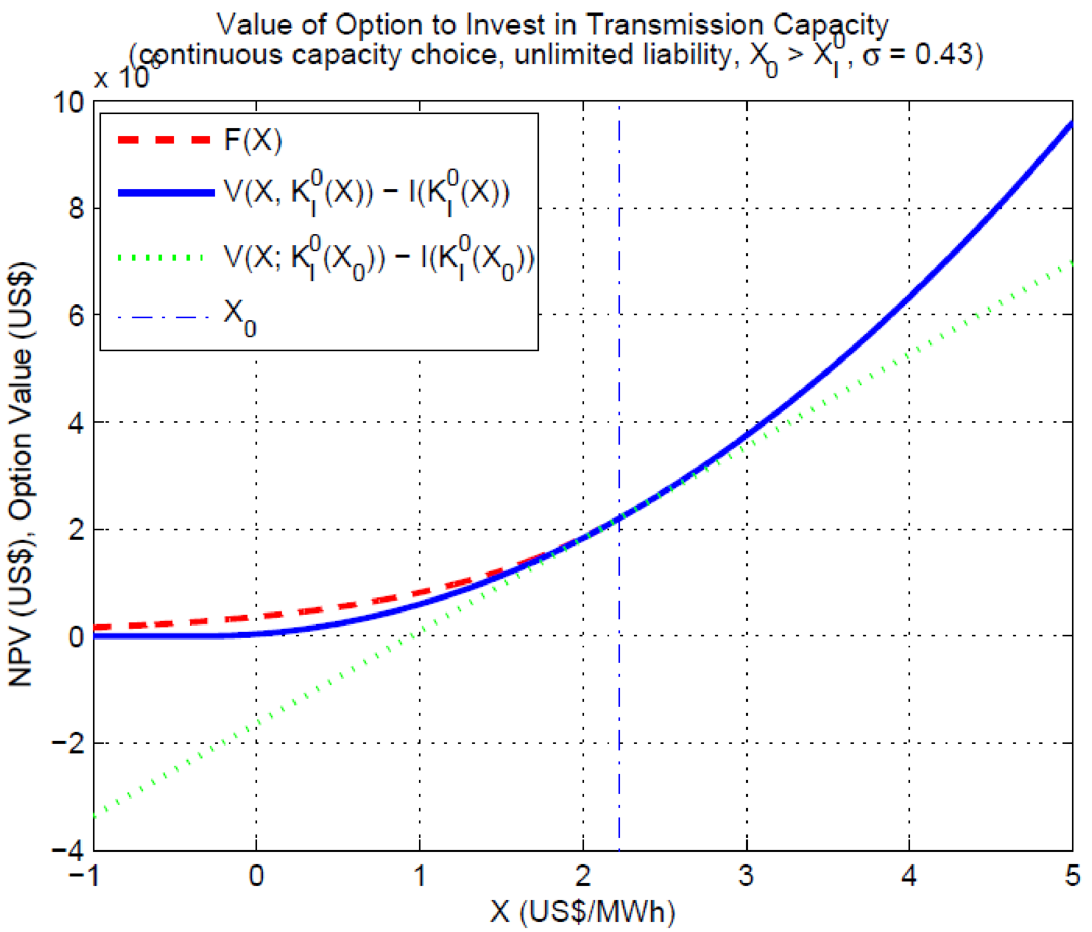
CONTINUOUS CAPACITY SCALING: Price Taker with Operational Flexibility

- Now, suppose that the firm may costlessly suspend and resume the project once installed
  - Expected project PV is $V(\theta, Q) = \frac{Q}{\rho^2}[\theta\rho + \alpha] + d_2 e^{\beta_2\theta}$ if $\theta \geq 0$ and $V(\theta, Q) = k_1 e^{\beta_1\theta}$ otherwise
  - Find k_1 and d_2 via VM and SP of the two branches of $V(\theta, Q)$ at $\theta = 0$
  - $k_1 = \frac{Q\beta_2}{(\beta_2-\beta_1)\rho}\left[\frac{\alpha}{\rho} - \frac{1}{\beta_2}\right]$ and $d_2 = \frac{Q\beta_1}{(\beta_2-\beta_1)\rho}\left[\frac{\alpha}{\rho} - \frac{1}{\beta_1}\right]$
- Inner extremum to determine capacity size, $Q^*(x)$: $\max_Q \frac{Q}{\rho^2}[\theta\rho + \alpha] + d_2(Q)e^{\beta_2\theta} - \eta Q^\gamma$
  - FONC implies $\frac{\rho\theta+\alpha}{\rho^2} + d_2'(Q)e^{\beta_2\theta} = \eta\gamma Q^{\gamma-1}$, where $d_2'(Q) = \frac{\beta_1}{\rho(\beta_2-\beta_1)}\left[\frac{\alpha}{\rho} - \frac{1}{\beta_1}\right]$
  - Thus, $Q^*(\theta) = \left\{\frac{1}{\eta\gamma}\left[\frac{\theta\rho+\alpha}{\rho^2} + \frac{\beta_1}{\rho(\beta_2-\beta_1)}\left(\frac{\alpha}{\rho} - \frac{1}{\beta_1}\right)e^{\beta_2\theta}\right]\right\}^{\frac{1}{\gamma-1}}$
- Option value is $F(\theta) = a_1 e^{\beta_1\theta}$, where θ^*, a_1, and $Q^*(\theta^*)$ must be found numerically

