TIØ 1: Financial Engineering in Energy Markets

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COURSE OUTLINE

- ★ Introduction (Chs 1–2)
- ★ Mathematical Background (Chs 3–4)
- ★ Investment and Operational Timing (Chs 5–6)
- ★ Entry, Exit, Lay-Up, and Scrapping (Ch 7)
- ★ Recent Theoretical Work I: Capacity Sizing
- ★ Recent Theoretical Work II: Risk Aversion and Multiple Risk Factors
- ★ Applications to the Energy Sector I: Capacity Sizing, Timing, and Operational Flexibility
- ★ Applications to the Energy Sector II: Modularity and Technology Choice

LECTURE OUTLINE

- ★ Optimal stopping time problem
- ★ Risk-averse decision makers
- ★ Analytical solutions with two sources of uncertainty

TRADITIONAL NPV APPROACH

- Example from McDonald (2002): oil extraction under certainty at a rate of one barrel per year forever
  - Current price of oil is $P_0 = 15$, discount rate is $\rho = 0.05$, growth rate of oil is $\alpha = 0.01$, operating cost is $c = 8$, and investment cost is $I = 180$
- Is it optimal to extract the oil now?
  - Assuming that the price of oil grows exponentially, the NPV from immediate extraction is $V(P_0) = \int_0^\infty e^{-\rho t} \left\{ P_0 e^{\alpha t} - c \right\} dt - I = \frac{P_0}{\rho - \alpha} - \frac{c}{\rho} - I = 215 - 180 = 35$
  - Since $V(P_0) > 0$, it is optimal to extract
- But, would it not be better to wait longer?
- Investment cost is being discounted, and the value of the oil is growing

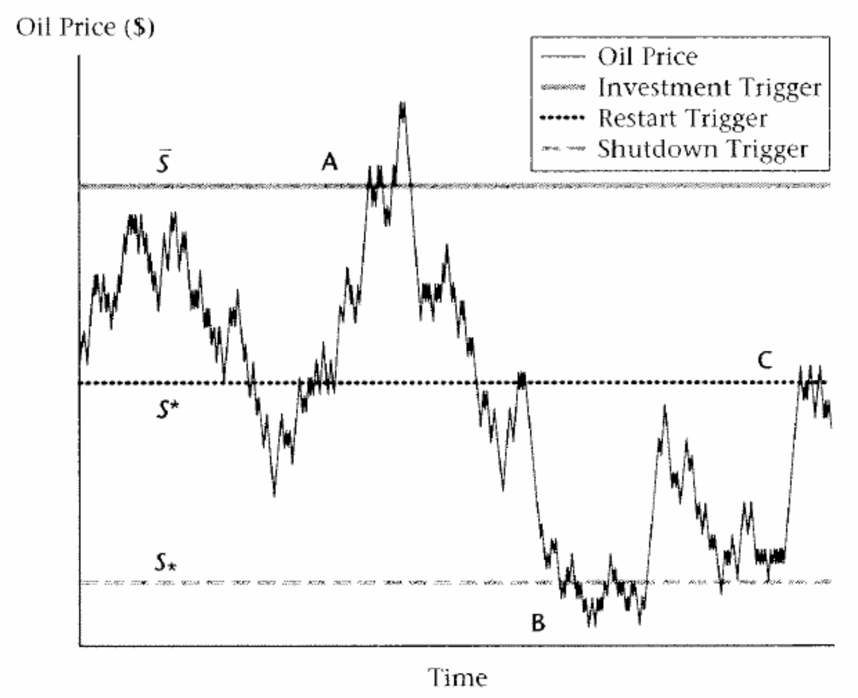
OPTIMAL INVESTMENT TIMING

- Think instead about value of perpetual investment opportunity
  - $F(P_0) = \max_T \int_T^\infty e^{-\rho t} \{P_0 e^{\alpha t} - c - \rho I\} dt = \max_T \frac{P_0}{\rho - \alpha} e^{(\alpha-\rho)T} - \frac{c}{\rho} e^{-\rho T} - I e^{-\rho T}$
  - $\Rightarrow T^* = \frac{1}{\alpha} \ln \left(\frac{c + \rho I}{P_0} \right) = 12.5163$
  - Or, invest when $P_{T^*} = 17$
  - Indeed, the initial value of the investment opportunity is $F(P_0) = 45.46 > 35 = V(P_0)$
- By delaying investment to the optimal time period, it is possible to maximise NPV
- How does this work when the price is stochastic?

OPTIMAL INVESTMENT UNDER UNCERTAINTY

★ Price process evolves according to a GBM, i.e., $dP_t = \alpha P_t dt + \sigma P_t dz_t$ with initial price $P_0 = p$

- Note that $(dP_t)^2 = \sigma^2 (P_t)^2 dt$



OPTIMAL INVESTMENT UNDER UNCERTAINTY

- If the project were started now, then its expected NPV is $V(p) = \mathcal{E}_p\left[\int_0^\infty e^{-\rho t}\left\{P_t - (c + \rho I)\right\} dt\right] = \frac{p}{\rho - \alpha} - \frac{c}{\rho} - I$

- Canonical real options problem:

$$F(p) = \sup_{\tau \in \mathcal{S}} \mathcal{E}_p\left[\int_\tau^\infty e^{-\rho t}\left\{P_t - (c + \rho I)\right\} dt\right]$$

$$\Rightarrow F(p) = \sup_{\tau \in \mathcal{S}} \mathcal{E}_p\left[e^{-\rho \tau} V(P_\tau)\right] = \max_{P_I \geq p}\left\{\left(\frac{p}{P_I}\right)^{\beta_1} V(P_I)\right\}$$

  - β_1 (β_2) is the positive (negative) root of $\frac{1}{2}\sigma^2\zeta(\zeta - 1) + \alpha\zeta - \rho = 0$

STOCHASTIC DISCOUNT FACTOR

★ Proposition: The conditional expectation of the stochastic discount factor, $\mathcal{E}_p\left[e^{-\rho\tau}\right]$, is the power function, $\left(\frac{p}{P_I}\right)^{\beta_1}$, where $\tau \equiv \min\{t : P_t \geq P_I\}$

★ Proof: Let $g(p) \equiv \mathcal{E}_p\left[e^{-\rho\tau}\right]$

- $g(p) = o(dt)e^{-\rho dt} + (1 - o(dt))e^{-\rho dt}\mathcal{E}_p\left[g(p + dP)\right]$
- $\Rightarrow g(p) = o(dt)e^{-\rho dt} + (1 - o(dt))e^{-\rho dt}\mathcal{E}_p\left[g(p) + dPg^{'}(p) + \frac{1}{2}(dP)^2 g^{''}(p) + o(dt)\right]$
- $\Rightarrow g(p) = o(dt) + e^{-\rho dt}g(p) + e^{-\rho dt}\alpha p g^{'}(p)dt + e^{-\rho dt}\frac{1}{2}\sigma^2 p^2 g^{''}(p)dt$
- $\Rightarrow g(p) = o(dt) + (1 - \rho dt)g(p) + (1 - \rho dt)\alpha p g^{'}(p)dt + (1 - \rho dt)\frac{1}{2}\sigma^2 p^2 g^{''}(p)dt$
- $\Rightarrow -\rho g(p) + \alpha p g^{'}(p) + \frac{1}{2}\sigma^2 p^2 g^{''}(p) = \frac{o(dt)}{dt}$
- $\Rightarrow g(p) = a_1 p^{\beta_1} + a_2 p^{\beta_2}$
- $\lim_{p \to 0} g(p) = 0 \Rightarrow a_2 = 0$ and $g(P_I) = 1 \Rightarrow a_1 = \frac{1}{P_I^{\beta_1}}$

OPTIMAL INVESTMENT THRESHOLD UNDER UNCERTAINTY

★ Solve for optimal investment threshold, P_I:

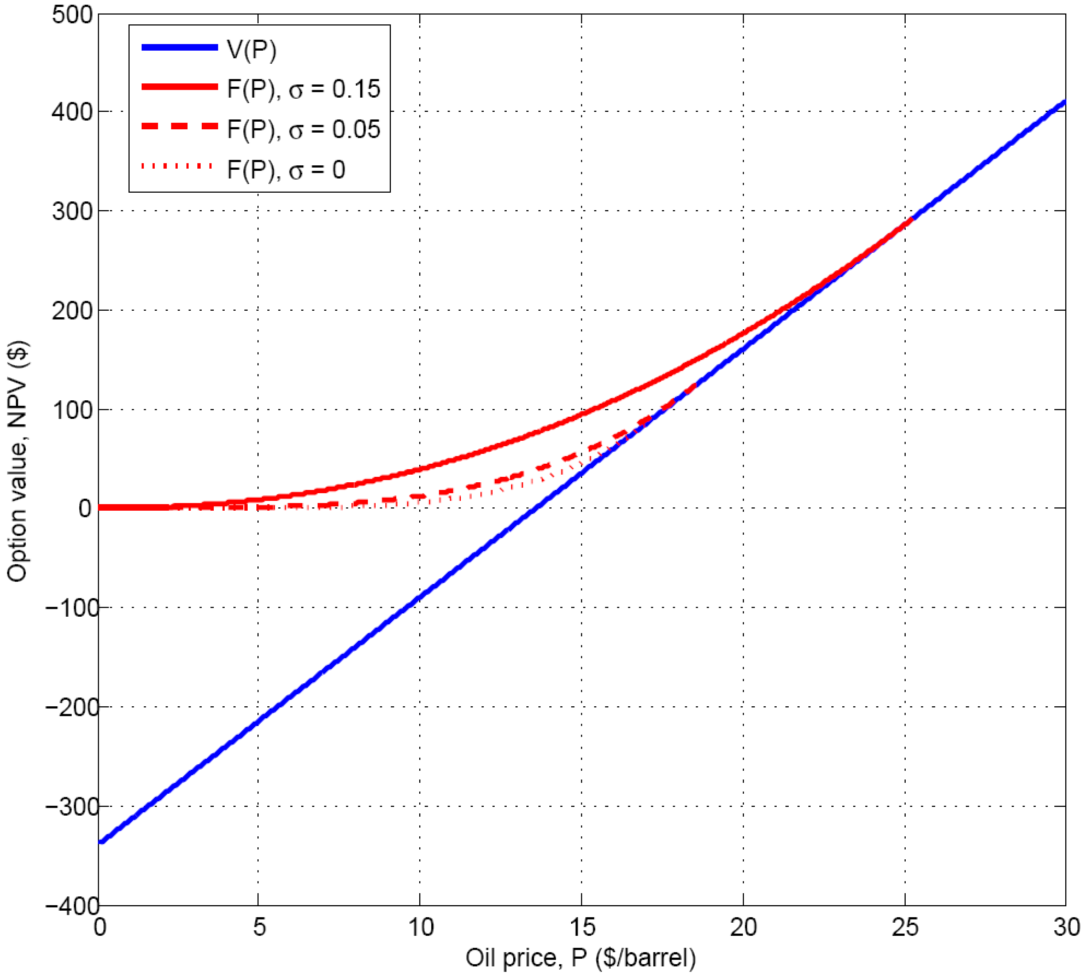
$$F(p) = \max_{P_I \geq p} \left\{ \left(\frac{p}{P_I} \right)^{\beta_1} V(P_I) \right\}$$

- First-order necessary condition yields $P_I = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\left(\frac{c}{\rho} + I\right)$
- Note that in the case without uncertainty, $\beta_1 = \frac{\rho}{\alpha} \Rightarrow P_I = c + \rho I$

★ For a level of volatility of $\sigma = 0.15$, $P_I = 25.28$, and the value of the investment opportunity is $F(p) = 94.35$

★ Compared to the case with certainty, the investment opportunity is worth more but is also less likely to be exercised

INVESTMENT THRESHOLDS AND VALUES



INVESTMENT UNDER UNCERTAINTY WITH ABANDONMENT

- If the project is abandoned after investment, then the expected incremental payoff is:

