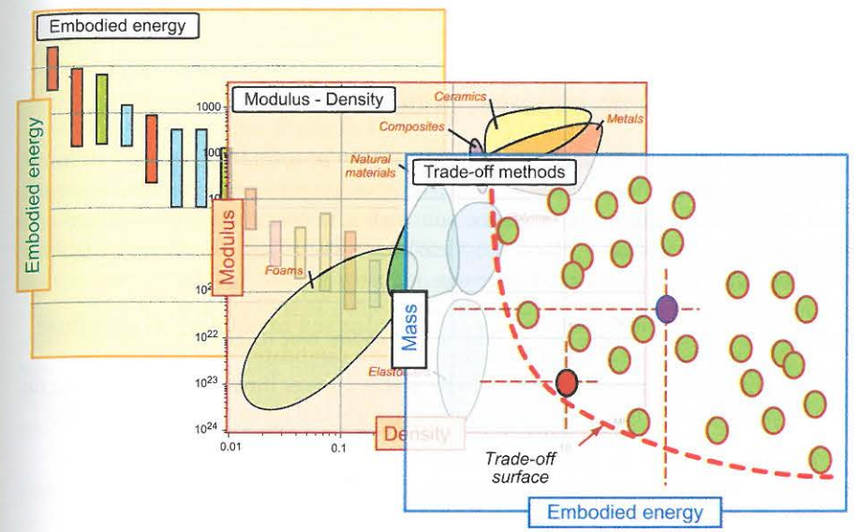
CHAPTER 9

Material selection strategies



CONTENTS

9.1 Introduction and synopsis
9.2 The selection strategy: choosing a car
9.3 Principles of materials selection
9.4 Selection criteria and property charts
9.5 Using indices for scaling
9.6 Resolving conflicting objectives: trade-off methods
9.7 Seven useful charts
9.8 Computer-aided selection
9.9 Summary and conclusions
9.10 Further reading
9.11 Appendix: deriving material indices
9.12 Exercises

9.1 Introduction and synopsis

Life is full of decisions. Which shoes to buy? Which restaurant to eat at? Which camera? Which bike? Which car? Which university? Most of us evolve strategies for reaching decisions, some involving emotional response ("It's cool/I just couldn't resist the color, and besides, Joe/Joanna has one"), others based on cold logic.

It is cold logic that we want here. The strategy then takes the following form:

- Assemble data for the characteristics of the thing you wish to select—make a *database*, mental or physical.
- Formulate the characteristics that the thing must have to satisfy your requirements—list the *constraints*. Those that meet the constraints become candidates for selection.
- Decide on the criterion you will use to rank the candidates that meet your constraints—choose and apply the *objective*.
- Research the top-ranked candidates more fully to satisfy yourself that nothing has been overlooked—seek *documentation*.

This chapter is about selecting materials using this constraints-objectives-documentation strategy. It follows naturally from the eco-audits of the previous

Materials and the Environment.
© 2013 Elsevier Inc. All rights reserved.

chapters—first the audit to identify the phase of life that matters most, then the selection to find the material that most effectively minimizes the eco-impact of that phase of life. We start by illustrating the strategy with a product rather than a material—the ideas are the same, but the material has added complications. We base the discussion around the selection of a car to meet given constraints and with two objectives, one of them that of minimizing carbon footprint.

9.2 The selection strategy: choosing a car

You need a new car. To meet your needs it must be a mid-sized four-door family sedan, gas-powered, and deliver at least 150 horsepower—enough to tow your sailboat. Given all of these requirements, you wish it to cost as little to own and emit as little CO_2 as possible (Figure 9.1, left-hand side). There are three *constraints* here, but they are not all of the same type.

- The requirements of *four-door family sedan* and *gasoline power* are *simple constraints*—a car **must** have these to be a candidate.
- The requirement of *at least* 150 hp places a lower limit but no upper one on power; it is a *limit constraint*—any car with 150 hp or more is acceptable.