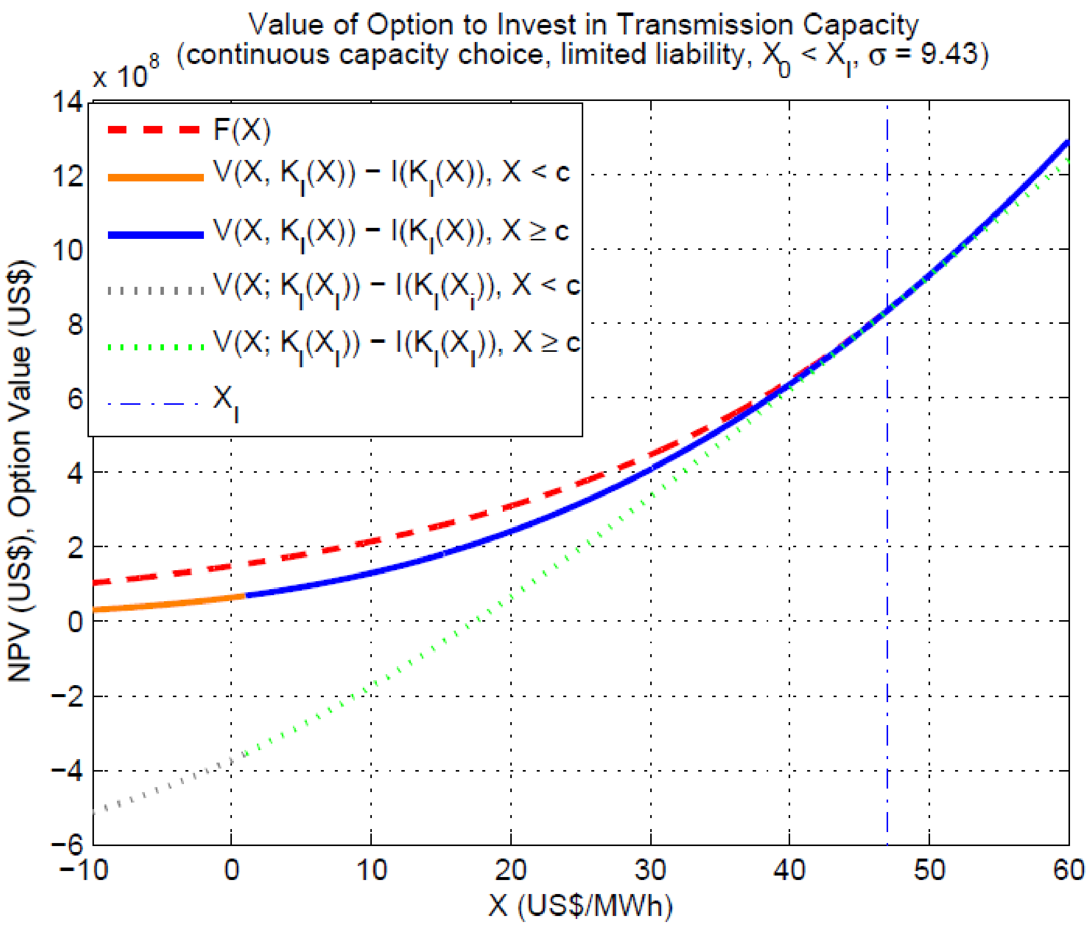
CONTINUOUS CAPACITY SCALING: Numerical Example without Operational Flexibility (Waiting Region)



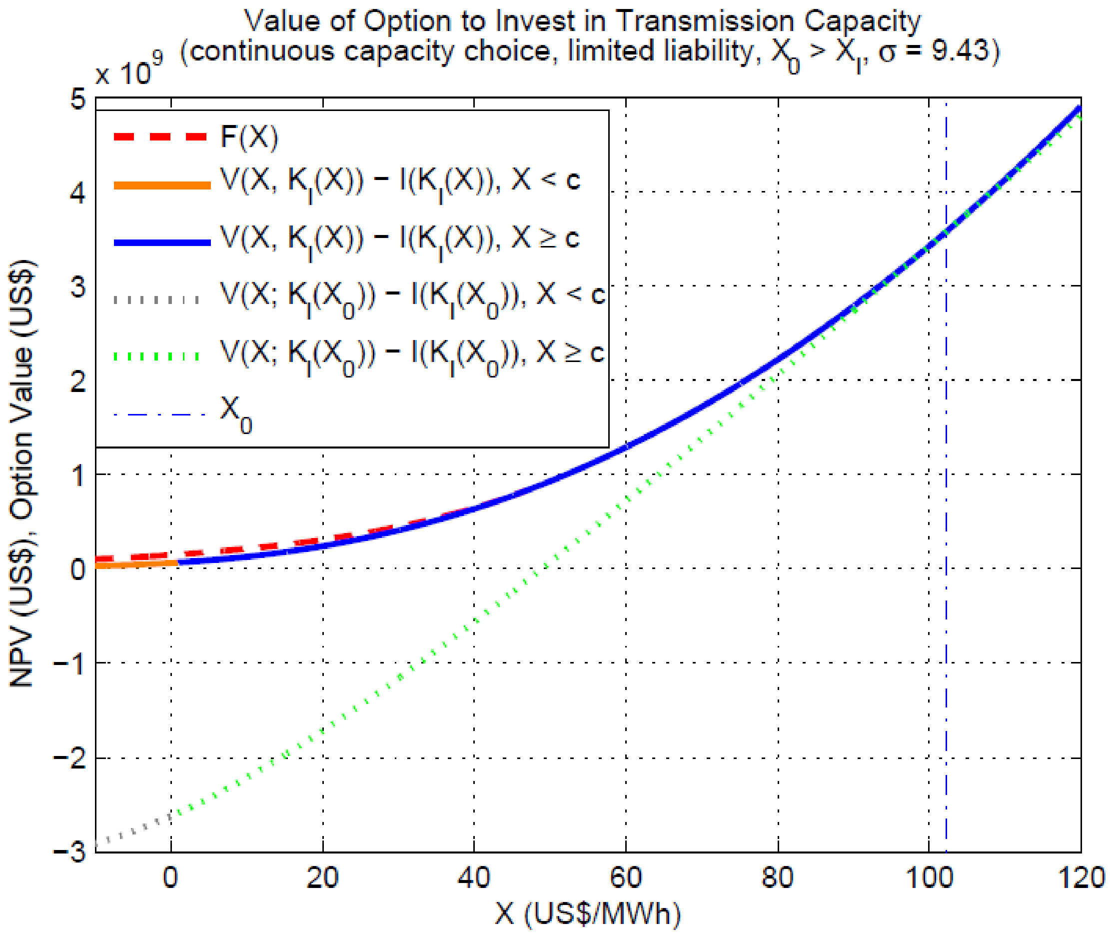
CONTINUOUS CAPACITY SCALING: Numerical Example without Operational Flexibility (Immediate Investment)



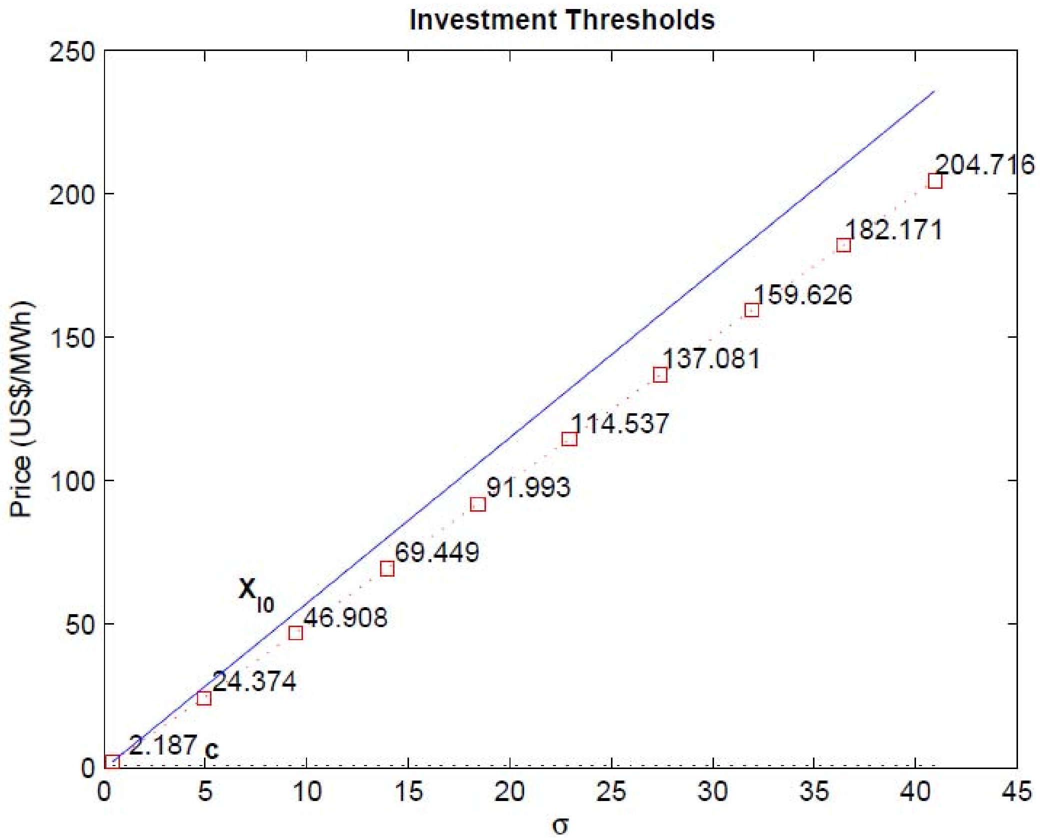
CONTINUOUS CAPACITY SCALING: Numerical Example with Operational Flexibility (Waiting Region)