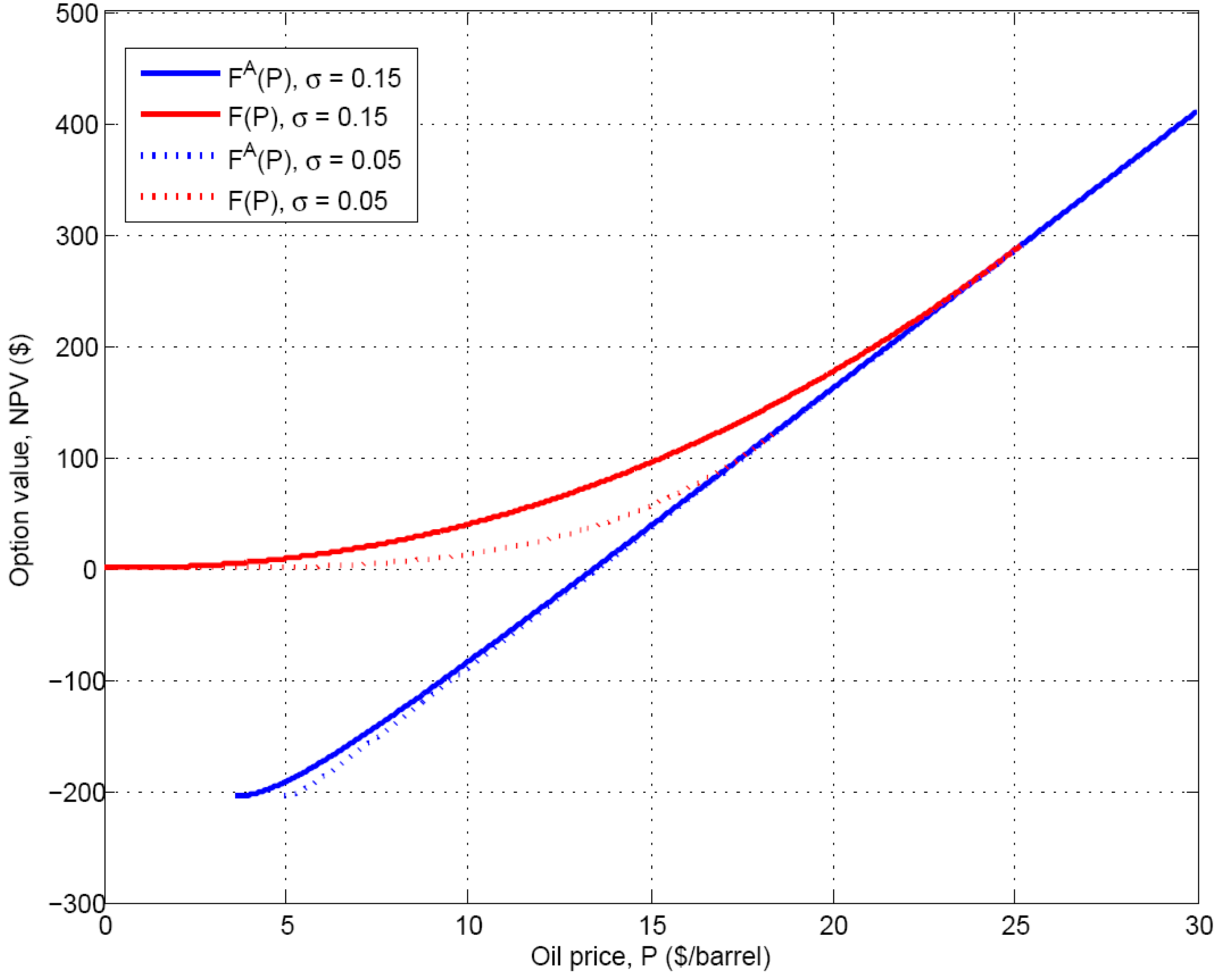
$$V^A(p) = \mathcal{E}_p\left[\int_0^\infty e^{-\rho t}\left\{(c-\rho K_s) - P_t\right\}dt\right] = \frac{c}{\rho} - K_s - \frac{p}{\rho-\alpha}$$

- Solve for optimal abandonment threshold, P_*:

$$F^A(p) = \max_{P_* \leq p}\left\{\left(\frac{p}{P_*}\right)^{\beta_2} V^A(P_*)\right\} + V(p)$$

  - First-order necessary condition yields $P_* = \frac{\beta_2}{\beta_2-1}(\rho-\alpha)\left(\frac{c}{\rho} - K_s\right)$
  - Solve numerically for P_I: $F(p) = \max_{P_I \geq p}\left\{\left(\frac{p}{P_I}\right)^{\beta_1}\left\{V(P_I) + \left(\frac{P_I}{P_*}\right)^{\beta_2} V^A(P_*)\right\}\right\}$

INVESTMENT THRESHOLDS AND VALUES WITH ABANDONMENT



INVESTMENT UNDER UNCERTAINTY WITH SUSPENSION AND RESUMPTION

- If the project is resumed from a suspended state, then the expected incremental payoff is:

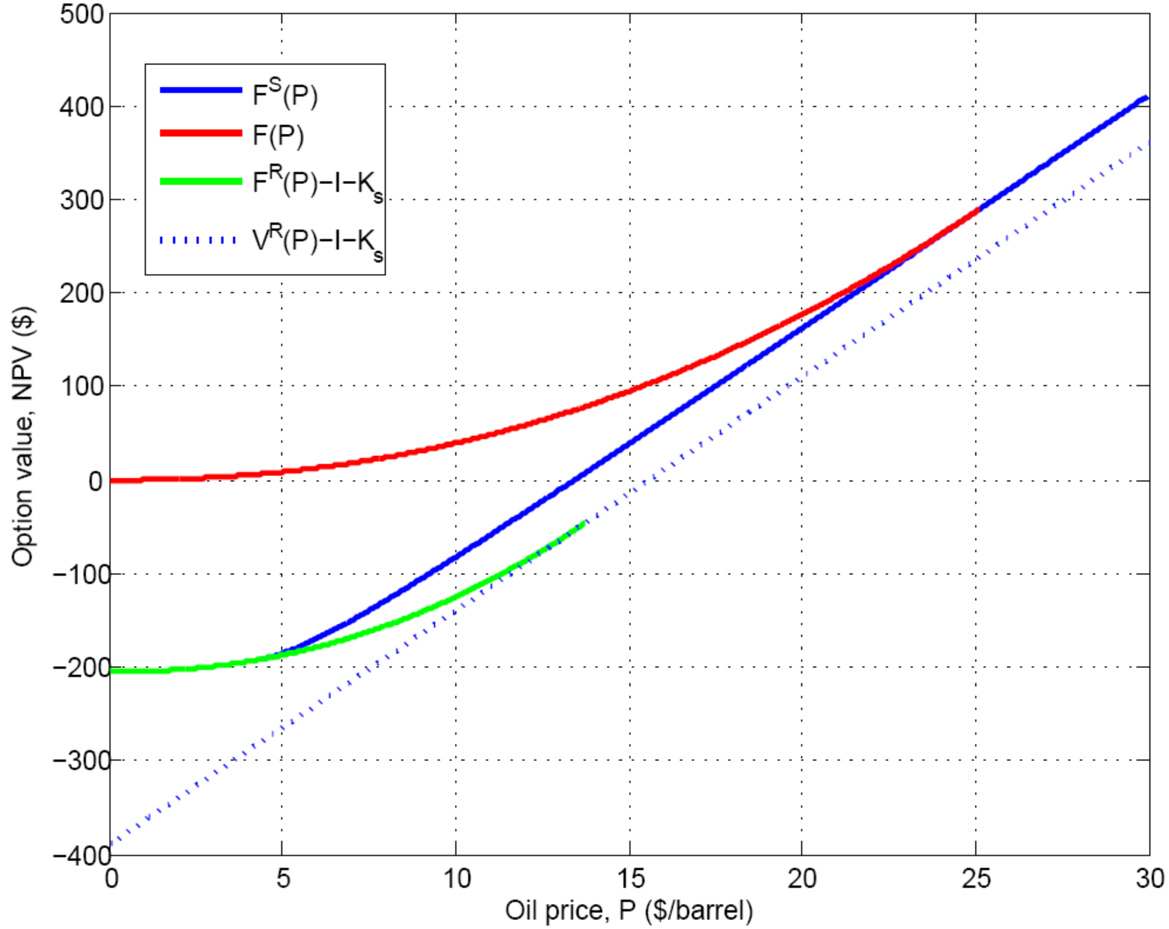
$$V^R(p) = \mathcal{E}_p\left[\int_0^\infty e^{-\rho t}\left\{P_t - (c + \rho K_r)\right\} dt\right] = \frac{p}{\rho - \alpha} - \frac{c}{\rho} - K_r$$

- Solve for optimal resumption threshold, P^*:

$$F^R(p) = \max_{P^* \geq p}\left\{\left(\frac{p}{P^*}\right)^{\beta_1} V^R(P^*)\right\}$$

  - First-order necessary condition yields $P^* = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\left(\frac{c}{\rho} + K_r\right)$
  - Substitute P^* back into $F^S(p)$ to solve numerically for P_* and then repeat for $F(p)$ to obtain P_I

INVESTMENT THRESHOLDS AND VALUES WITH RESUMPTION



INVESTMENT WITH INFINITE SUSPENSION AND RESUMPTION OPTIONS

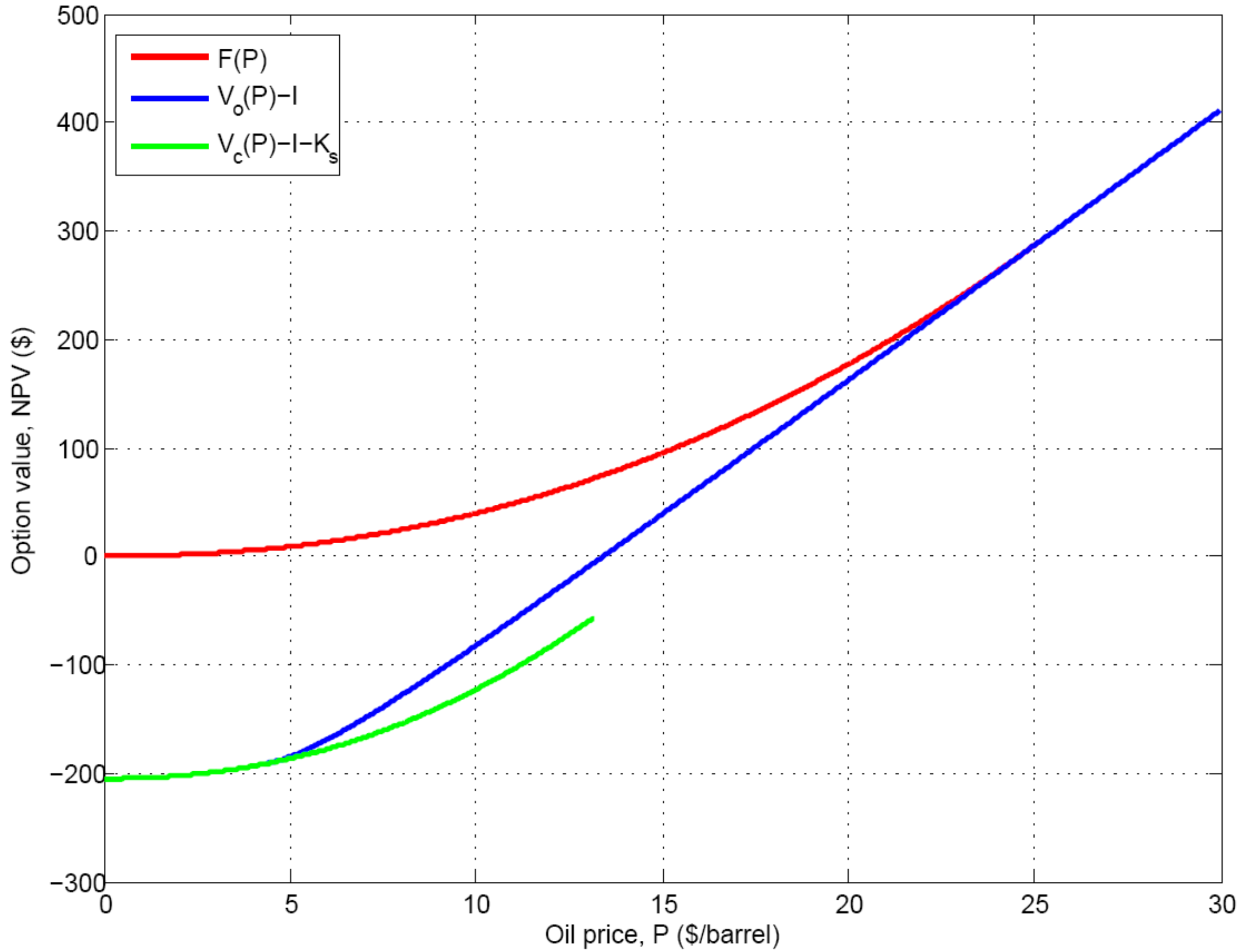
- Start with the expected value of a suspended project: $V_c(p,\infty,\infty;P_*,P^*) = \left(\frac{p}{P^*}\right)^{\beta_1}(V_o(P^*,\infty,\infty;P_*,P^*) - K_r)$

- Also note the expected value of an active project: $V_o(p,\infty,\infty;P_*,P^*) = \frac{p}{\rho-\alpha} - \frac{c}{\rho} + \left(\frac{p}{P_*}\right)^{\beta_2}\left(\frac{c}{\rho} - K_s - \frac{P_*}{\rho-\alpha} + V_c(P_*,\infty,\infty;P_*,P^*)\right)$