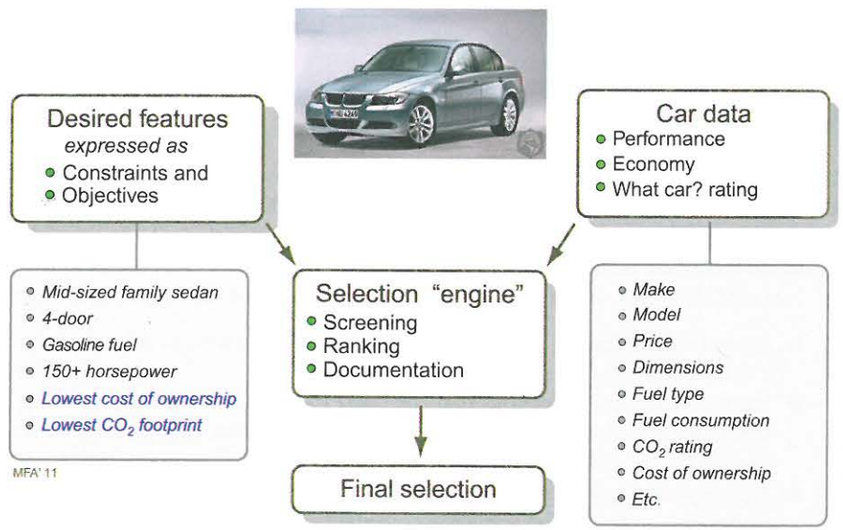


FIGURE 9.1 *Selecting a car. The requirements are expressed as constraints and objectives (objectives in blue). Records containing data for cars are* screened *using the constraints and* ranked *by the objectives to find the most attractive candidates. These are then explored further by examining* documentation.

The wish for minimum cost of ownership is an *objective*, a criterion of excellence. The most desirable cars, from among those that meet the constraints, are those that minimize this objective. The wish to minimize the CO_2 emission is a second objective, one that may not be compatible with the first.

To proceed, you need information about available cars (Figure 9.1, right-hand side). Car makers' web sites, dealers, car magazines, and advertisements in the national press list such information. It includes car type and size, number of doors, fuel type, engine power, and price; car magazines go further and estimate the cost of ownership, meaning the sum of running costs, taxes, insurance, servicing, and depreciation, listing it as \$/mile or €/km.

Now: decision time (Figure 9.1, central box). The selection engine (you, in this example) uses the constraints to *screen out*, from all the available cars, those that are not four-door gasoline-powered family sedans with 150 hp. Many cars meet these constraints; the list is still long. You need a way to order it so that the best choices are at the top. That is what the objective is for: it allows you to *rank* the surviving candidates by cost of ownership—those with the lowest values are ranked most highly. Rather than just choosing the one at the top, it is better to keep the top three or four and seek further *documentation*, exploring their other features in depth (delivery time, size of trunk, service frequency, security . . .) weighing the small differences in cost against the desirability of these features.

But we have overlooked a second objective, listed in blue on the left of Figure 9.1. You are an environmentally responsible person; you wish to minimize the CO_2 rating as well as the cost of ownership. Choosing to meet two objectives is more complicated than to meet just one. The problem is that the car that best satisfies one objective—minimizing cost, for example—may not be the one that minimizes the other—CO_2—and vice versa, so it is not possible to minimize both at the same time. A compromise has to be reached, and that needs *trade-off methods*.

Figure 9.2 shows the method. Its axes are the two objectives: cost of ownership and CO_2 rating. Suppose a friend has recommended a particular car, shown as an orange dot at the center of the diagram. Your research has revealed cars with combinations of cost and carbon shown by the other dots. Several—the purple ones in the upper right—have higher values of both; they lie in the "unacceptable" quadrant. Several—the blue ones—either have lower cost or lower carbon, but not both. One—the green one—is both cheaper *and* produces less carbon; it ranks more highly than the orange dot by both objectives. It is the obvious choice.

Or is it? That depends on the value you attach to a low carbon footprint. If you think it is a good idea so long as you don't have to pay a premium for it, then the car marked "Choice if cost matters most" is the best. If instead you are ready to pay whatever it takes to minimize CO_2 emission, then the car marked "Choice if carbon matters most" is the one to go for.

All three choices lie on the boundary of the occupied region of the figure together with several others that are compromises between them. The envelope of these—the broken line—is called the *trade-off line*. Cars that lie on or near this line have the best compromise combination of cost and carbon. So even if we can't reach a single

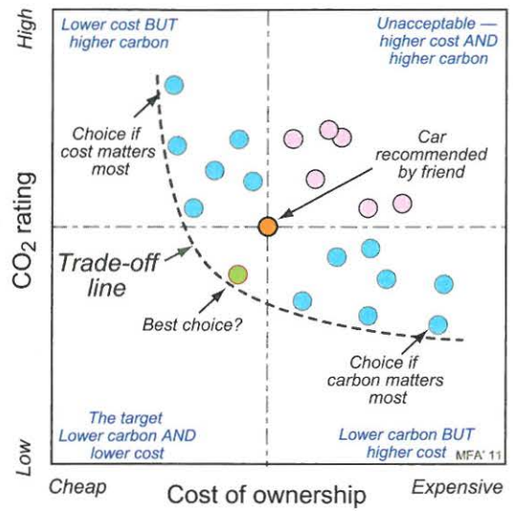


FIGURE 9.2 *The trade-off between carbon footprint and cost of ownership*

definitive choice (at least not yet) we have made progress. The viable candidates are those on or close to the trade-off line. All others are definitely less good.