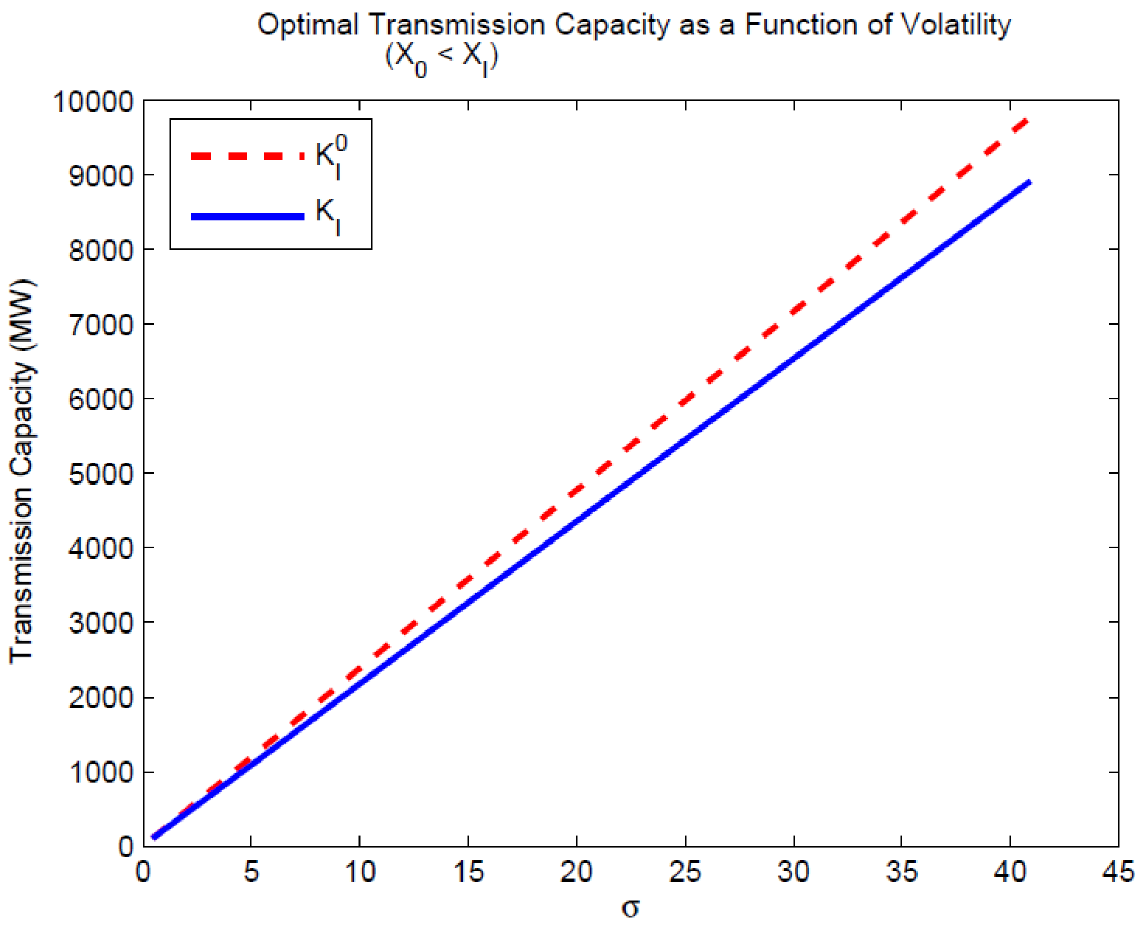
CONTINUOUS CAPACITY SCALING: Numerical Example with Operational Flexibility (Immediate Investment)



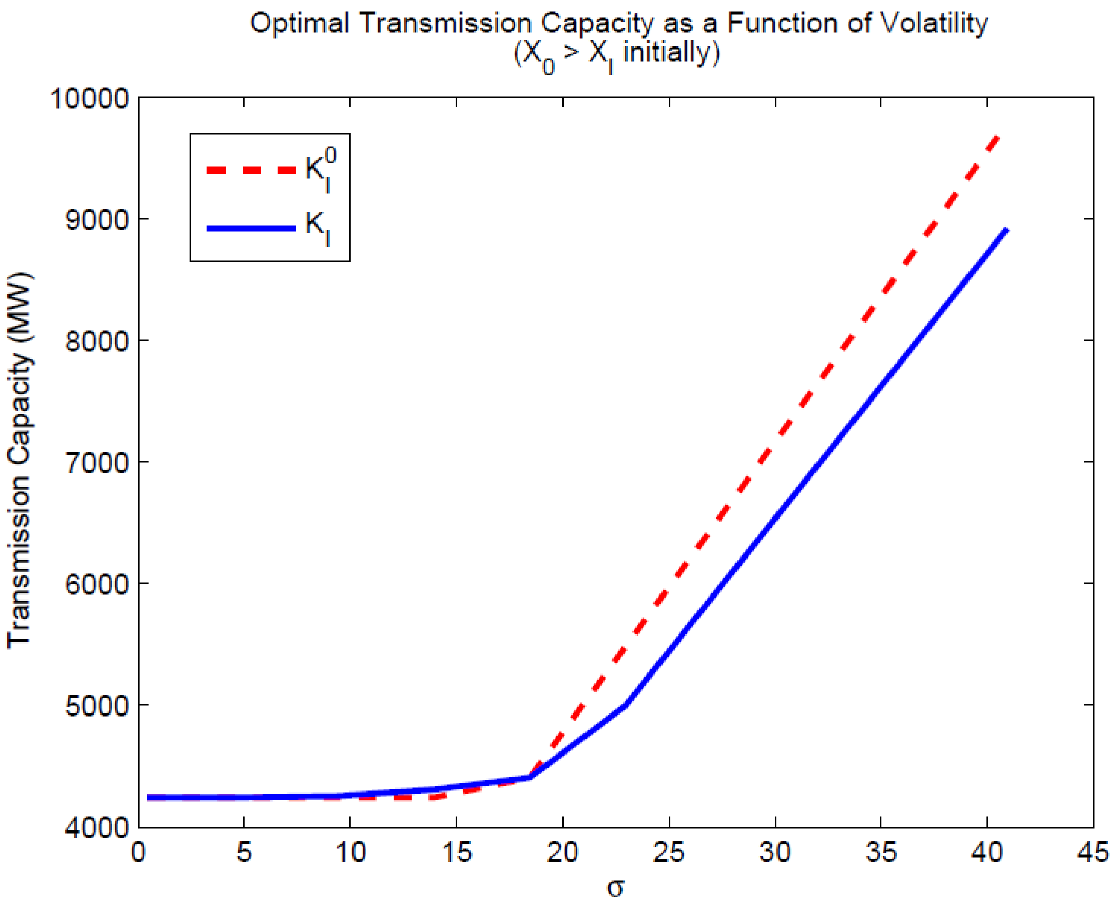
CONTINUOUS CAPACITY SCALING: Investment Thresholds



