AALTO UNIVERSITY
Systems analysis laboratory

Mat-2.4136 Special Topics in Decision Making: Aggregation functions
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Exercise 5: Conjunctive and disjunctive aggregation functions

1. Prove that any t-norm has zero-divisor if and only if it has nilpotent elements (prop. 3.19 in the book)

2. Consider two t-norms T_1 and T_2. Prove that their convex linear combination, i.e. $\lambda T_1(x, y) + (1 - \lambda)T_2$ $\lambda \in [0, 1]$ is not, in general, a t-norm, but it is always a conjunctive aggregation function.

3. What t-norm do you generate if you use the additive generator $g(t) = \left(\frac{1}{t} - 1\right)^{\lambda}$? Here $\lambda > 0$ is a parameter determining the attitudinal character of the t-norm.

4. And if you use $\frac{1}{\lambda}\ln(1 + \lambda(1 - t))$ with $\lambda > -1$? Can you, for the t-norm obtained with this generator, and the one obtained in the previous point, find some special or limiting cases with respect to λ?

5. Given two t-norms T_1 and T_2, is it always possible to say $T_1 > T_2$ or $T_1 < T_2$ for all $(x, y) \in [0, 1]^2$? Prove it or find a counterexample.

6. Let T be a t-norm and S its dual t-conorm obtained by using a strong negation N. Show that the following laws hold:

   - $N(T(x, y)) = S(N(x), N(y))$ for all $x, y \in [0, 1]$
   - $N(S(x, y)) = T(N(x), N(y))$ for all $x, y \in [0, 1]$