Methods like this are used as a tool for decision making in many fields: in deciding between design options for new products, in optimizing the operating methods for a new plant, in guiding where the site of a new town will be—and in selecting materials. We turn to that next.

9.3 Principles of materials selection

Selecting materials involves seeking the best match between design requirements and the properties of the materials that might be used to make it. Figure 9.3 shows the strategy of the last section applied to selecting materials for a portable bike shed. On the left is the list of requirements that the material must meet, expressed as constraints and objectives. The constraints include the ability to be molded, weather resistance, adequate stiffness, and strength. The objectives include that it needs to be as light and as cheap as possible. On the right is the database of material attributes, drawn from suppliers' data sheets, handbooks, web-based sources, or software specifically designed for materials selection or from Chapter 15. The comparison "engine" applies the constraints on the left to the materials on the right and ranks the survivors using an objective, delivering a short list of viable candidates, just as with the cars. If two or more objectives are active, trade-off methods like the one in Figure 9.2 resolve the conflict.

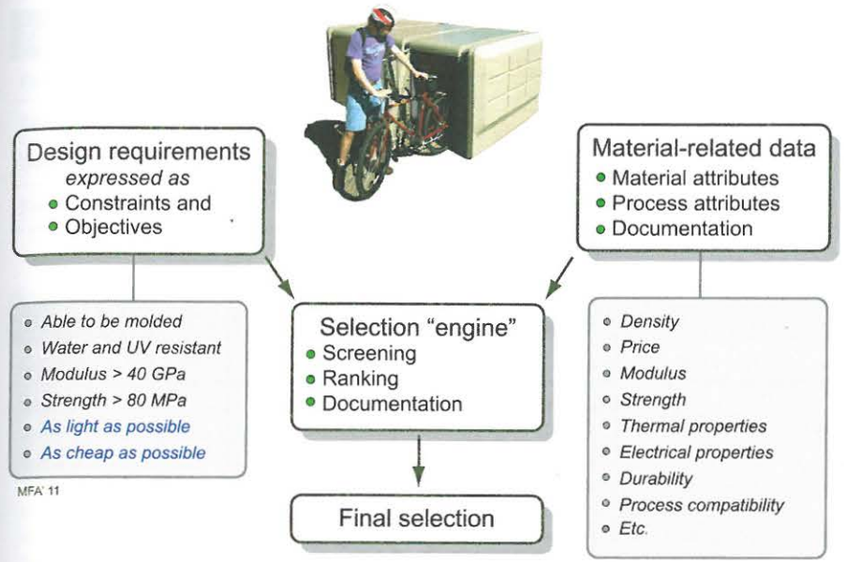


FIGURE 9.3 *Selecting a material for a portable bike shed. The requirements are expressed as constraints and objectives (objectives in blue). Records containing data for materials are* screened *using the constraints and* ranked *by the objectives to find the most attractive candidates. These are then explored further by examining* documentation.

There is, however, a complication. The requirements for the car were straightforward—doors, fuel type, power—all of these are explicitly listed by the manufacturer. The design requirements for a component of a product specify what it should *do* but not what *properties* its materials should have. So the first step is one of *translation*: converting the design requirements into constraints and objectives that can be applied to the materials database (Figure 9.4). The next task is that of *screening*, as with cars, eliminating the materials that cannot meet the constraints. This is followed by the *ranking* step, ordering the survivors by their ability to meet a criterion of excellence, such as that of minimizing cost, embodied energy, or carbon footprint. The final task is to explore the most promising candidates in depth, examining how they are used at present, case histories of failures, and how best to design with them, the step we called *documentation*. Now let's take a closer look at each step.

9.3.1 Translation. There is a story about a man—his name was Claude Shannon—who invented a product with only one function. It was a box with a switch on the front. When you pressed the switch, the box opened, a hand came

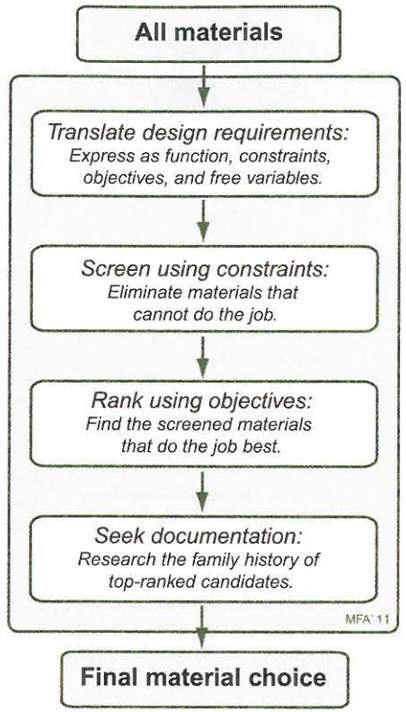


FIGURE 9.4 *The strategy. There are four steps: translation, screening, ranking, and documentation. All can be implemented in software, allowing large populations of materials to be investigated.*

out, switched off the switch, and went back inside. If you pressed the button after that, nothing happened. Just one function. Once.