CONTINUOUS CAPACITY SCALING: Optimal Capacity (Waiting Region)



CONTINUOUS CAPACITY SCALING: Optimal Capacity (Immediate Investment)



DISCRETE CAPACITY SCALING: Dixit (1993)

★ Suppose that there are N discrete projects

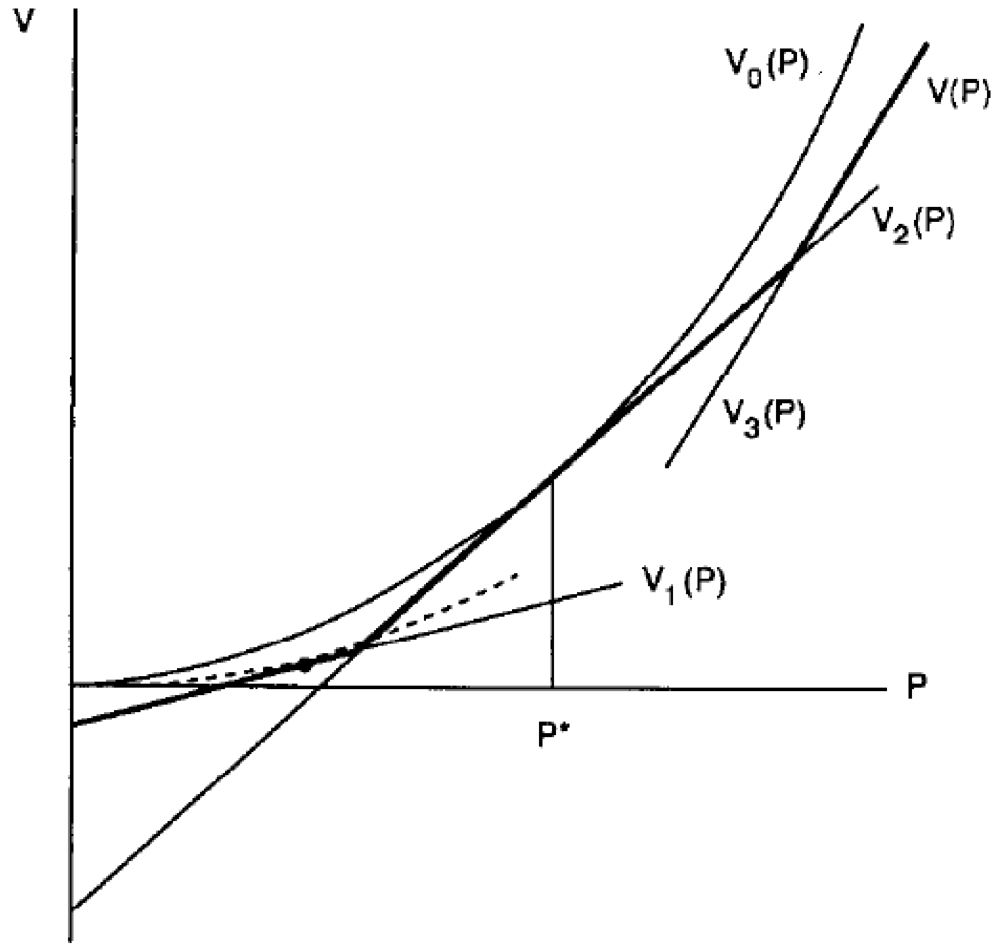
- Each has capital cost K_i and output scale X_i, $i = 1, \ldots, N$, ranked in increasing order
- Assume output price follows GBM, i.e., $dP_t = \mu P_t dt + \sigma P_t z_t$
- Discount rate is $\rho > \mu \geq 0$

★ Expected NPV of project i given initial price p is $V_i(p) = \frac{pX_i}{\rho - \mu} - K_i$

★ With N projects, the expected value of a now-or-never investment is $V(p) = \max_{i=1,\ldots,N} \{V_i(p)\}$

★ Option value of investment is $F(p) = Bp^{\beta_1}$, where β_1 is the positive root of $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0$

DISCRETE CAPACITY SCALING: Dixit (1993)



DISCRETE CAPACITY SCALING: Dixit (1993)