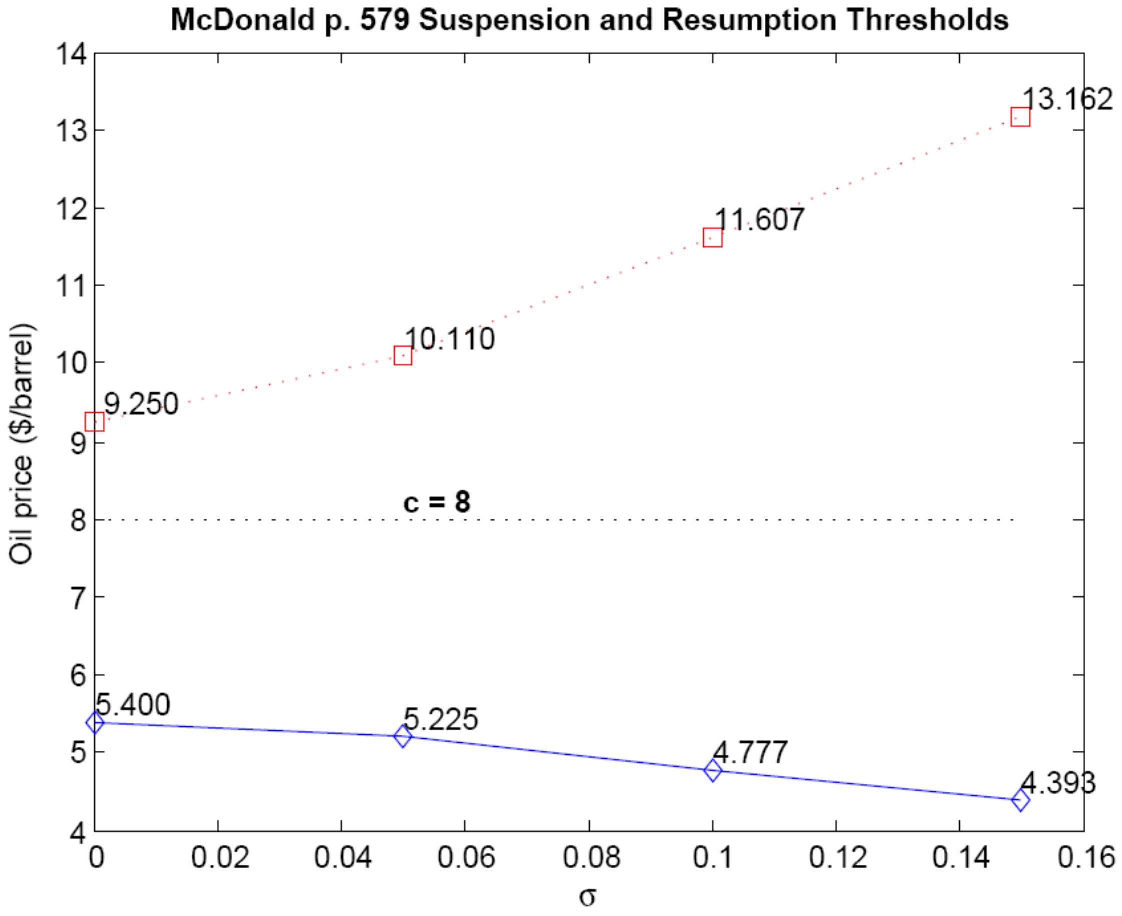
  - Solve the two equations numerically, i.e., start with initial thresholds and successively iterate until convergence

- Finally, solve for P_I numerically: $F(p,\infty,\infty;P_*,P^*) = \max_{P_I \geq p}\left(\frac{p}{P_I}\right)^{\beta_1}\{V_o(P_I,\infty,\infty;P_*,P^*) - I\}$

INVESTMENT THRESHOLDS AND VALUES WITH COMPLETE FLEXIBILITY



INVESTMENT THRESHOLDS WITH COMPLETE FLEXIBILITY



NUMERICAL RESULTS: Data from McDonald (2002)

★ $P_0 = 15, c = 8, \rho = 0.05, \alpha = 0.01, I = 180, K_s = 25, K_r = 25$

σ	N_s	N_r	P_I	P_*	P^*	$F(P_0)$
0.05	0	0	18.5846	-	-	56.0527
0.10	0	0	21.5927	-	-	74.6799
0.15	0	0	25.2791	-	-	94.3469
0.05	1	0	18.5846	4.9396	-	56.0527
0.10	1	0	21.5821	4.2514	-	74.7062
0.15	1	0	25.1587	3.6315	-	94.6154
0.05	1	1	18.5846	5.2246	10.1122	56.0527
0.10	1	1	21.5784	4.7702	11.7489	74.7153
0.15	1	1	25.1233	4.3625	13.7548	94.6946
0.05	∞	∞	18.5846	5.2246	10.1104	56.0527
0.10	∞	∞	21.5784	4.7766	11.6070	74.7154
0.15	∞	∞	25.1219	4.3926	13.1619	94.6977

INCORPORATION OF RISK AVERSION

- Hugonnier and Morellec (2007) take the perspective of a risk-averse decision maker with the perpetual option to invest in a project without operational flexibility
- Chronopoulos, De Reyck, and Siddiqui (2011) consider a case with operational flexibility
  - Includes embedded options to shut down and re-start the project (infinitely) many times after initial investment
  - Solve for optimal investment and operational thresholds along with option value of investment opportunity
- Take the approach of McDonald and Siegel (1986) to solve nested optimal stopping time problems
  - Specify a CRRA utility-of-wealth function
  - Apply result from Karatzas and Shreve (1999) concerning the discounted expected value of a function of a GBM process
  - Solve embedded sub-problems backwards

RISK-AVERSE PROBLEM FORMULATION: Assumptions

- Decision maker has the perpetual right to start the project at any time for deterministic investment cost, I
- Price process evolves according to a GBM, i.e., $dP_t = \alpha P_t dt + \sigma P_t dz_t$ with initial price $P_0 = p$
  - An active project incurs a deterministic operating cost of c
- Utility-of-wealth function is $U(w) = \frac{w^{1-\gamma}}{1-\gamma}$ for $0 \leq \gamma < 1$
- The project may also entail (infinitely) many embedded options to shut down and re-start costlessly
- Risk-free and subjective interest rates are r and ρ, respectively (both greater than α)

RISK-AVERSE PROBLEM: Timeline of Cash Flows without Operational Flexibility

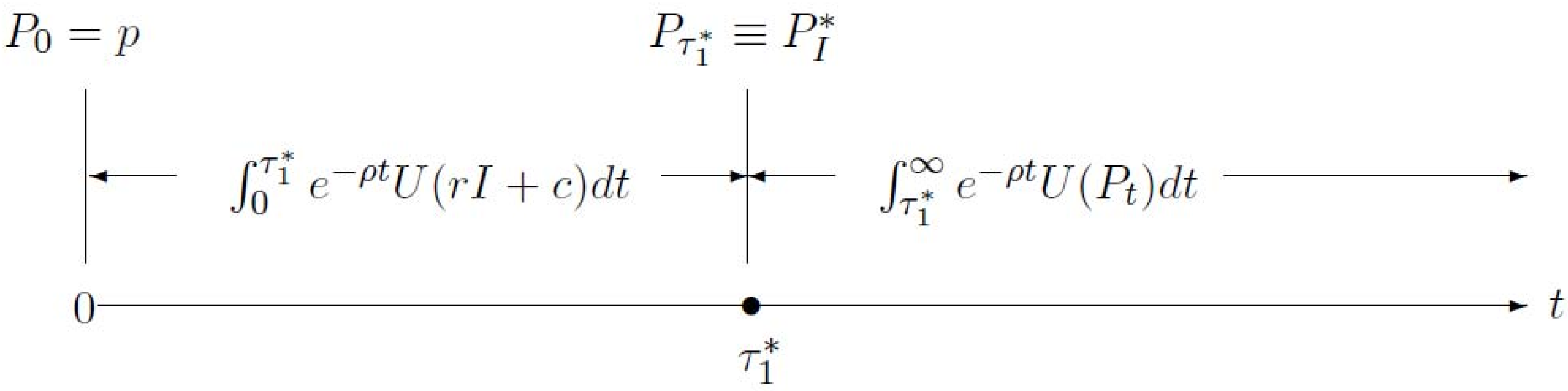


Figure 1: Perpetual Investment under Risk Aversion

- Initially hold a CD of size $I + \frac{c}{r}$ that earns the risk-free rate of return and, at time τ_1^*, is exchanged for a stream of risky instantaneous cash flows, P_t
  - The discounted conditional lifetime expected utility of cash flows is $\int_0^{\tau_1^*} e^{-\rho t} U(rI + c)dt + \mathcal{E}_p \left[\int_{\tau_1^*}^{\infty} e^{-\rho t} U(P_t)dt \right] = \int_0^{\infty} e^{-\rho t} U(rI + c)dt + \mathcal{E}_p \left[e^{-\rho \tau_1^*} \right] V_0(P_{\tau_1^*})$, where $V_0(p) = \mathcal{E}_p \left[\int_0^{\infty} e^{-\rho t} \left\{ U(P_t) - U(rI + c) \right\} dt \right]$

RISK-AVERSE PROBLEM: No Operational Flexibility

- From Karatzas and Shreve (1999), the expected NPV of active project is $V_0(P_{\tau_1}) = \mathcal{E}_{P_{\tau_1}}\left[\int_0^\infty e^{-\rho t}\left(U\left(P_t\right) - U\left(rI + c\right)\right)dt\right] = \frac{\beta_1\beta_2 p^{1-\gamma}}{\rho(1-\gamma)(1-\beta_2-\gamma)(1-\beta_1-\gamma)} - \frac{(c+rI)^{1-\gamma}}{\rho(1-\gamma)}$

- Value of investment opportunity: $F_0(p) = \sup_{\tau_1 \in \mathcal{S}} \mathcal{E}_p\left[e^{-\rho\tau_1}\right] V_0\left(P_{\tau_1}\right) = \max_{P_I \geq p}\left(\frac{p}{P_I}\right)^{\beta_1} V_0(P_I)$

- Optimal investment threshold is $P_I^*(\gamma) = (c + rI)\left[\frac{\beta_2 - 1 + \gamma}{\beta_2}\right]^{\frac{1}{1-\gamma}}$

RISK-AVERSE PROBLEM: Effect of Risk Aversion on Investment Threshold

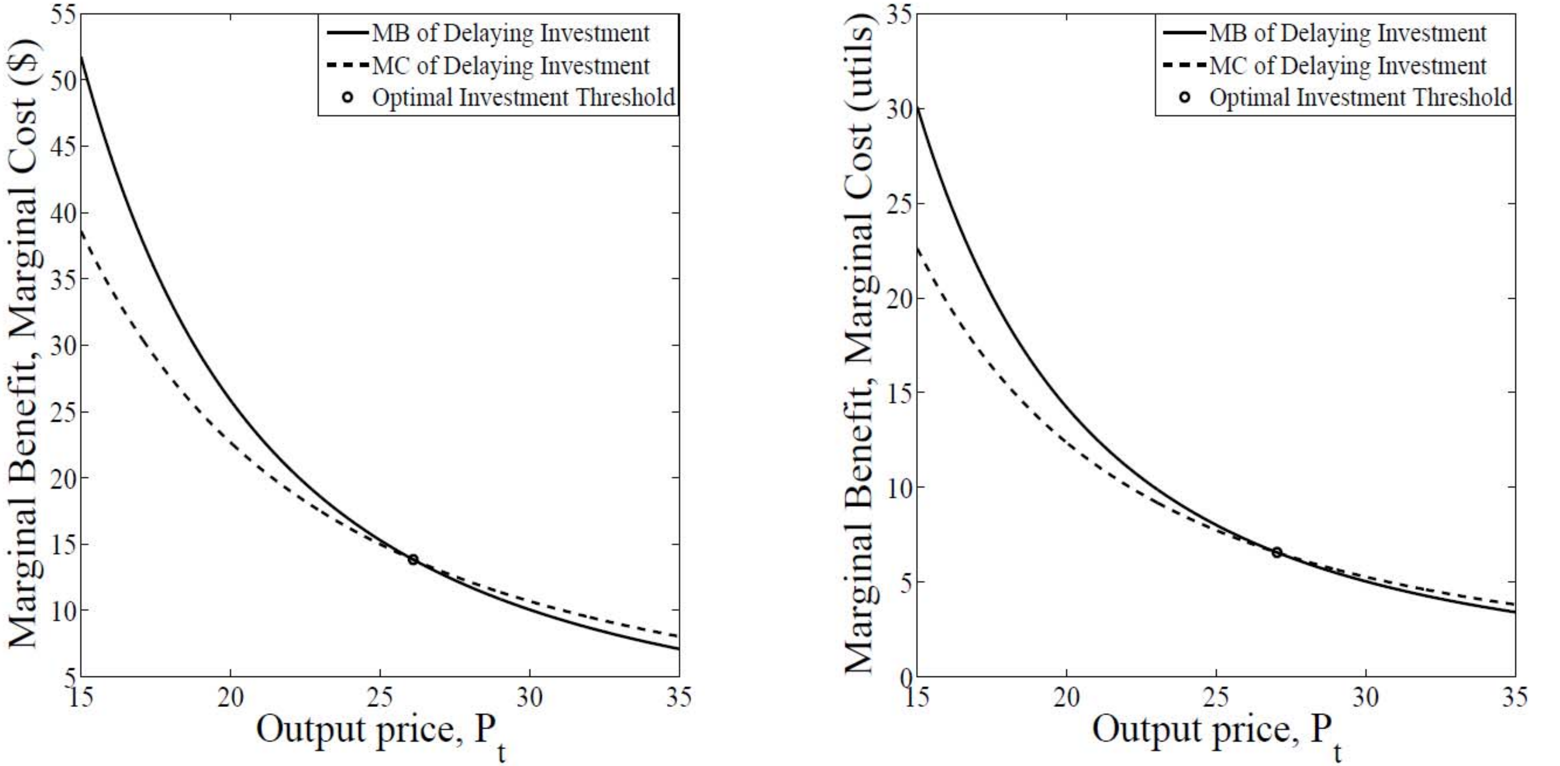


Figure 5: Marginal benefit versus marginal cost under risk neutrality (left) and risk aversion, $\gamma = 0.25$, (right) for an irreversible investment opportunity