Some products are like that: air bags, for example, or disposable diapers. But most of the things engineers design function more than once, and many have more than one function. Typical functions are to support a load, to contain a pressure, to transmit heat, to provide electrical insulation, and so forth. This must be achieved subject to *constraints*: that certain dimensions are fixed, that the component must carry the design loads without failure, must insulate against or conduct heat or electricity, must function safely in a certain range of temperature and in a given environment, and many more. In designing the component, the designer has one or more *objectives*: to make it as cheap as possible, perhaps, or as light, or as environmentally benign, or some combination of these. Certain parameters can be adjusted in order to optimally meet the objective—the designer is free to vary dimensions that are not constrained by design requirements and, most importantly, free to

Table 9.1 Function, constraints, objectives, and free variables

Function	What does the component do?
Constraints	What non-negotiable conditions must be met?
Objective	What is to be maximized or minimized?
Free variables	What parameters of the problem is the designer free to change?

Table 9.2 Examples of common constraints and objectives*

Common constraints	Common objectives
Must be	**Minimize**
■ Electrically conducting	■ Cost
■ Optically transparent	■ Mass
■ Corrosion resistant	■ Volume
■ *Non-toxic*	■ Thermal losses
■ *Non-restricted substance*	■ Electrical losses
■ *Able to be recycled*	■ *Resource depletion*
■ *Biodegradable*	■ *Energy consumption*
	■ *Carbon emissions*
Must meet a target value of	■ *Waste*
■ Stiffness	■ *Environmental impact*
■ Strength	■ *Water use*
■ Fracture toughness	
■ Thermal conductivity	
■ Service temperature	

**Environment-related constraints and objectives are italicized.*

choose the material for the component. We call these *free variables*. Constraints, objectives, and free variables (Table 9.1) define the boundary conditions for selecting a material and—in the case of load-bearing components—the choice of shape for its cross-section.

It is important to be clear about the distinction between constraints and objectives. A constraint is an essential condition that must be met, usually expressed as an upper or lower limit on a material property. An objective is a quantity for which an extreme value (a maximum or minimum) is sought, frequently the minimization of cost, mass, volume, or—of particular relevance here—environmental impact (Table 9.2).

The outcome of the translation step is a list of the design-limiting properties and the constraints they must meet. The first step in relating design requirements to material properties is therefore a clear statement of function, constraints, objectives, and free variables.

Translation of the design requirements for the helmet visor

Example: A material is required for the visor of a safety helmet to provide maximum facial protection.

Translation: To allow clear vision the visor must be *optically transparent*. To protect the face from the front, from the sides, and from below, it must be doubly curved, requiring that the material can *be molded*. We thus have two constraints: transparency and ability to be molded.

Fracture of the visor would expose the face to damage: "maximizing facial protection" therefore translates into maximizing resistance to fracture. The material property that measures resistance to fracture is the *fracture toughness*, K_{1c}. The objective is therefore to maximize K_{1c}.

9.3.2 Screening. Constraints are gates: meet the constraint and you pass through the gate, fail to meet it and you are shut out. Screening does just that: it eliminates candidates that cannot do the job at all because one or more of their attributes lies outside the limits set by the constraints. As examples, the requirement that "the component must function in boiling water" or that "the component must be non-toxic" imposes obvious limits on the attributes of maximum service temperature and toxicity that successful candidates must meet. The left-hand column of Table 9.2 lists common constraints.

9.3.3 Ranking: material indices. To rank the materials that survive the screening step we need criteria of excellence—what we have called objectives. The right-hand column of Table 9.2 lists common objectives. Each is a measure of performance. Performance is sometimes limited by a single property, sometimes by a combination of them. Thus the best materials to minimize thermal losses (an objective) are the ones with the smallest values of the thermal conductivity, λ. The best materials to minimize DC electrical losses (another objective) are those with the lowest electrical resistivity ρ_e—provided, of course, that they also meet all other constraints imposed by the design. Here the objective is met by selecting the material with an extreme (here, the lowest) value of a single property. Often, though, it is not one property but a group of properties that are relevant. Thus the best materials for a

light stiff tie-rod are those with the smallest value of the group, ρ/E, where ρ is the density and E is Young's modulus. Those for a strong beam of lowest embodied energy are those with the lowest value of $H_m\rho/\sigma_y^{2/3}$ where H_m is the embodied energy of the material and σ_y is its yield strength. The property or property group that maximizes performance for a given design is called its *material index*.