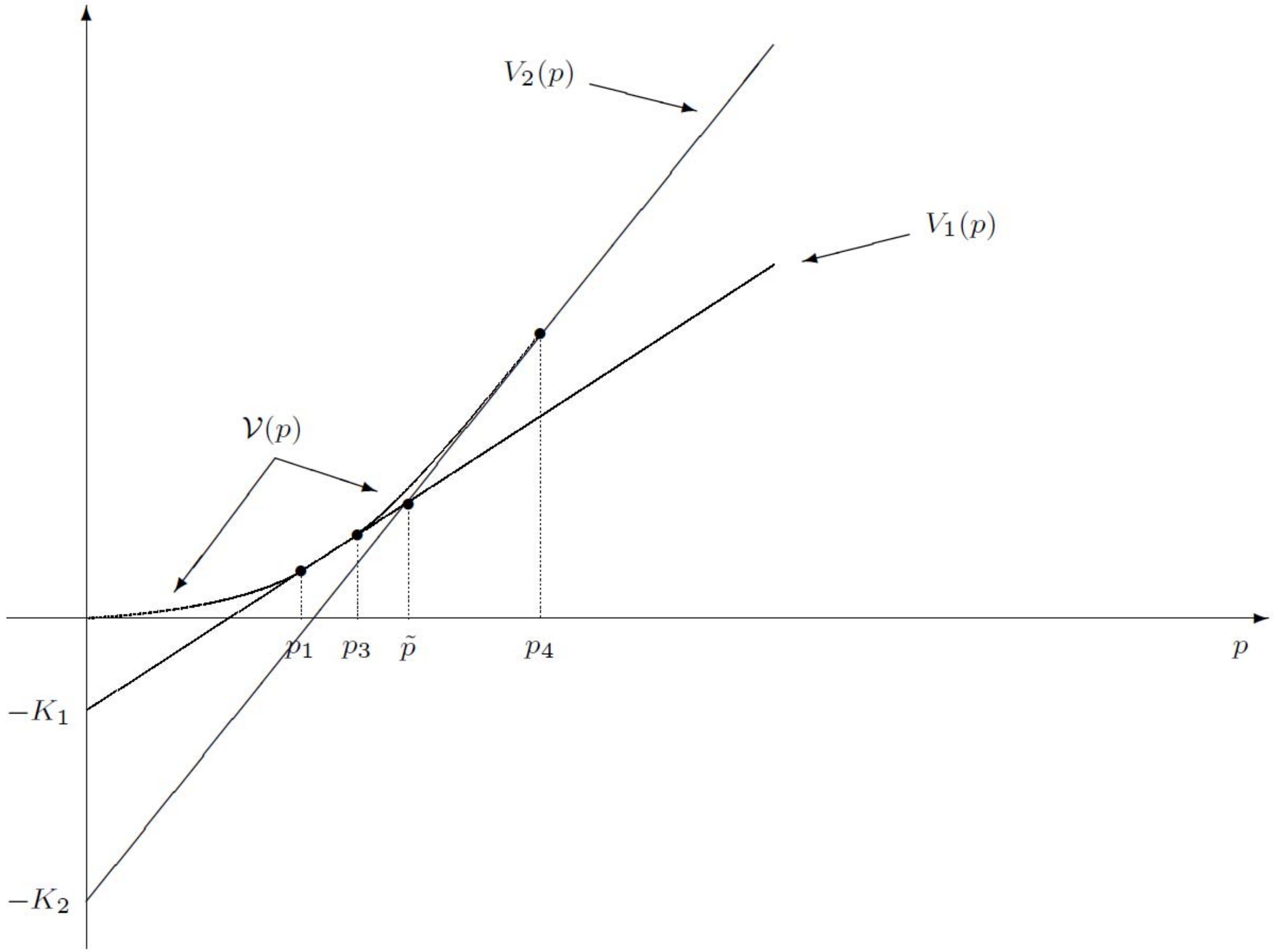
★ Procedure for finding p^* and B via VM and SP conditions with $V(p)$

1. Calculate p_i^* and B_i for each project independently: $p_i^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{K_i(\rho-\mu)}{X_i}$, $B_i = \left(\frac{\beta_1 - 1}{K_i}\right)^{\beta_1 - 1}\left(\frac{X_i}{\beta_1(\rho-\mu)}\right)^{\beta_1}$
2. Select project j with largest B_i, i.e., the largest $\frac{X_i^{\beta_1}}{K_i^{\beta_1 - 1}}$ ($j \equiv \arg\max_{i=1,\ldots,N}\{B_i\}$)
3. If $p < p_j^*$, the wait until price hits p_j^* and invest in project j; otherwise, invest immediately in the project with the highest $V_i(p)$

★ However, what if $j = 2$ and the current price is equal to the indifference price between projects 2 and 3, $\tilde{p}_{23}$?

★ Décamps, Mariotti, and Villeneuve (2006) argue that it would never be optimal to invest at this point

DISCRETE CAPACITY SCALING: DMV (2006)

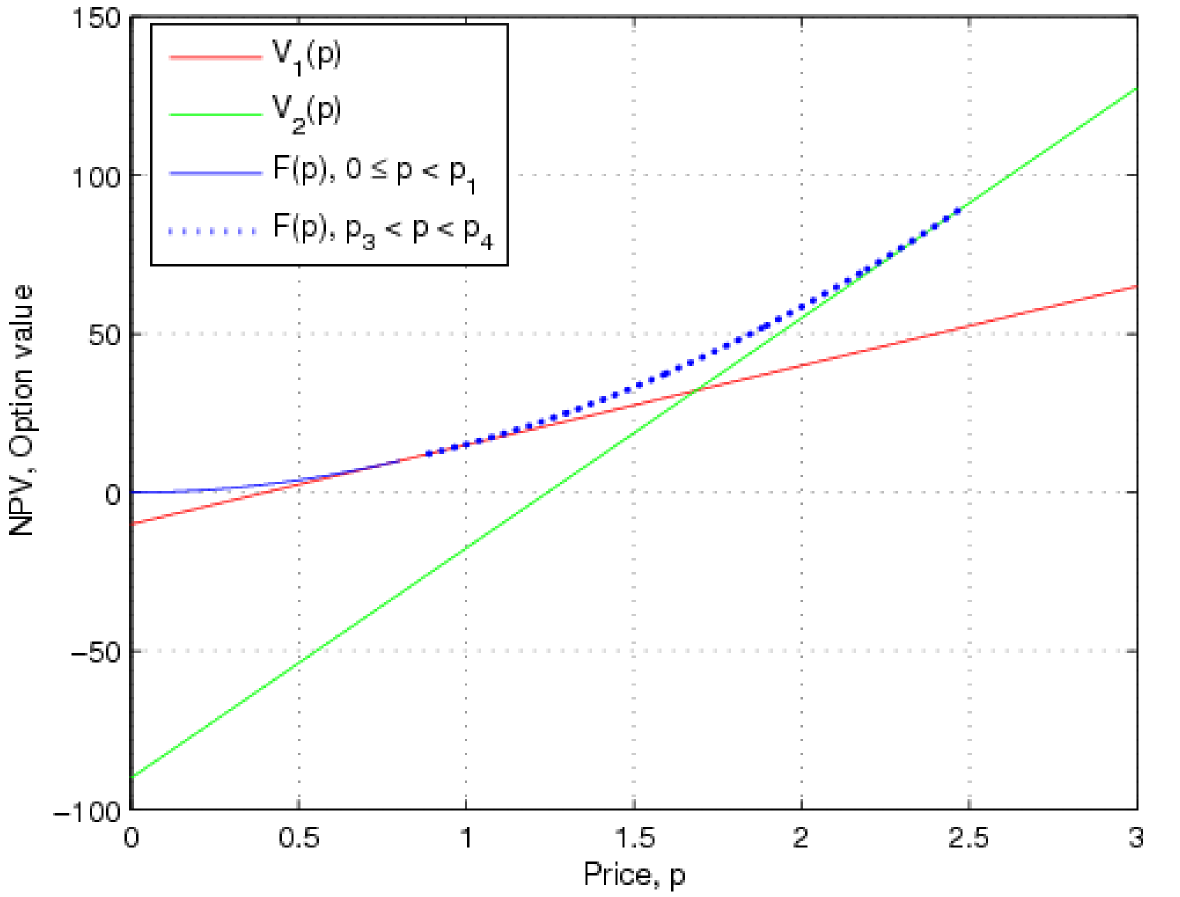


DISCRETE CAPACITY SCALING: DMV (2006)