RISK-AVERSE PROBLEM: Timeline of Cash Flows with Single Abandonment Option

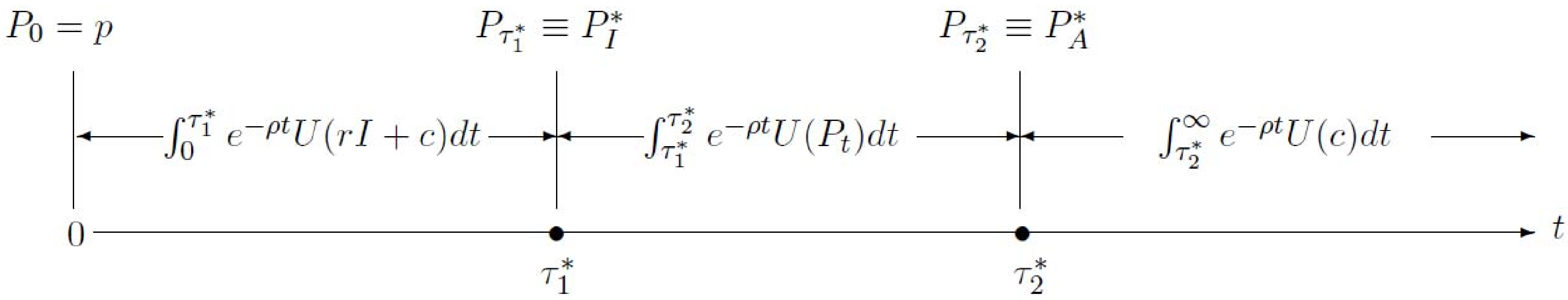


Figure 2: Investment under Risk Aversion with a Single Abandonment Option

★ Now, allow for abandonment at time τ_2^*

- The discounted conditional lifetime expected utility of cash flows is $\int_0^{\infty} e^{-\rho t} U(rI + c)dt + \mathcal{E}_p \left[e^{-\rho\tau_1^*}\right] V_0(P_{\tau_1^*}) + \mathcal{E}_p \left[e^{-\rho\tau_2^*}\right] V_1(P_{\tau_2^*})$, where $V_1(p) = \mathcal{E}_p \left[\int_0^{\infty} e^{-\rho t} \{U(c) - U(P_t)\} dt\right]$

RISK-AVERSE PROBLEM: Single Abandonment Option

- Expected discounted utility of cash flows at time τ_1 is $V_0(P_{\tau_1}) + \sup_{\tau_2 \geq \tau_1} \mathcal{E}_{P_{\tau_1}} \left[e^{-\rho(\tau_2 - \tau_1)} V_1(P_{\tau_2}) \right]$

- Value of investment opportunity: $F_1(p) = \sup_{\tau_1 \in \mathcal{S}} \mathcal{E}_p \left[e^{-\rho \tau_1} \left\{ V_0\left(P_{\tau_1}\right) + \sup_{\tau_2 \geq \tau_1} \mathcal{E}_{P_{\tau_1}} \left[e^{-\rho(\tau_2 - \tau_1)} V_1(P_{\tau_2}) \right] \right\} \right]$
  - $\Rightarrow F_1(p) = \max_{P_I \geq p} \left(\frac{p}{P_I} \right)^{\beta_1} \left[V_0(P_I) + F_A(P_I) \right]$, where $F_A(P_I) = \max_{P_A \leq P_I} \left(\frac{P_I}{P_A} \right)^{\beta_2} V_1(P_A)$
  - Optimal abandonment threshold is $P_A^*(\gamma) = c \left[\frac{\beta_1 - 1 + \gamma}{\beta_1} \right]^{\frac{1}{1-\gamma}}$
  - FONC for investment: $\frac{\beta_2}{1-\beta_2-\gamma} \left(P_I^* \right)^{1-\gamma} + (c + rI)^{1-\gamma} - \left(\frac{P_I^*}{P_A^*} \right)^{\beta_2} \frac{\rho(\beta_1 - \beta_2)}{\beta_1} (1-\gamma) V_1(P_A^*) = 0$

RISK-AVERSE PROBLEM: Effect of Abandonment Option on Investment Threshold

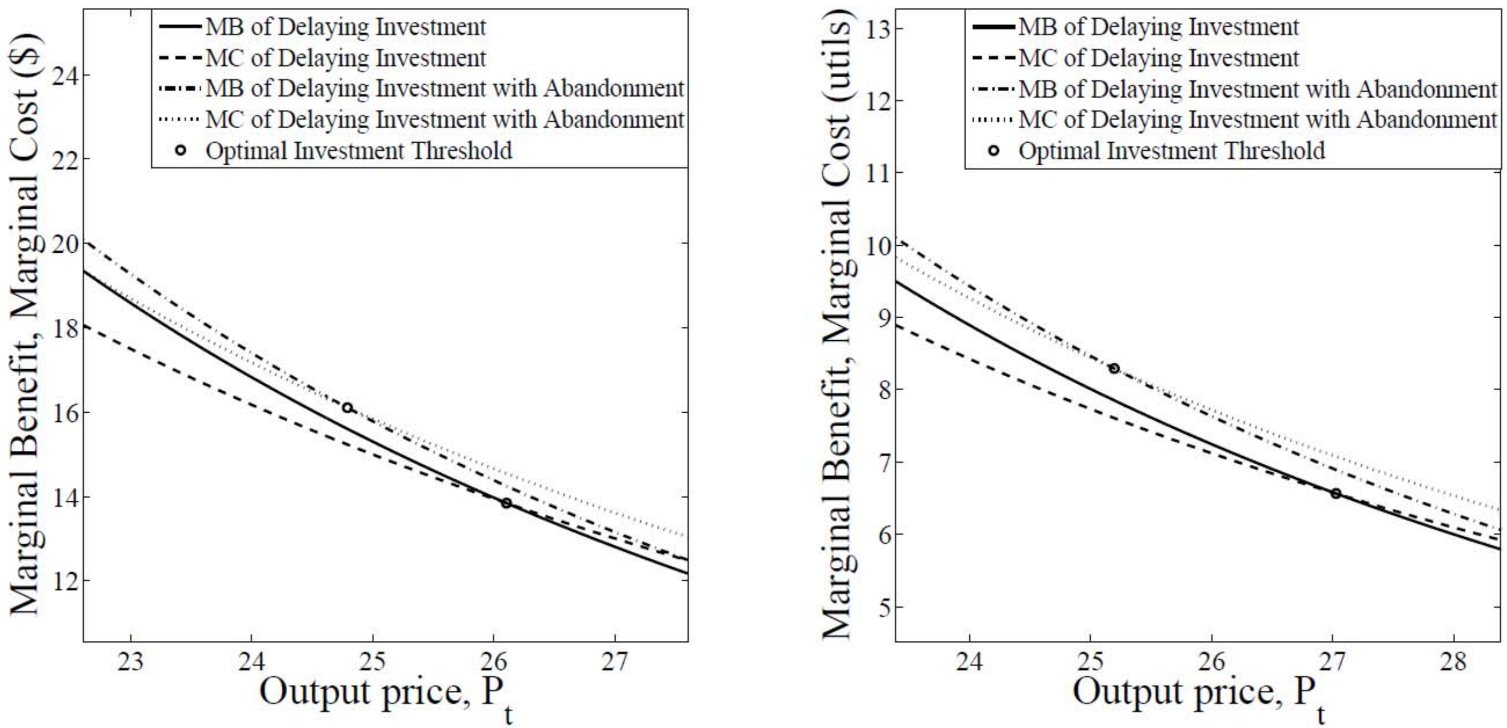


Figure 10: Marginal benefit versus marginal cost under risk neutrality (left) and risk aversion, $\gamma = 0.25$ (right) for an investment opportunity with an embedded abandonment option

RISK-AVERSE PROBLEM: Timeline of Cash Flows with Single Suspension and Resumption Option

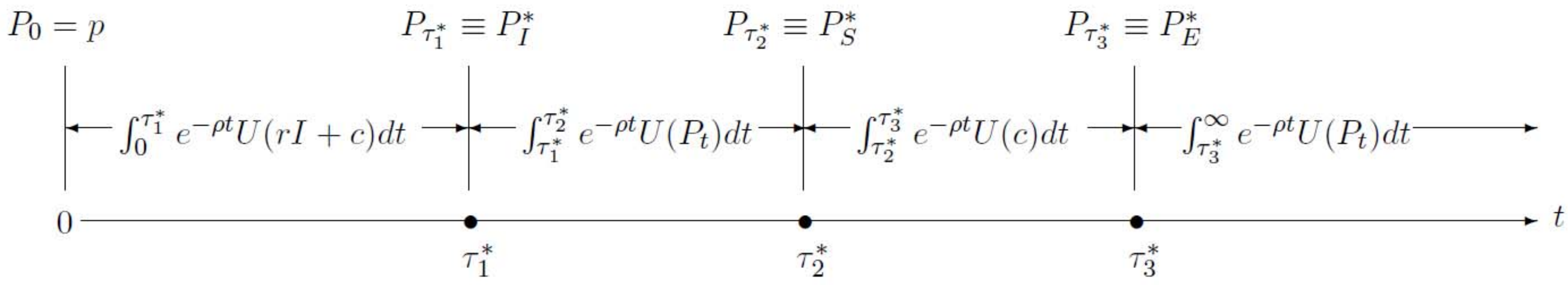


Figure 3: Investment under Risk Aversion with One Suspension and One Resumption Option

- With subsequent resumption option at τ_3^*
  - The discounted conditional lifetime expected utility of cash flows is $\int_0^\infty e^{-\rho t} U(rI + c) dt + \mathcal{E}_p \left[e^{-\rho \tau_1^*} \right] V_0(P_{\tau_1^*}) + \mathcal{E}_p \left[e^{-\rho \tau_2^*} \right] V_1(P_{\tau_2^*}) + \mathcal{E}_p \left[e^{-\rho \tau_3^*} \right] V_2(P_{\tau_3^*})$, where $V_2(p) = \mathcal{E}_p \left[\int_0^\infty e^{-\rho t} \left\{ U(P_t) - U(c) \right\} dt \right]$

RISK-AVERSE PROBLEM: Single Suspension and Resumption Option

- Time-τ_1 expected discounted utility of cash flows: $V_0(P_{\tau_1}) + \sup_{\tau_2 \geq \tau_1} \mathcal{E}_{P_{\tau_1}} \left[e^{-\rho(\tau_2 - \tau_1)} \left[V_1(P_{\tau_2}) + \sup_{\tau_3 \geq \tau_2} \mathcal{E}_{P_{\tau_2}} \left[e^{-\rho(\tau_3 - \tau_2)} V_2(P_{\tau_3}) \right] \right] \right]$

- Value of investment opportunity: $F_2(p) = \max_{P_I \geq p} \left(\frac{p}{P_I} \right)^{\beta_1} \left[V_0(P_I) + F_S(P_I) \right]$
    - $F_S(P_I) = \max_{P_S \leq P_I} \left(\frac{P_I}{P_S} \right)^{\beta_2} \{ V_1(P_S) + F_E(P_S) \}$
    - $F_E(P_S) = \max_{P_E \geq P_S} \left(\frac{P_S}{P_E} \right)^{\beta_1} V_2(P_E)$

- Optimal resumption threshold is $P_E^*(\gamma) = c \left[\frac{\beta_2 - 1 + \gamma}{\beta_2} \right]^{\frac{1}{1-\gamma}}$

RISK-AVERSE PROBLEM: Effect of Suspension and Resumption Options on Investment Threshold