Table 9.3 lists indices for stiffness and strength-limited design for three generic components: a tie, a beam, and a panel, for each of five objectives. The first three relate to design for the environment. Selecting materials with the objective of minimizing volume uses as few materials as possible, conserving resources. Selecting with the objective of minimizing mass is central to the eco-design of transportation systems (or indeed of anything that moves) because fuel consumption for transportation scales with weight. Selecting with the objective of minimizing embodied energy is important when large quantities of material are used, as they are in construction of buildings, bridges, roads, and other infrastructure. The fifth column, selection with the objective of minimizing cost, is always with us. Table 9.4 lists indices for thermal design. The first is a single property, the thermal conductivity, λ; materials with the lowest values of λ minimize heat loss at steady state, that is, when the temperature gradient is constant. The other two guide

Table 9.3 Indices for stiffness and strength-limited design

	Configuration	Objective: to minimize				
		Volume	Mass	Embodied energy	Carbon footprint	Material cost
Stiffness-limited design	Tie	$1/E$	ρ/E	$H_m\rho/E$	$CO_2.\rho/E$	$C_m\rho/E$
	Beam	$1/E^{1/2}$	$\rho/E^{1/2}$	$H_m\rho/E^{1/2}$	$CO_2.\rho/E^{1/2}$	$C_m\rho/E^{1/2}$
	Panel	$1/E^{1/3}$	$\rho/E^{1/3}$	$H_m\rho/E^{1/3}$	$CO_2.\rho/E^{1/3}$	$C_m\rho/E^{1/3}$
Strength-limited design	Tie	$1/\sigma_y$	ρ/σ_y	$H_m\rho/\sigma_y$	$CO_2.\rho/\sigma_y$	$C_m\rho/\sigma_y$
	Beam	$1/\sigma_y^{2/3}$	$\rho/\sigma_y^{2/3}$	$H_m\rho/\sigma_y^{2/3}$	$CO_2.\rho/\sigma_y^{2/3}$	$C_m\rho/\sigma_y^{2/3}$
	Panel	$1/\sigma_y^{1/2}$	$\rho/\sigma_y^{1/2}$	$H_m\rho/\sigma_y^{1/2}$	$C_m\rho/\sigma_y^{1/2}$	$CO_2.\rho/\sigma_y^{1/2}$

Density, ρ (kg/m^3); Elastic (Young's) modulus, E (GPa); Yield strength, σ_y (MPa); Carbon footprint CO_2 (kg/kg); Price, C_m ($/kg); Embodied energy/kg of material, H_m (MJ/kg)

Table 9.4 Indices for thermal design

	Objective: to minimize		
Objective	Steady-state heat loss	Thermal inertia	Heat loss in a thermal cycle
Index	λ	$C_p\rho$	$(\lambda C_p\rho)^{1/2}$

Thermal conductivity λ (W/m · K); Specific heat, C_p (J/kg · K); Thermal diffusivity, $a = \lambda/C_p\rho$ (m^2/s)

material choice when the temperature fluctuates. The symbols are defined under each table.

There are many such indices, each associated with maximizing some aspect of performance. They provide criteria of excellence that allow ranking of materials by their ability to perform well in the given application. Their derivation is described more fully in the appendix to this chapter and their use is illustrated by case studies in Chapter 10. All can be plotted on material property charts that identify the best candidates. The charts for the indices of Tables 9.3 and 9.4 appear later in this chapter (Section 9.6).

To summarize, then: *screening* uses constraints to isolate candidates that are capable of doing the job; *ranking* uses an objective to identify the candidates that can do the job best.

Screening and ranking for the helmet visor

Example: A search for transparent materials that can be molded delivers the following list. The first four are thermoplastics, the last two, glasses. Fracture toughness values come from the data sheets of Chapter 15.

Material	Average fracture toughness K_{1c} MPa · m$^{1/2}$
Polycarbonate (PC)	3.4
Cellulose acetate (CA)	1.7
Polymethyl methacrylate (Acrylic, PMMA)	1.2
Polystyrene (PS)	0.9
Soda-lime glass	0.6
Borosilicate glass	0.6

The constraints have reduced the number of viable materials to six candidates. When ranked by fracture toughness, the top-ranked candidates are PC, CA, and PMMA.

9.3.4 Documentation. The outcome of the steps so far is a ranked short list of candidates that meet the constraints and are ranked most highly by the objective. You could just choose the top-ranked candidate, but what hidden weaknesses might it have? What is its reputation? Has it a good track record? To proceed further, we seek a detailed profile of each: its documentation (Figure 9.4, bottom).