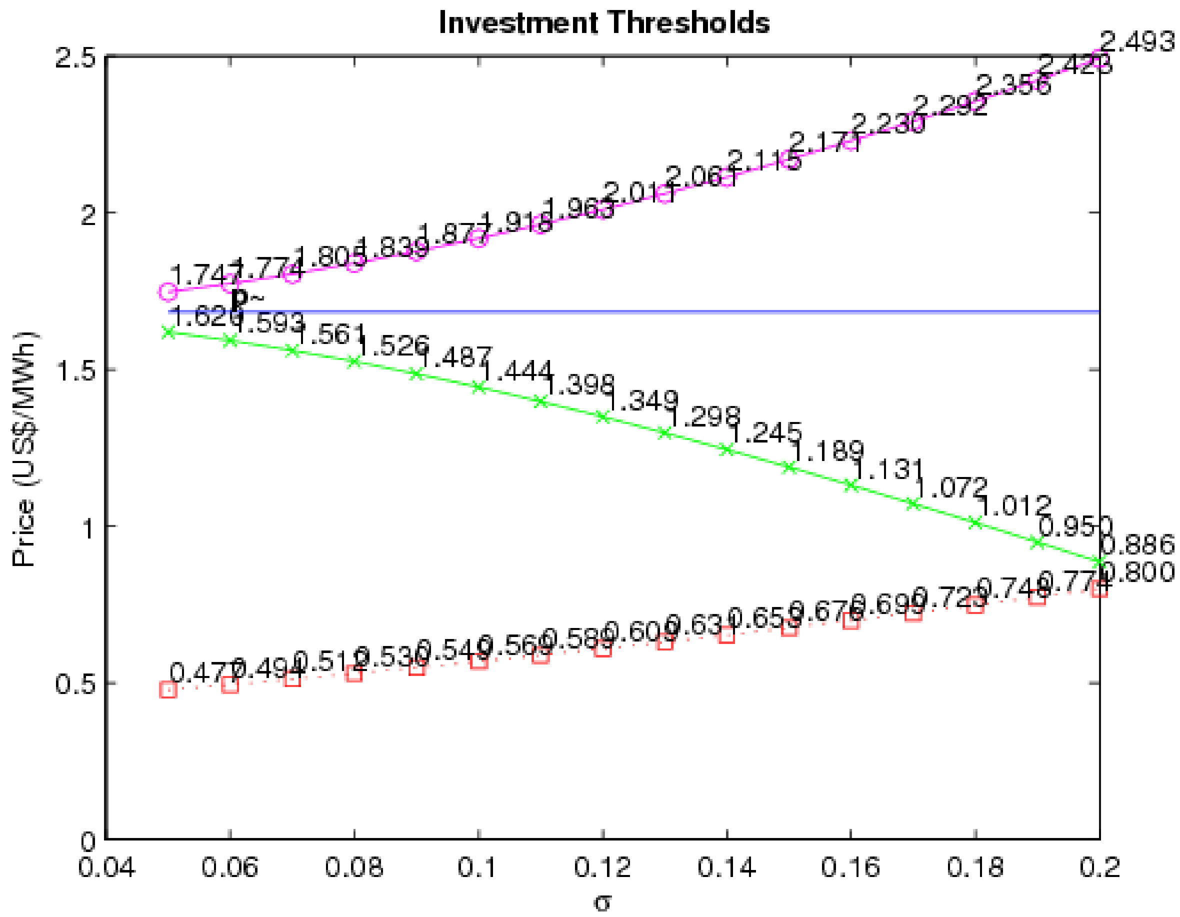
- Consider only two projects such that $B_1 > B_2$ and $\tilde{p} = \frac{(\rho-\mu)(K_2-K_2)}{X_2-X_1}$ is the indifference point
  - Invest in project 1 if $p \in [p_1, p_3]$ and invest in project 2 if $p \in [p_4, \infty)$
  - Otherwise, if $p \in [0, p_1)$, then wait for project 1; or, if $p \in (p_3, p_4)$, then wait for project 2 (1) if price increases (decreases)

- Option value is $F(p) = B_1 p^{\beta_1}$ for $0 \leq p < p_1$ and $F(p) = D_1 p^{\beta_1} + D_2 p^{\beta_2}$ for $p_3 < p < p_4$, where β_2 is the negative root of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$
  1. $F(p_3) = V_1(p_3) \Rightarrow D_1 p_3^{\beta_1} + D_2 p_3^{\beta_2} = \frac{p_3 X_1}{\rho-\mu} - K_1$
  2. $F'(p_3) = V_1'(p_3) \Rightarrow \beta_1 D_1 p_3^{\beta_1 - 1} + \beta_2 D_2 p_3^{\beta_2 - 1} = \frac{X_1}{\rho-\mu}$
  3. $F(p_4) = V_2(p_4) \Rightarrow D_1 p_4^{\beta_1} + D_2 p_4^{\beta_2} = \frac{p_4 X_2}{\rho-\mu} - K_2$
  4. $F'(p_4) = V_2'(p_4) \Rightarrow \beta_1 D_1 p_4^{\beta_1 - 1} + \beta_2 D_2 p_4^{\beta_2 - 1} = \frac{X_2}{\rho-\mu}$