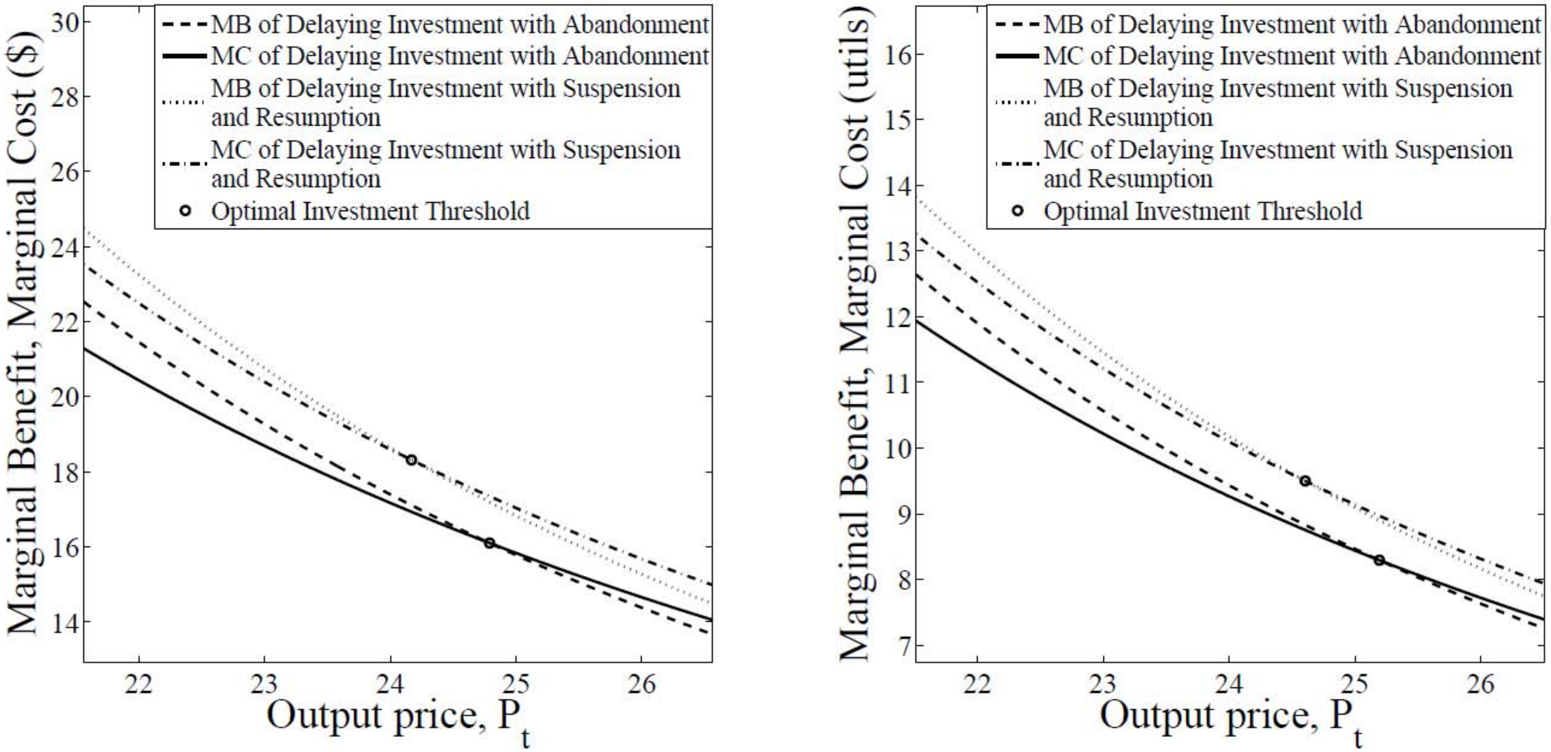


Figure 12: Marginal benefit versus marginal cost under risk neutrality (left) and risk aversion, $\gamma = 0.25$, (right) for an investment opportunity with a suspension and resumption option

RISK-AVERSE PROBLEM: Complete Operational Flexibility

★ At the resumption threshold, P_E, the expected utility of cash flows of an active firm is $V_o(P_E, \infty, \infty; P_S, P_E) = V_2(P_E) + \left(\frac{P_E}{P_S}\right)^{\beta_2} V_1(P_S) + \left(\frac{P_E}{P_S}\right)^{\beta_2} \left(\frac{P_S}{P_E}\right)^{\beta_1} V_2(P_E) + \cdots$

- $\Rightarrow V_o(P_E, \infty, \infty; P_S, P_E) = \sum_{i=0}^{\infty} \left\{ \left(\frac{P_E}{P_S}\right)^{\beta_2} \left(\frac{P_S}{P_E}\right)^{\beta_1} \right\}^i V_2(P_E) + \left(\frac{P_E}{P_S}\right)^{\beta_2} \sum_{i=0}^{\infty} \left\{ \left(\frac{P_E}{P_S}\right)^{\beta_2} \left(\frac{P_S}{P_E}\right)^{\beta_1} \right\}^i V_1(P_S) = \frac{1}{1 - \left(\frac{P_E}{P_S}\right)^{\beta_2} \left(\frac{P_S}{P_E}\right)^{\beta_1}} \left\{ V_2(P_E) + \left(\frac{P_E}{P_S}\right)^{\beta_2} V_1(P_S) \right\}$
- $V_c(P_S, \infty, \infty; P_S, P_E) = \left(\frac{P_S}{P_E}\right)^{\beta_1} V_o(P_E, \infty, \infty; P_S, P_E)$
-

$$F_\infty(p) = \max_{P_I \geq p} \left(\frac{p}{P_I}\right)^{\beta_1} \left[\mathcal{E}_{P_I} \left[\int_0^\infty e^{-\rho t} \left\{ U(P_t) - U(c + rI) \right\} dt \right] + \left(\frac{P_I}{P_S}\right)^{\beta_2} V_c(P_S, \infty, \infty; P_S, P_E) \right]$$

RISK-AVERSE PROBLEM: Value Curves without Operational Flexibility

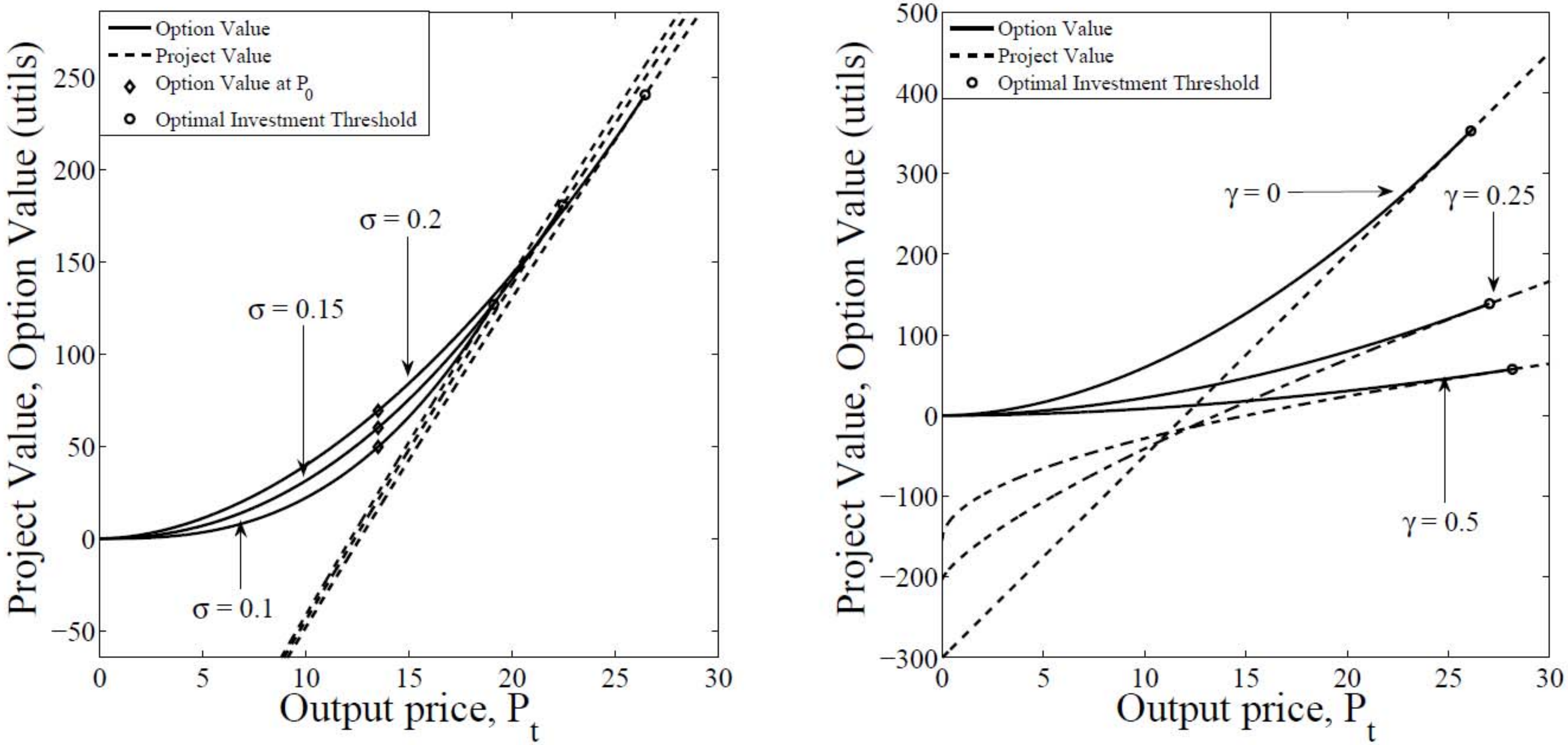


Figure 6: Option value and project value versus P_t for $\gamma = 0.25$ and $\sigma = 0, 0.15, 0.2$ (left), and option value and project value versus P_t for $\sigma = 0.2$ and $\gamma = 0, 0.25, 0.5$ (right)

RISK-AVERSE PROBLEM: Investment Thresholds without Operational Flexibility

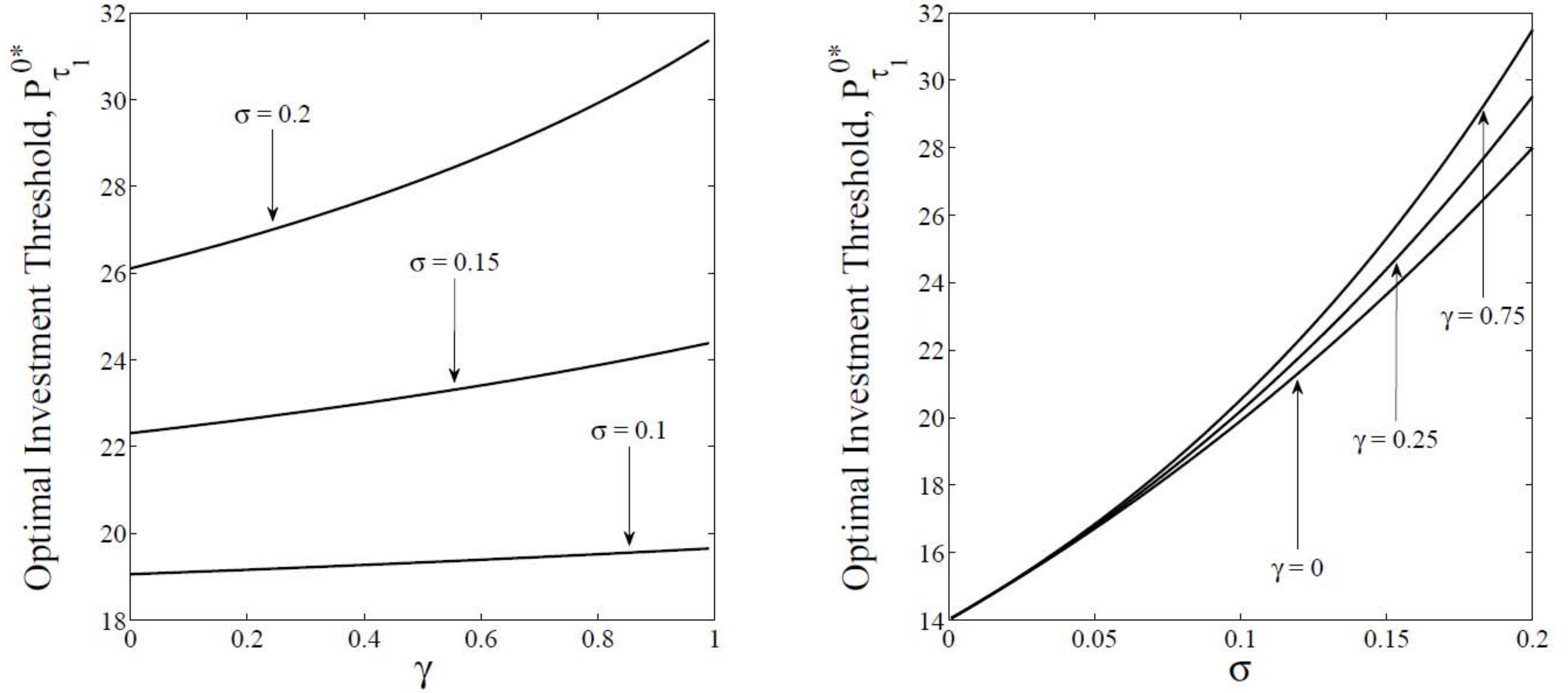


Figure 4: Optimal investment threshold versus γ for $\sigma = 0.1, 0.15, 0.2$ (left), and optimal investment threshold versus σ for $\gamma = 0, 0.25, 0.5$ (right).