What form does documentation take? Typically, it is descriptive, graphical, or pictorial: case studies of previous uses of the material, failure analyses, details of its corrosion behavior in particular environments, of its availability and pricing, warnings of its environmental impact or toxicity, or descriptions of how it is recycled.

Such information is found in handbooks, suppliers' data sheets, web sites of environmental agencies, and other high-quality web sites. Documentation helps narrow the short list to a final choice, allowing a definitive match to be made between design requirements and material choice.

Why are all these steps necessary? Without screening and ranking, the candidate pool is enormous and the volume of documentation is overwhelming. Dipping into it, hoping to stumble on a good material, gets you nowhere. But once a small number of potential candidates have been identified by the screening and ranking steps, detailed documentation can be sought for these few alone, and the task becomes viable.

Documentation for materials for the helmet visor

Example: At this point it helps to know how the three top-ranked candidates listed in the previous Examples box are used. A quick web search reveals the following.

Polycarbonate: Safety shields and goggles; lenses; light fittings; safety helmets; laminated sheet for bulletproof glazing.

Cellulose acetate: Eyeglasses frames; lenses; goggles; tool handles; covers for television screens; decorative trim and steering wheels for cars.

PMMA, Plexiglas: Lenses of all types; cockpit canopies and aircraft windows; containers; tool handles; safety spectacles; lighting; automotive tail lights.

This is encouraging: all three materials have a history of use for goggles and protective screening. The one that ranked highest in our list—polycarbonate—has a history of use for protective helmets. We select this material, confident that, with its high fracture toughness, it is the best choice.

9.4 Selection criteria and property charts

Material property charts were introduced in Chapter 6. They are of two types: *bar charts* and *bubble charts*. A bar chart is simply a plot of one or a group of properties—Chapter 6 has several of them. Bubble charts plot two properties or groups of properties. Constraints and objectives can be plotted on them.

Screening: constraints on charts. As we have seen, design requirements impose non-negotiable demands ("constraints") on the material of which it is made. These limits can be plotted as horizontal or vertical lines on material property charts. Figures 9.5 and 9.6 show two schematic examples. The first is a bar chart of embodied energy. A selection line has been placed to impose the limit *embodied energy* <10 MJ/kg; all the materials below the line meet the constraint. The second shows a schematic of the *modulus-density* chart. We suppose that the design imposes limits on these of *modulus* >10 GPa and *density* <2,000 kg/m^3, shown

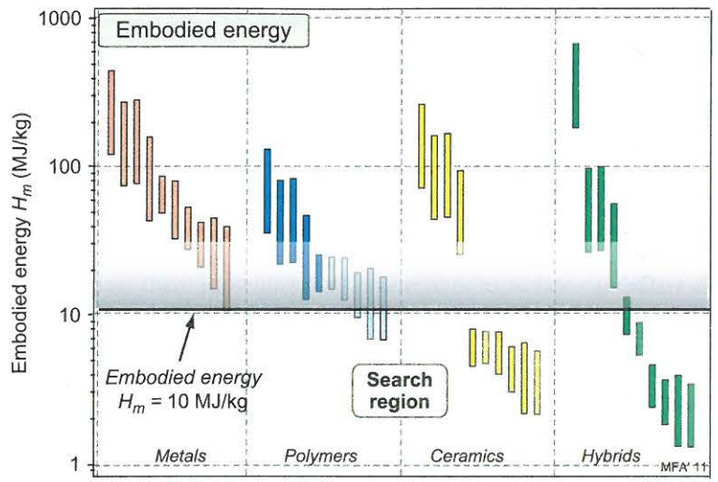


FIGURE 9.5 *Screening using a bar chart. Here we seek materials with embodied energies less than 10 MJ/kg. The materials in the "Search region" below the selection line meet the constraint.*

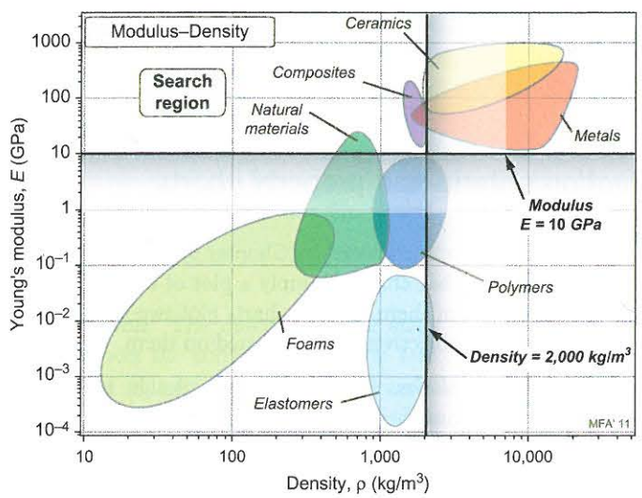


FIGURE 9.6 *Screening using a bubble chart. Two constraints are plotted: modulus >10 GPa and density <2,000 kg/m³. The materials in the "Search region" at the upper-left meet both constraints.*

on the figure. All materials in the window defined by the limits, labeled "Search region," meet both constraints.