DISCRETE CAPACITY SCALING: DMV (2006) Example



DISCRETE CAPACITY SCALING: DMV (2006) Example

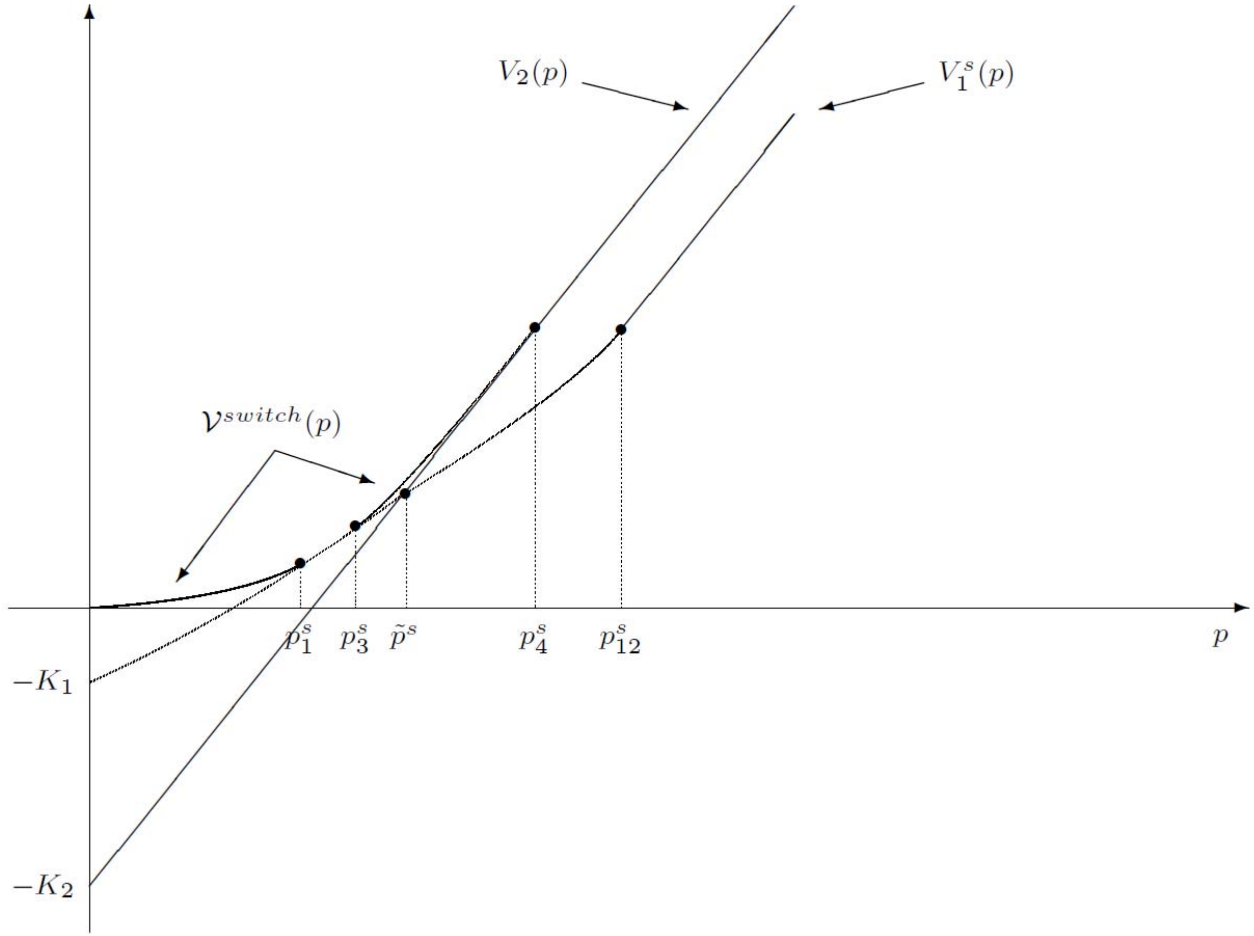


DISCRETE CAPACITY SCALING: DMV (2006) Example

Table 1. Numerical illustration for $\mu = 0$, $\rho = 0.04$, $\sigma = 0.2$, $X_1 = 1$ and $K_1 = 10$

	$\tilde{p}$	p_3	p_4	Δ
$X_2 = 1.9, K_2 = 40$	1.333	0.948	1.727	9.4%
$X_2 = 2.9, K_2 = 90$	1.684	0.886	2.493	29.8%
$X_2 = 3.9, K_2 = 160$	2.069	0.862	3.286	52.7%

DISCRETE CAPACITY SCALING: DMV (2006) with Switching



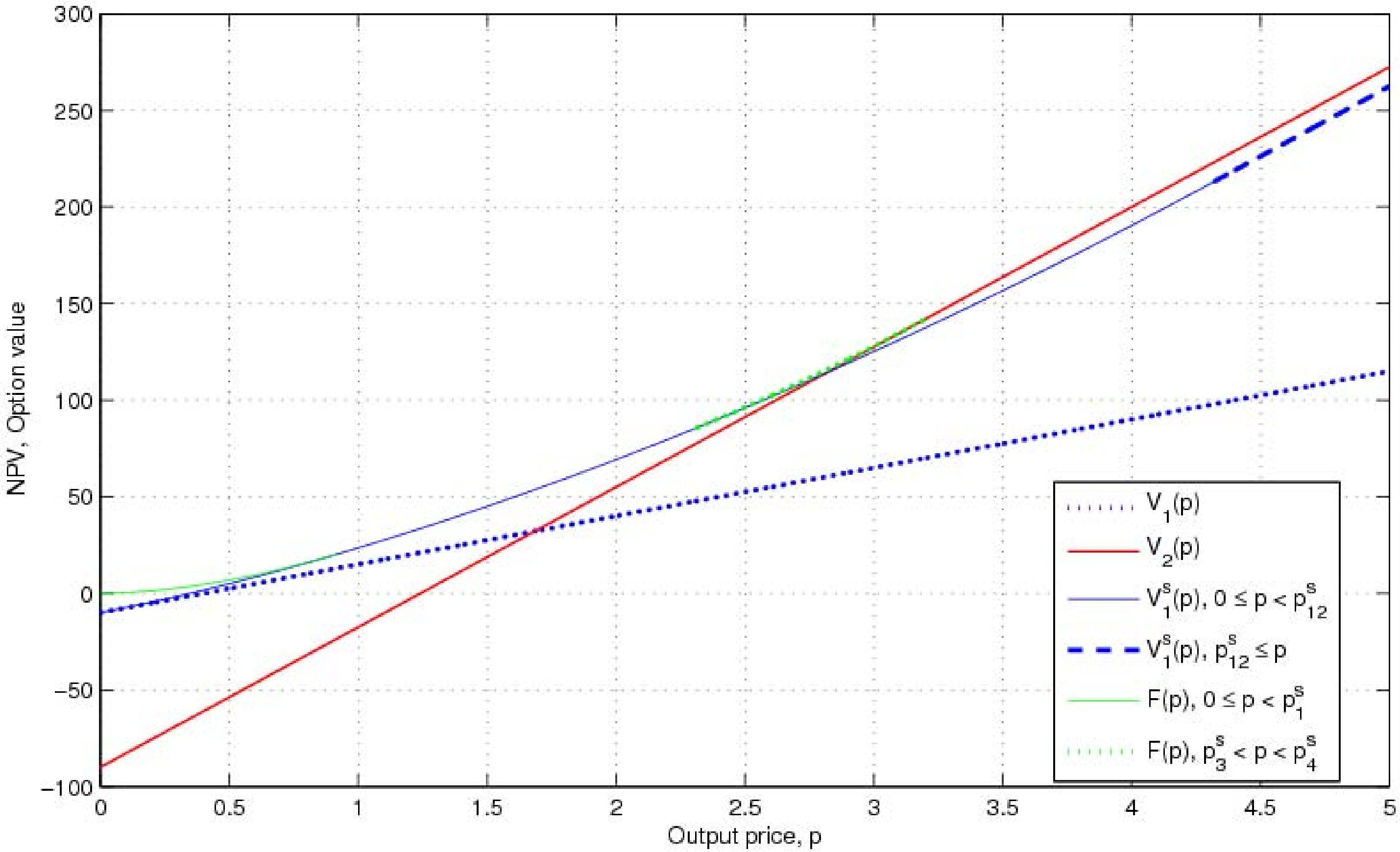
DISCRETE CAPACITY SCALING: DMV (2006) with Switching