RISK-AVERSE PROBLEM: Results Summary with Abandonment

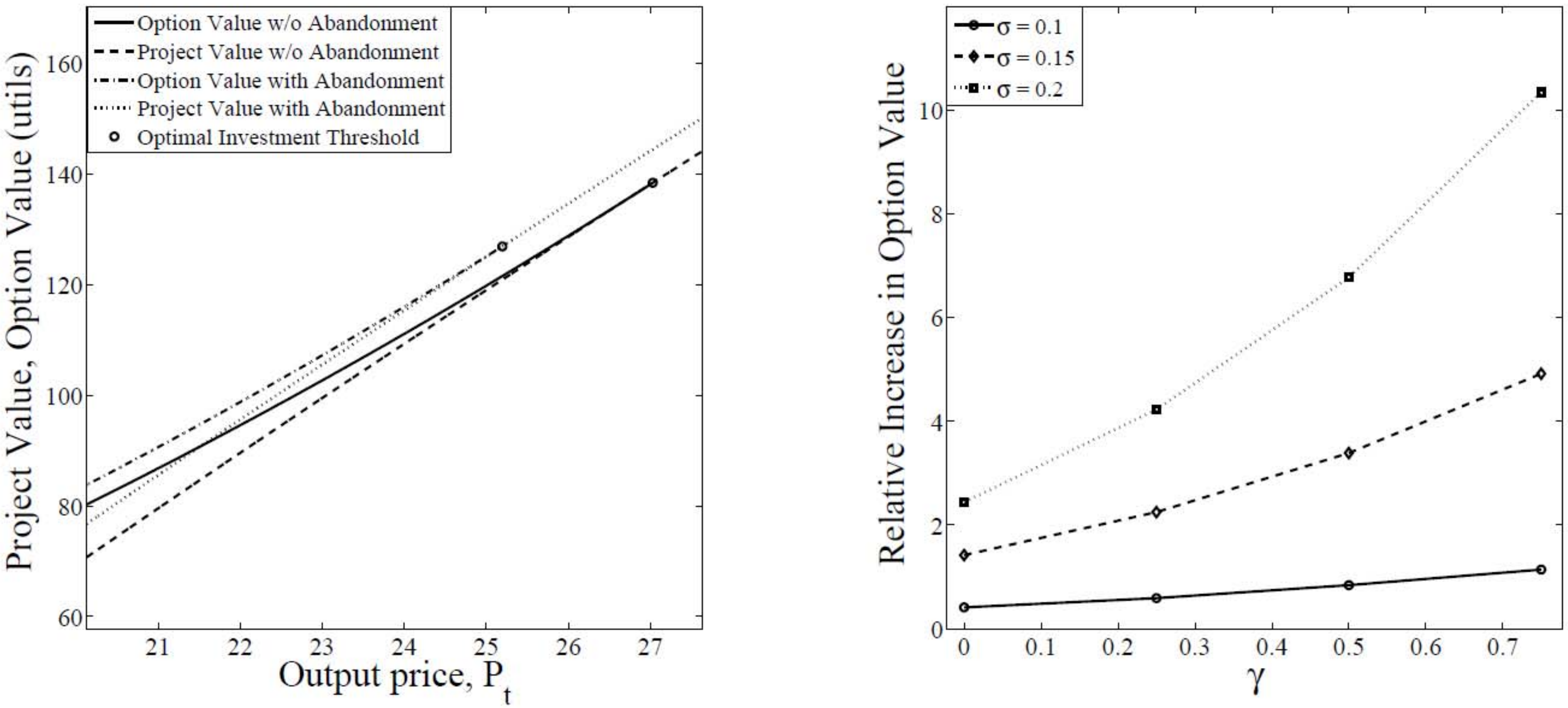


Figure 7: Effect of the abandonment option on optimal investment threshold and option value

RISK-AVERSE PROBLEM: Abandonment Thresholds

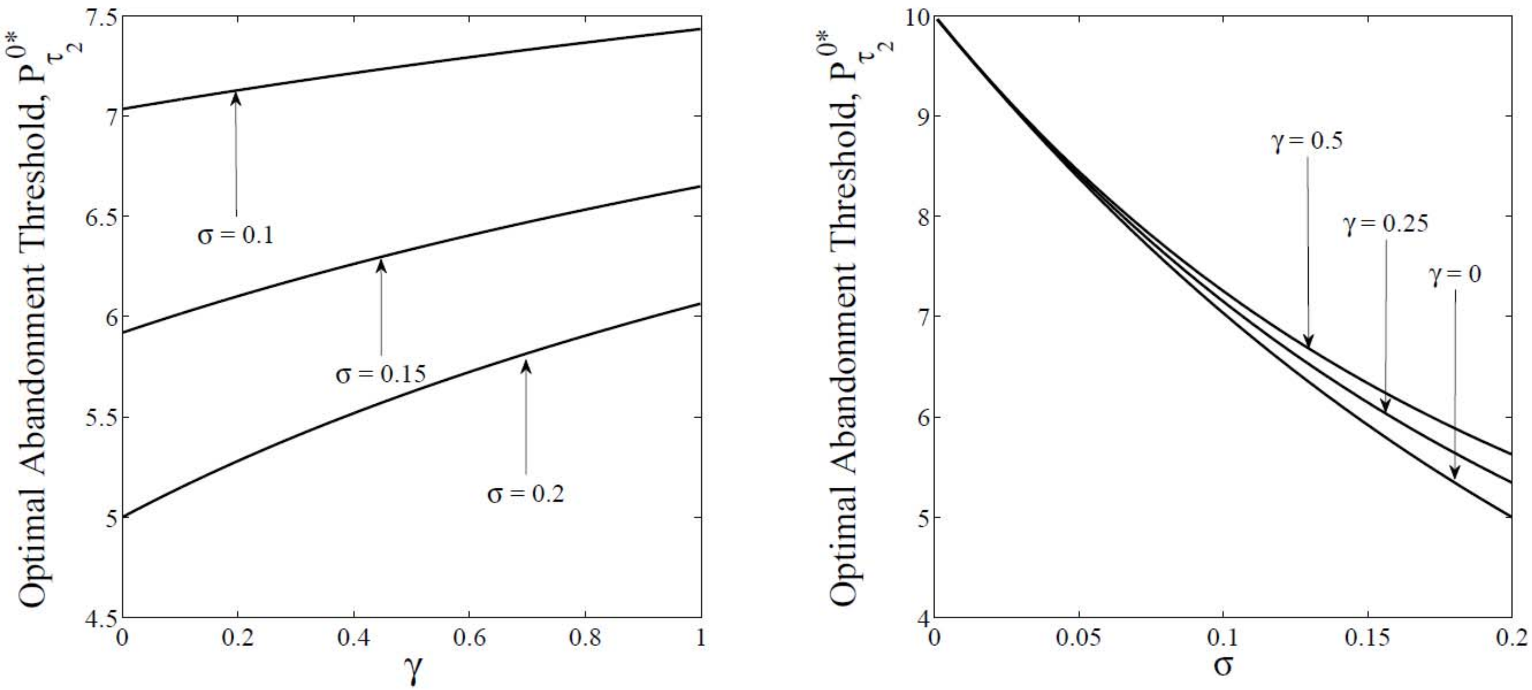


Figure 8: Optimal abandonment threshold versus γ for $\sigma = 0.1, 0.15, 0.2$ (left), optimal abandonment threshold versus σ for $\gamma = 0, 0.25, 0.5$ (right)

RISK-AVERSE PROBLEM: Results Summary with Single Suspension and Resumption

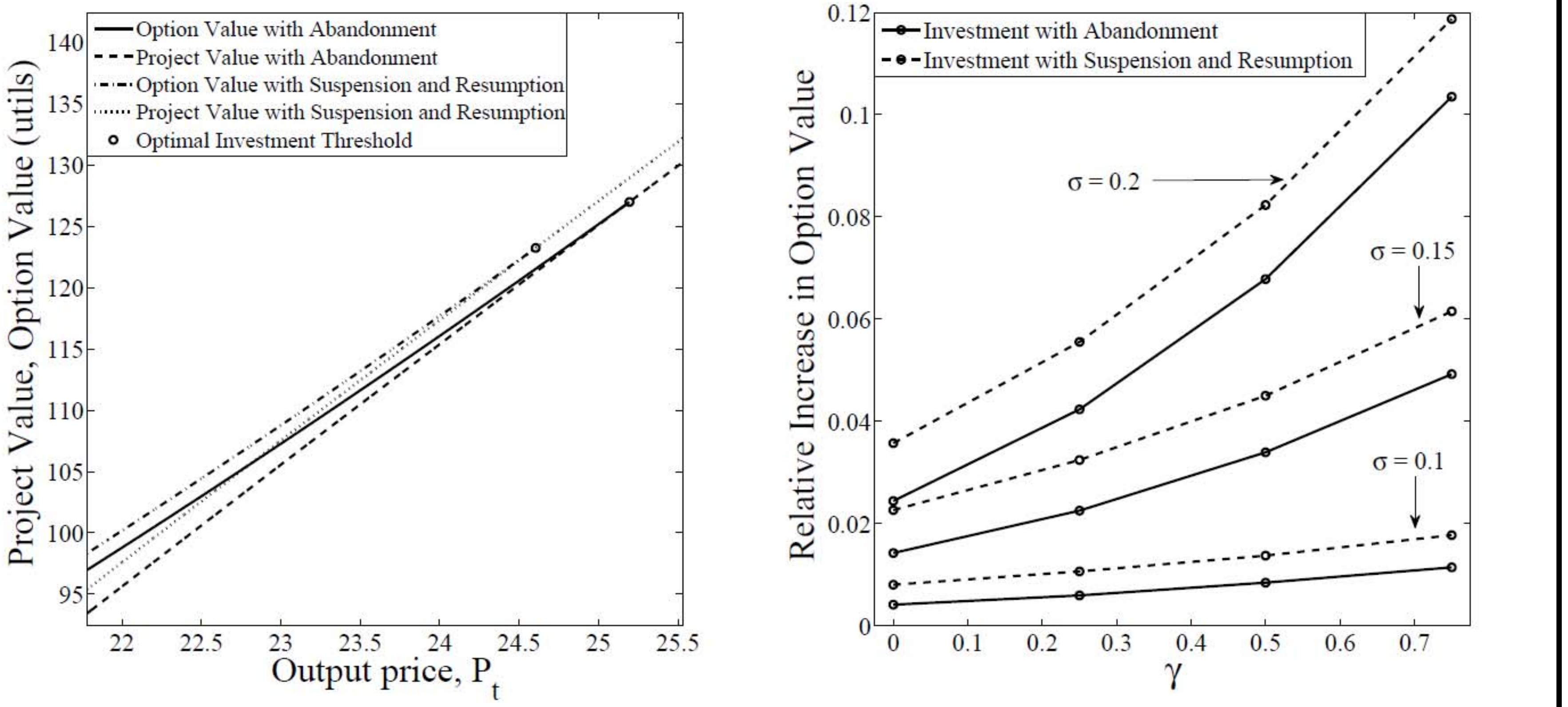


Figure 11: Effect of the resumption option on optimal investment threshold and option value

RISK-AVERSE PROBLEM: Impact of Operational Flexibility and Risk Aversion on Optimal Decision Thresholds

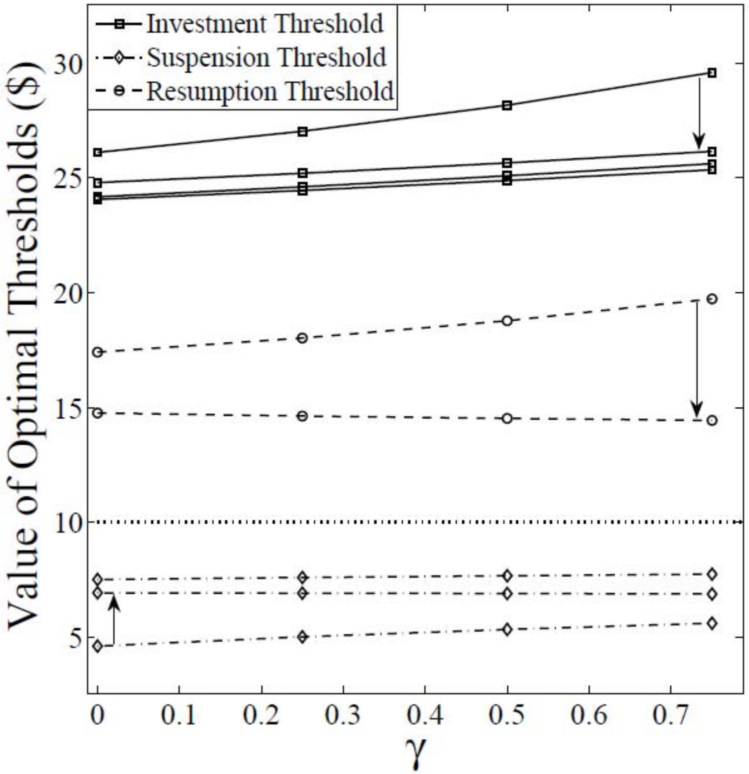


Figure 14: Impact of operational flexibility and risk aversion on optimal decision thresholds