Ranking: indices on charts. Material indices measure performance; they allow ranking of the materials that meet the constraints of the design. We use the design of light, stiff components as examples; the other material indices are used in a similar way.

Figure 9.7 shows a schematic of the $E-\rho$ chart shown earlier. The logarithmic scales allow all three of the indices—$M = \rho/E$, $\rho/E^{1/3}$ and $\rho/E^{1/2}$—listed in Table 9.3 of the previous section to be plotted onto it. Consider the first of these:

$$M = \frac{\rho}{E} \tag{9.1}$$

taking logs

$$\log(E) = \log(\rho) - \log(M) \tag{9.2}$$

For a given value of M, this is the equation of a straight line of slope 1 on a plot of $\log(E)$ against $\log(\rho)$, as shown on the figure. Similarly, the condition

$$M = \frac{\rho}{E^{1/3}} = \text{constant, } C \tag{9.3}$$

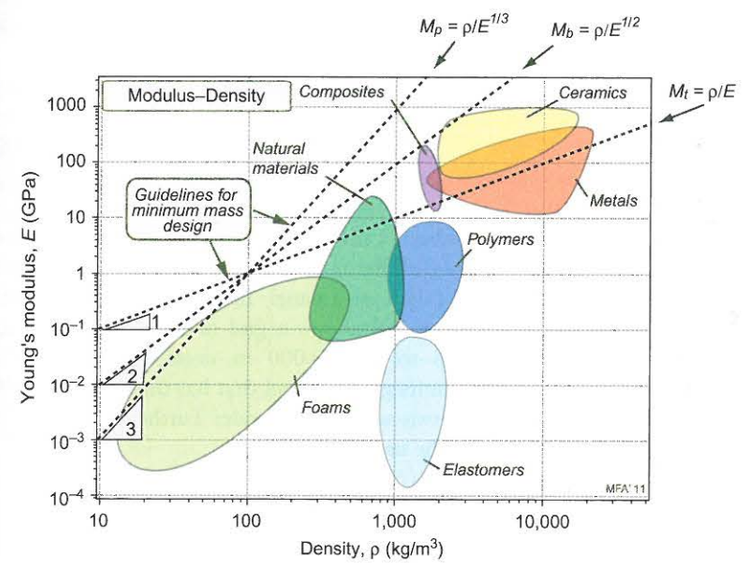


FIGURE 9.7 *A schematic E–ρ chart showing guidelines for three material indices for stiff, lightweight structures*

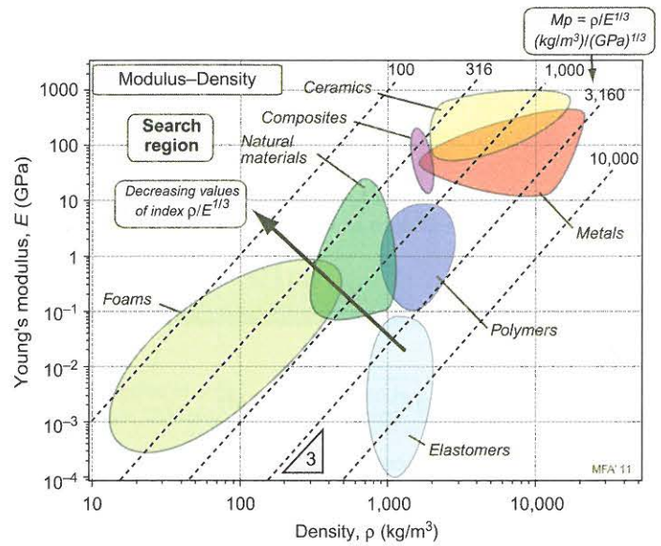


FIGURE 9.8 *A schematic E–ρ chart showing a grid of lines for the index $\rho/E^{1/3}$. The units are $(kg/m^3)/(GPa)^{1/3}/(kg/m^3)$.*

becomes, on taking logs,

$$\log(E) = 3\log(\rho) - 3\log(C) \tag{9.4}$$

This is another straight line, this time with a slope of 3, also shown. By inspection, the third index, $\rho/E^{1/2}$, will plot as a line of slope 2. We refer to these lines as *selection guidelines*. They give the slope of the family of parallel lines belonging to that index, each line corresponding to a different value of the index, M. Selection guidelines are marked on the charts that appear later in this chapter.

