## Assignment 7

You are purchasing an apartment based on 6 criteria (objectives). You have narrowed down to 4 candidates in different locations in Helsinki and Espoo (see Table 1). Criteria 1, 2 and 6 are to be minimized, 3 and 5 to be maximized. Sauna is a preferred feature.

Table 1: Apartments

| Criterion; Location | Lauttasaari | Munkkiniemi | Soukka | Leppävaara |
| :---: | :---: | :---: | :---: | :---: |
| 1 Distance to Otaniemi | 4 | 4 | 15 | 5 |
| 2 Price | 150000 | 150000 | 130000 | 140000 |
| 3 Area | 45 | 45 | 60 | 60 |
| 4 Sauna | no | yes | no | yes |
| 5 Rooms | 2 | 2 | 3 | 2 |
| 6 Distance to Kamppi | 3 | 5 | 20 | 12 |

a) Solve the problem through lexicographic ordering. In Table 1, the criteria are listed in their order of importance (Distance to Otaniemi is the most important). Which apartment is selected?
b) Solve the problem using an additive value function: value of alternative $x^{j}$ is $V\left(x^{j}\right)=$ $\sum_{i=1}^{6} w_{i} v_{i}\left(x_{i}^{j}\right)$ where $x_{i}^{j}$ is the performance of alternative $x^{j}$ with regard to $i$ :th criterion. Use linear criterion-specific value functions $v_{i}$ such that the worst and best performances are given values 0 and 1 , respectively, i.e., $v_{i}\left(x_{i}^{*}\right)=1$ and $v_{i}\left(x_{i}^{0}\right)=0$. Use rank order centroid weights, i.e., the weight of $i$-th most important criterion is $w_{i}=(1 / 6) \sum_{j=i}^{6}(1 / j)$. In Table 1, the criteria are listed in their order of importance (Distance to Otaniemi is the most important). Which apartment has the highest value now?
c) Use goal programming $\min \sum \tilde{w}_{i}\left|x_{i}^{j}-z_{i}\right|$ with the goal of an utopian apartment $z_{i}=x_{i}^{*}$. Find positive weights $\tilde{w}$ such that (i) they sum up to one and (ii) the goal programming model is consistent with the additive value function, i.e., $\sum_{i=1}^{6} w_{i} v_{i}\left(x_{i}^{j}\right)=\alpha \sum_{i=1}^{6} \tilde{w}_{i}\left|x_{i}^{j}-z_{i}\right|+\beta$ for some $\alpha$ and $\beta$, whenever $x_{i}^{j} \in\left[x_{i}^{0}, x_{i}^{*}\right] \forall i=1, \ldots, 6$ (i.e., the models must be consistent with each other for any alternative whose performances are within the range specified by the alternatives in the assignment). Which apartment has the smallest objective function value in the goal programming model?