RISK-AVERSE PROBLEM: Results Summary with Complete Flexibility

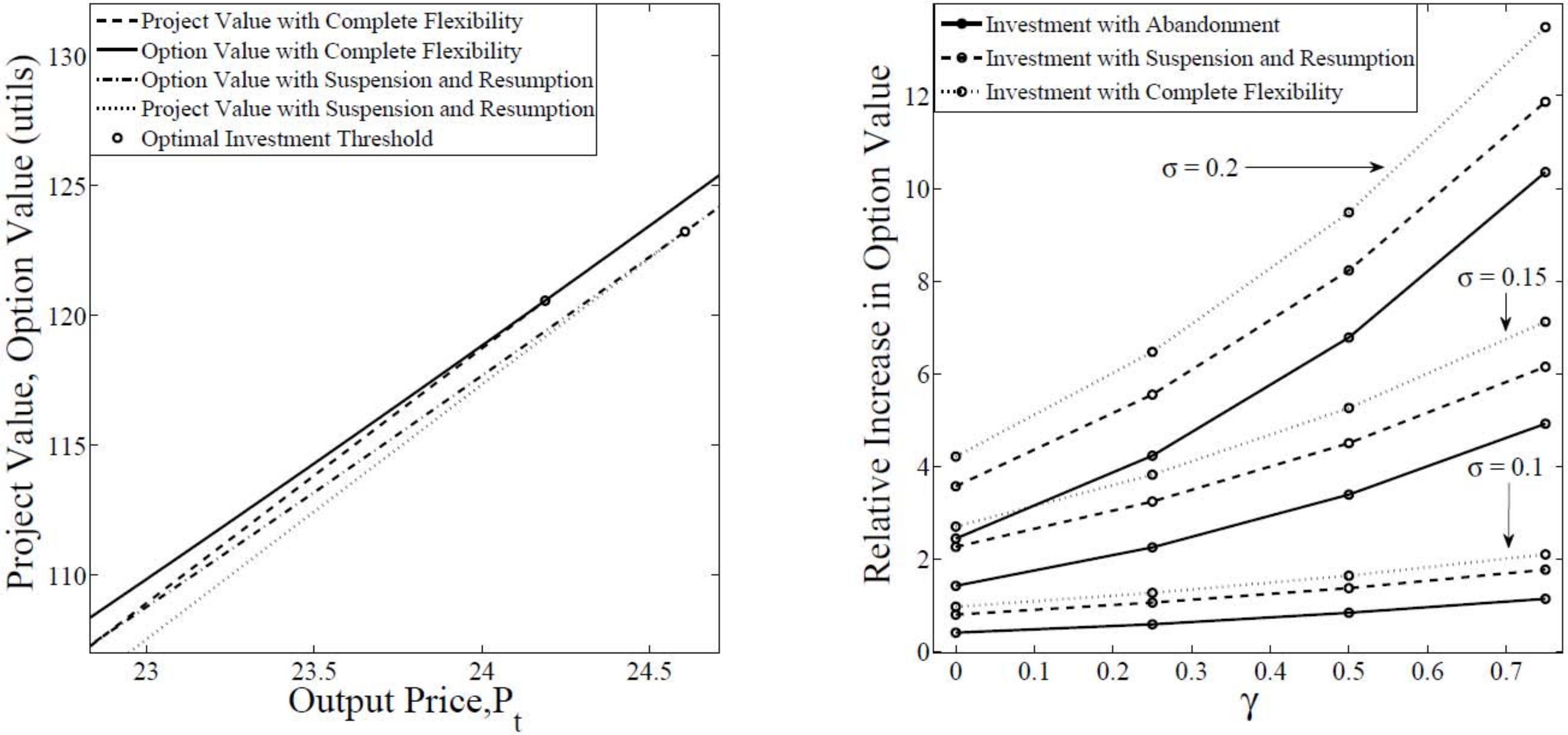


Figure 15: Impact of complete flexibility on the optimal investment threshold and option value

TWO SOURCES OF UNCERTAINTY: Analytical Solutions

- For a perpetual investment problem with payoff of the form $V(P,C) = \frac{P}{\rho-\alpha} - \frac{C}{\rho}$, where both P_t and C_t follow correlated GBMs, use homogeneity to convert the resulting PDE to an ODE and solve analytically for the free boundary, $P^*(C)$ (Dixit and Pindyck (1994))

- But, what if the payoff is of the form $V(P,C) = \frac{P}{\rho-\alpha} - \frac{C}{\rho} - I$?
  - Homogeneity no longer holds because of the I term
  - Pindyck (2002) examines an environmental control problem and proposes an analytical solution of the form $F(P,C) = aP^{\beta}C^{\eta}$
  - Adkins and Paxson (2008) formalise the proof with geometric interpretation
  - Heydari, Ovenden, and Siddiqui (2011) apply this technique to a problem with CCS retrofits

PROBLEM FORMULATION: Assumptions

- Long-term electricity (E_t in \$/MWh$_e$), coal (F_t in \$/MWh), and CO_2 (C_t in \$/t) prices are exogenous and evolve according to correlated GBMs, i.e.,
  - $dE_t = \alpha_E E_t dt + \sigma_E E_t dz_E$, $dF_t = \alpha_F F_t dt + \sigma_F F_t dz_F$, $dC_t = \alpha_C C_t dt + \sigma_C C_t dz_C$, and $\mathcal{E}[dz_i dz_j] = \rho_{ij} dt \; \forall i, j$

- In response to CO_2 emissions restrictions, the plant owner may retrofit with CCS for an investment cost of I^{ccs} (in \$) to obtain a reduction in the emissions rate, ϵ_C (in t/MWh$_e$), along with an increase in the heat rate, ϵ_F (in MWh/MWh$_e$)

PROBLEM FORMULATION: Assumptions (continued)

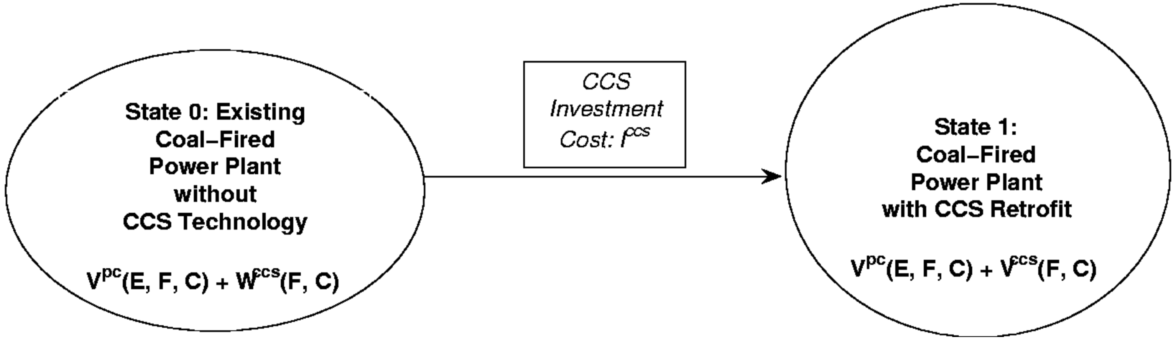
- ★ Annual electricity production of plant, Q (in MWh$_e$), is unaffected by retrofit decision

- ★ Retrofit occurs instantaneously upon decision

- ★ Infinite lifetime for the plant regardless of retrofit option

- ★ The exogenous discount rate is μ

PROBLEM FORMULATION: CCS Retrofit Decision



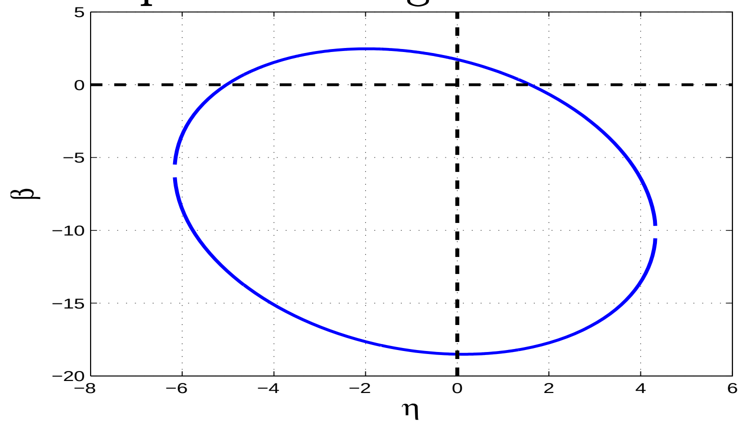
★ First, determine the PV of benefits from the CCS retrofit:

- $V^{pc}(E,F,C) = Q\mathcal{E}\left[\int_0^\infty (E_t e^{-\mu t} - \epsilon_F F_t e^{-\mu t} - \epsilon_C C_t e^{-\mu t})dt \,|E,F,C\right]$
  $\Rightarrow \mathrm{V}^{pc}(E,F,C) = Q\left[\frac{E}{\mu-\alpha_E} - \frac{\epsilon_F F}{\mu-\alpha_F} - \frac{\epsilon_C C}{\mu-\alpha_C}\right]$
- $V^{pc}(E,F,C) + V^{ccs}(F,C) = Q\left[\frac{E}{\mu-\alpha_E} - \frac{\epsilon_F^{ccs} F}{\mu-\alpha_F} - \frac{\epsilon_C^{ccs} C}{\mu-\alpha_C}\right]$
  $\Rightarrow \mathrm{V}^{ccs}(F,C) = Q\left[\frac{(\epsilon_F - \epsilon_F^{ccs})F}{\mu-\alpha_F} + \frac{(\epsilon_C - \epsilon_C^{ccs})C}{\mu-\alpha_C}\right]$

PROBLEM FORMULATION: CCS Retrofit Option Value

- Use the Bellman Equation to solve for the option value to retrofit to CCS:
  - $\mu W^{ccs} dt = \mathcal{E}[dW^{ccs}]$
    $\Rightarrow \frac{1}{2}\sigma_F^2 F^2 W_{FF}^{ccs} + \frac{1}{2}\sigma_C^2 C^2 W_{CC}^{ccs} + \rho\sigma_F\sigma_C FCW_{FC}^{ccs} + \alpha_F FW_F^{ccs} + \alpha_C CW_C^{ccs} - \mu W^{ccs} = 0$

- Guess $W^{ccs}(F, C) = aF^\beta C^\eta$
  - $H(\beta, \eta) = \frac{1}{2}\sigma_F^2\beta(\beta-1) + \frac{1}{2}\sigma_C^2\eta(\eta-1) + \rho\sigma_F\sigma_C\beta\eta + \alpha_F\beta + \alpha_C\eta - \mu = 0$
  - The roots of H fall on an ellipse that passes through all four axes (Adkins and Paxson (2008))



PROBLEM FORMULATION: CCS Retrofit Option Value

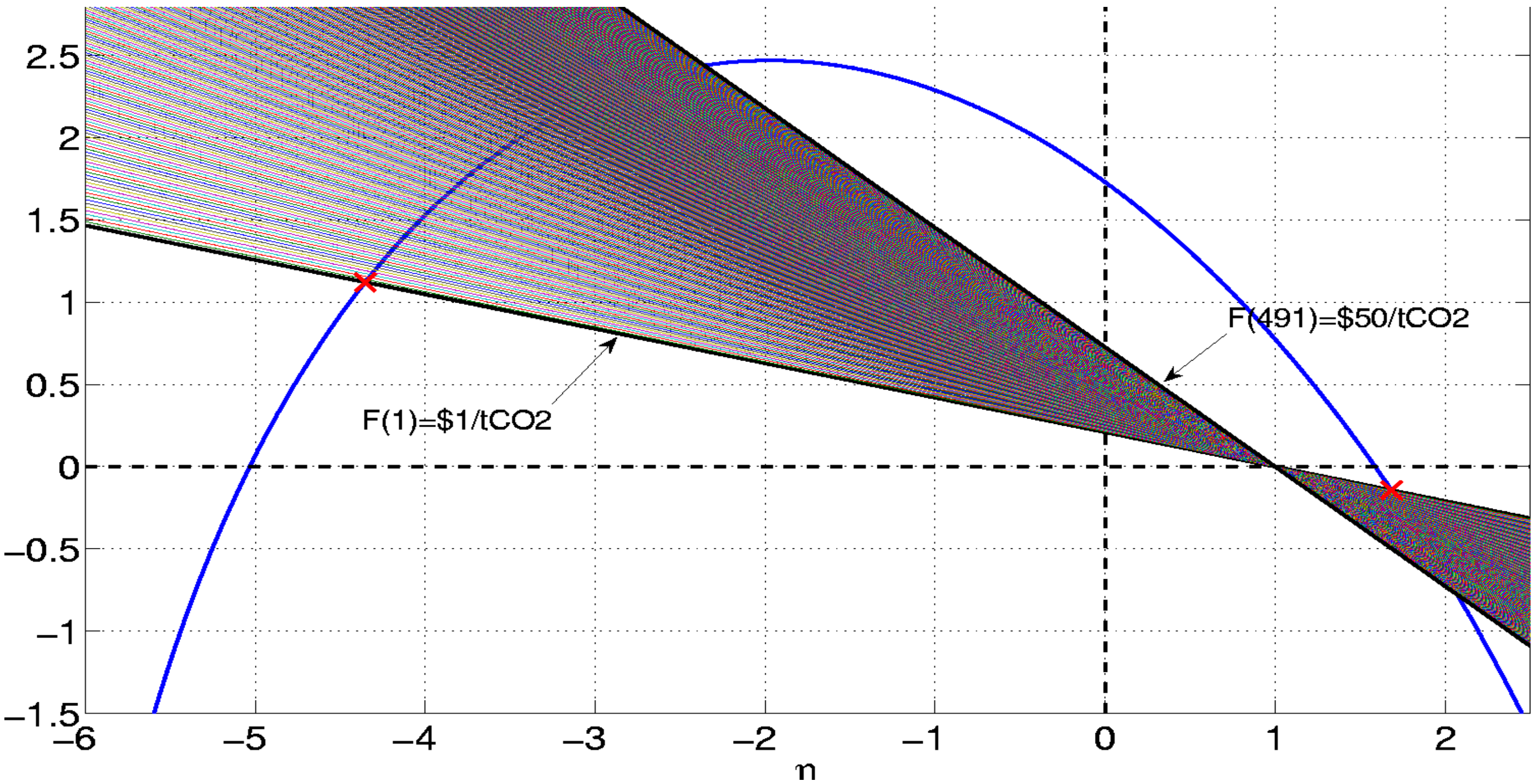
★ Value-matching and smooth-pasting conditions