- Now, suppose that project 1 comes with a switching option, i.e., it is possible to pay the investment cost K_2 and switch to project 2
  - Expected NPV of project 2 is still $V_2(p) = \frac{pX_2}{\rho-\mu} - K_2$
  - Expected NPV of project 1 is now $V_1^s(p) = \frac{pX_1}{\rho-\mu} - K_1 + B_{12}^s p^{\beta_1}$ for $p < p_{12}^s$ and $V_1^s(p) = \frac{pX_2}{\rho-\mu} - K_1 - K_2$ otherwise
  - VM and SP of the two branches of $V_1^s(p)$ yields $p_{12}^s = \left[\frac{K_2(\rho-\mu)}{X_2-X_1}\right]\left(\frac{\beta_1}{\beta_1-1}\right)$ and $B_{12}^s = \left(\frac{K_2}{\beta_1-1}\right)(p_{12}^s)^{-\beta_1}$
  - Indifference price between projects 1 and 2, $\tilde{p^s}$, must now be found numerically
- Option value of independent investment in project 2 is the same as before: $F_2(p) = B_2 p^{\beta_1}$
- But, option value of independent investment in project 1 with switching option is $F_1^s(p) = B_1^s p^{\beta_1}$ for $0 \leq p < p_1^s$, where A_1^s and p_1^s depend on whether $\frac{X_1}{K_1} > \frac{X_2-X_1}{K_2}$

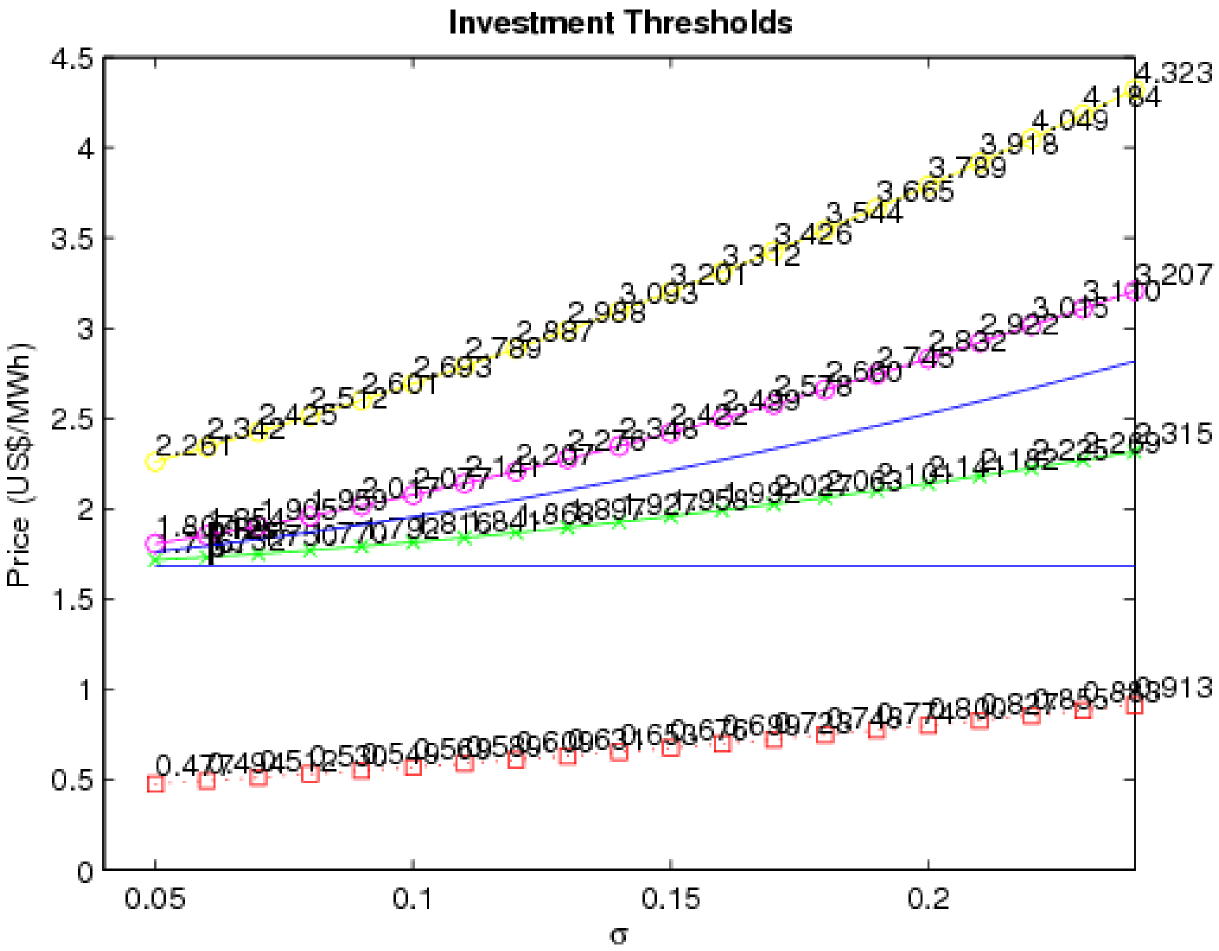
DISCRETE CAPACITY SCALING: DMV (2006) with Switching

- If $\frac{X_1}{K_1} > \frac{X_2 - X_1}{K_2}$, then invest first in project 1 with switching option and wait for opportunity to upgrade to project 2
  - Thus, $p_1^s = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{K_1(\rho-\mu)}{X_1} = p_1 < p_{12}^s$ and $B_1^s = B_1 + B_{12}^s$
- Otherwise, $p_1^s = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{(K_1+K_2)(\rho-\mu)}{X_2}$ and $B_1^s = \frac{X_2 \left(p_1^s\right)^{1-\beta_1}}{\beta_1(\rho-\mu)}$
- Finally, check whether $B_2 > B_1^s$
  - If so, then ignore project 1 with the switching option
  - Otherwise, find dichotomous option value via VM and SP
    1. $F(p_3^s) = V_1^s(p_3^s) \Rightarrow D_1^s \left(p_3^s\right)^{\beta_1} + D_2^s \left(p_3^s\right)^{\beta_2} = \frac{p_3^s X_1}{\rho-\mu} - K_1 + B_{12}^s \left(p_3^s\right)^{\beta_1}$
    2. $F'(p_3^s) = {V^s}'_1(p_3^s) \Rightarrow \beta_1 D_1^s \left(p_3^s\right)^{\beta_1 - 1} + \beta_2 D_2^s \left(p_3^s\right)^{\beta_2 - 1} = \frac{X_1}{\rho-\mu} + \beta_1 B_{12}^s \left(p_3^s\right)^{\beta_1 - 1}$
    3. $F(p_4^s) = V_2(p_4^s) \Rightarrow D_1^s \left(p_4^s\right)^{\beta_1} + D_2^s \left(p_4^s\right)^{\beta_2} = \frac{p_4^s X_2}{\rho-\mu} - K_2$
    4. $F'(p_4^s) = V_2'(p_4^s) \Rightarrow \beta_1 D_1^s \left(p_4^s\right)^{\beta_1 - 1} + \beta_2 D_2^s \left(p_4^s\right)^{\beta_2 - 1} = \frac{X_2}{\rho-\mu}$

DISCRETE CAPACITY SCALING: DMV (2006) with Switching Example



DISCRETE CAPACITY SCALING: DMV (2006) with Switching Example



QUESTIONS