- $W^{ccs}(F, C^*(F)) = V^{ccs}(F, C^*(F)) - I^{ccs}$
- $W_F^{ccs}(F, C^*(F)) = V_F^{ccs}(F, C^*(F))$
- $W_C^{ccs}(F, C^*(F)) = V_C^{ccs}(F, C^*(F))$
- This system gives us a linear relationship between β and η: $\beta = \frac{Q(\epsilon_F - \epsilon_F^{ccs})(\eta-1)F}{(\mu-\alpha_F)I^{ccs} - Q(\epsilon_F - \epsilon_F^{ccs})F}$

★ Impose this line on the ellipse $H(\beta, \eta) = 0$

- Two sets of solutions:
  $\beta_1 < 0$ and $\eta_1 > 0$
  $\beta_2 > 0$ and $\eta_2 < 0$
- Hence, $W^{ccs}(F, C) = a_1 F^{\beta_1} C^{\eta_1} + a_2 F^{\beta_2} C^{\eta_2}$
- For low values of C, the option value is worthless, i.e., $a_2 = 0$, which implies $W^{ccs}(F, C) = a_1 F^{\beta_1} C^{\eta_1}$

PROBLEM FORMULATION: CCS Retrofit Option Value



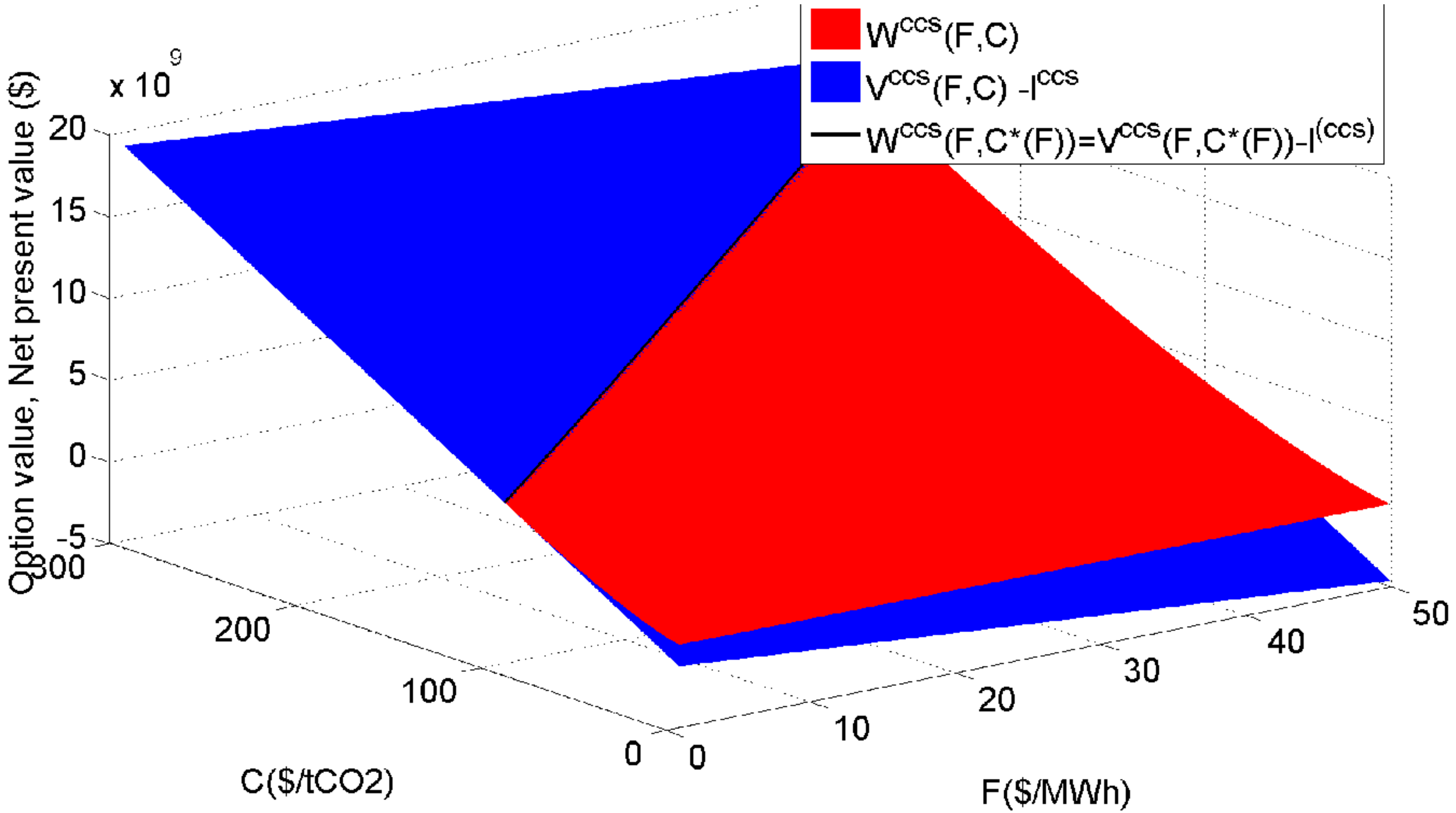
NUMERICAL EXAMPLE: Data

μ	α_F	α_C	σ_F	σ_C	ρ	ϵ_F	ϵ_C	Q
0.09	0.05	0.05	0.20	0.40	0.20	2.20	0.735	4380 GWh_e

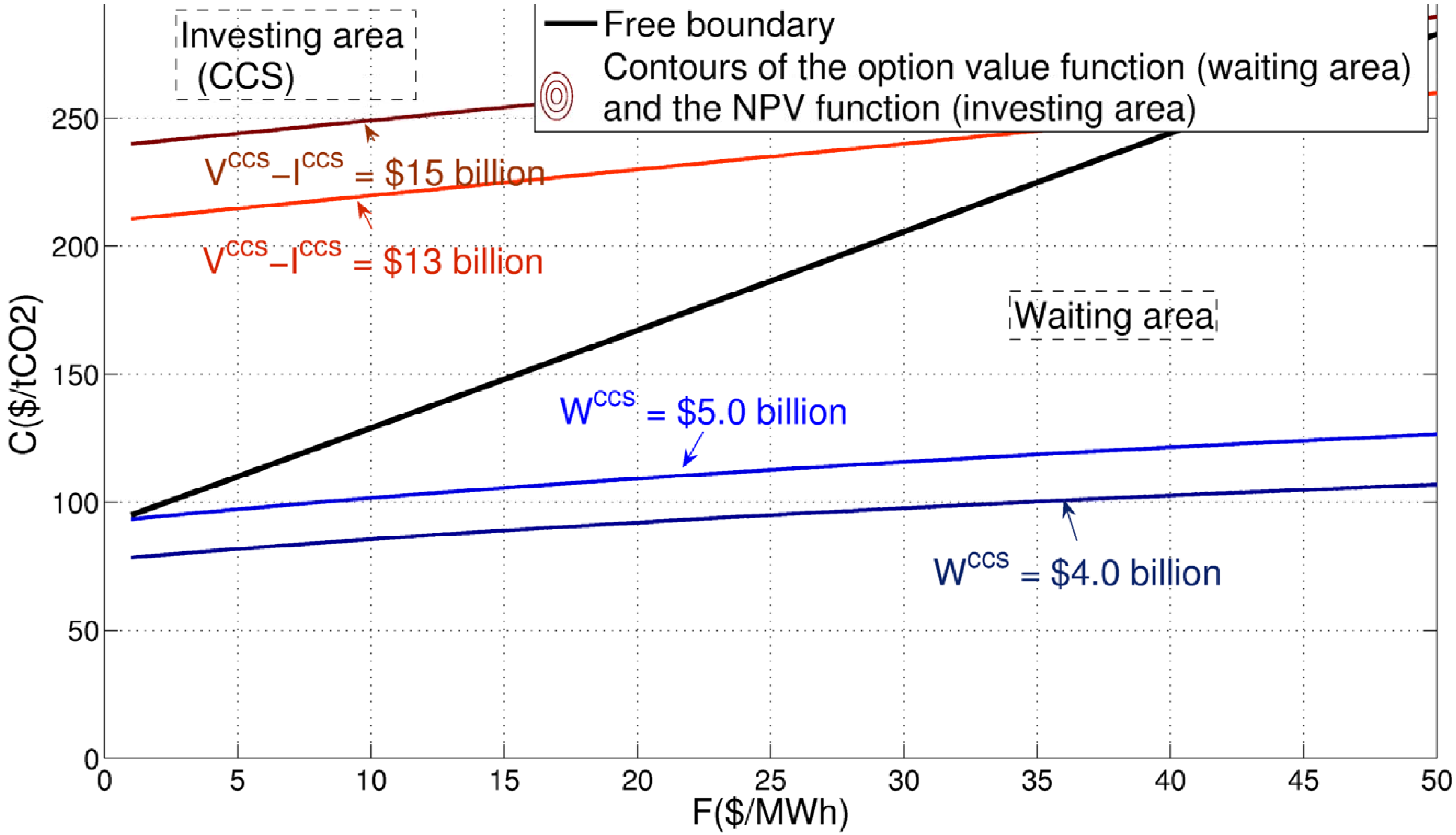
ϵ_F^{ccs}	ϵ_C^{ccs}	I^{ccs}
0.112	\$1.3 billion	

F_0	C_0
$\$15.50/MWh$	$\$31.81/t$

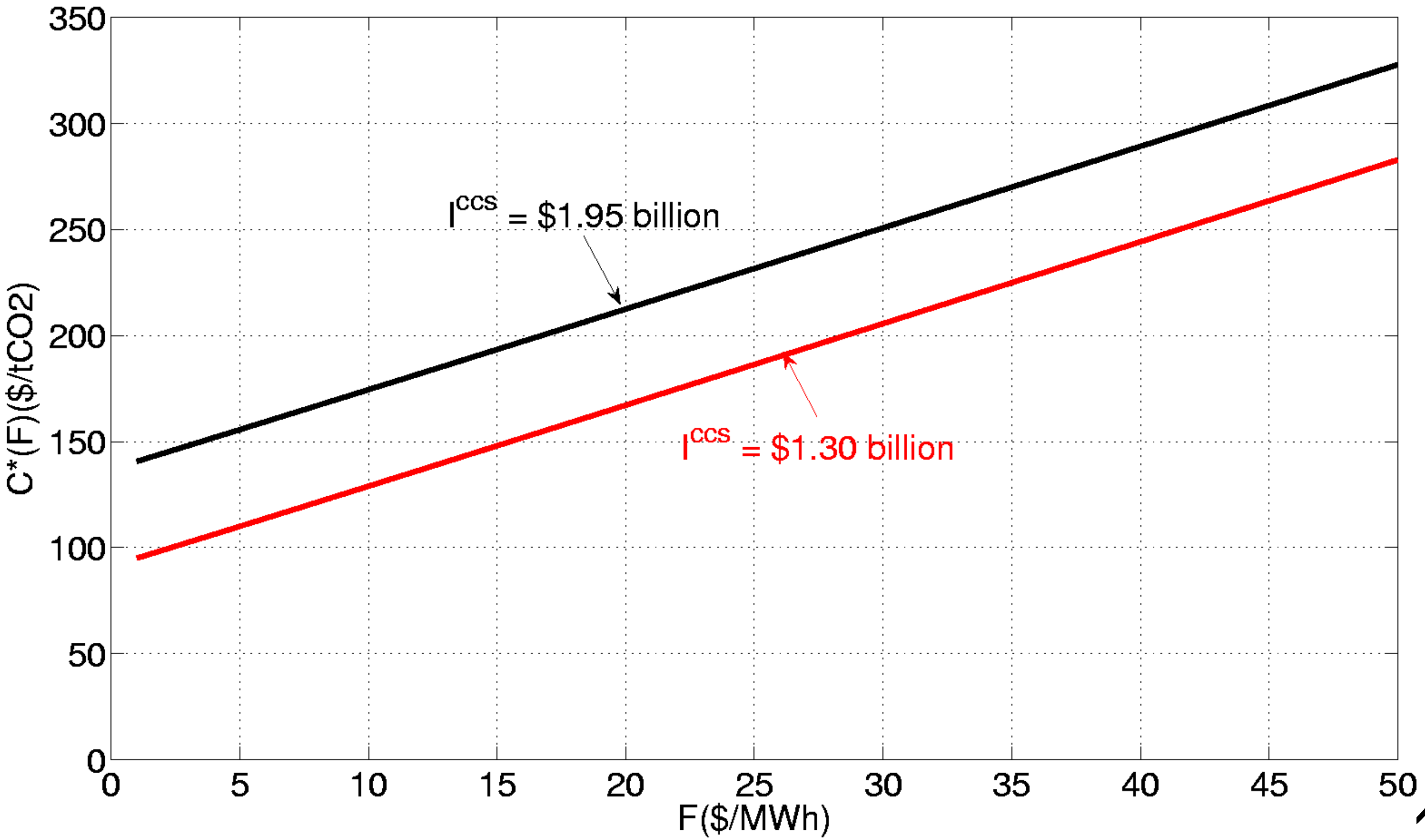
NUMERICAL EXAMPLE: CCS
Retrofit Option Values



NUMERICAL EXAMPLE: CCS Retrofit Thresholds



NUMERICAL EXAMPLE: Sensitivity Analysis



QUESTIONS