It is now easy to read off the subset of materials that maximize performance for each loading geometry. For example, all the materials that lie on a line of constant $M = \rho/E^{1/3}$ perform equally well as a light, stiff panel; those above the line perform better, those below, less well. Figure 9.8 shows a grid of lines corresponding to values of $M = \rho/E^{1/3}$ from $M = 100$ to $M = 10{,}000$ in units of $(\mathrm{kg \cdot m^{-3}})/\mathrm{GPa}^{1/3}$. A material with $M = 100$ in these units gives a panel that has one tenth the weight of one with $M = 1{,}000$. The first two texts listed under Further reading develop numerous case studies illustrating the use of the method.

9.5 Using indices for scaling

Most of the products we use today were designed when the dominant objectives were cost, performance, and safety. It is only now that the objective of minimizing

environmental impact has been added to this list. An eco-audit or full LCA of these products identifies the phase of life that is causing the most damage, and it often suggests that replacing the materials with which the product is made by a set that are lighter, or stronger, or have lower embodied energies, or are easier to recycle would reduce the eco-burden.

But substitution is not that simple. The density of aluminum is one third of that of steel, so you might think that the weight of an aluminum car body-in-white (BiW) would be one third of that of a steel BiW. But aluminum is less than half as stiff and (depending on the alloy and heat treatment) about half as strong as steel. If the BiW is to function as well after the substitution as it did before, the section thickness of the aluminum components must be increased to compensate for the lesser properties. Thick sections are heavier than thin ones, so the reduction in weight is not nearly as large as it at first appeared. And there is something else to remember: aluminum costs, per kg, three times more than steel. Substitution is seldom cost-neutral. What, then, are the scaling laws when one material is replaced by another? And what is the gain in eco-performance when property-compensation is properly included? Indices can tell us.

The factor by which the mass of a tie, beam, or panel is changed by substitution is given by the ratio of the index for the new material to that of the old one. The factors by which embodied energy, carbon footprint, or material cost are changed by substitution are similarly given by the ratios of the relevant indices, giving the scaling factors in Table 9.5. The index describing the performance of a material as

Table 9.5 Scaling laws for stiffness- and strength-limited design

	Configuration	Eco-performance gain by substitution			
		Volume*	Mass*	Embodied energy*	Material cost*
Stiffness-limited design	Tie	$\left(\frac{E_o}{E_1}\right)$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{E_o}{E_1}\right)$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)$
	Beam	$\left(\frac{E_o}{E_1}\right)^{1/2}$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/2}$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/2}$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/2}$
	Panel	$\left(\frac{E_o}{E_1}\right)^{1/3}$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/3}$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/3}$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/3}$
Strength-limited design	Tie	$\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)$
	Beam	$\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{2/3}$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{2/3}$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{2/3}$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{2/3}$
	Panel	$\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{1/2}$	$\frac{\rho_1}{\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{1/2}$	$\frac{H_{m,1}\rho_1}{H_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{1/2}$	$\frac{C_{m,1}\rho_1}{C_{m,o}\rho_o}\cdot\left(\frac{\sigma_{y,o}}{\sigma_{y,1}}\right)^{1/2}$

** The subscript "o" refers to the original material; the subscript "1" refers to the substitute. The scaling laws for carbon footprint are the same as those for embodied energy with H_m replaced by CO_2.*

Table 9.6 Scaling laws for thermal design

Objective	Eco-performance gain by substitution		
	Fixed steady-state heat loss*	Volume for given heat capacity*	Heat loss in a thermal cycle*
Scaling law	$\frac{t_1}{t_o} = \frac{\lambda_1}{\lambda_o}$	$\frac{V_1}{V_o} = \frac{C_{p,1}\,\rho_1}{C_{p,o}\,\rho_o}$	$\frac{Q_1}{Q_o} = \left(\frac{\lambda_1\,C_{p,1}\,\rho_1}{\lambda_o\,C_{p,o}\,\rho_o}\right)^{1/2}$

Wall thickness t, (m); Volume V, (m^3); Heat Q (kJ)

thermal insulation is simply the thermal conductivity, λ (Table 9.4). The factor by which the insulation thickness must change to maintain the same level of heat loss when a new material is chosen is given by the ratio of the index for the new and the old. Change in thermal mass and thermal loss in a heat cycle, similarly, scale with the ratios of the indices, giving the scaling laws in Table 9.6.

Weight savings by materials substitution

Example: A steel beam, loaded in bending, is to be replaced by an aluminum one to save weight. The beam stiffness must remain unchanged. What is the maximum potential weight savings that this substitution allows? Here are the material properties.

Material	Density ρ (kg/m^3)	Modulus E (GPa)
Steel	7,850	210
Aluminum	2,710	70

Answer: From Table 9.5, the ratio of the mass after substitution to that before is

$$\frac{m_1}{m_o} = \frac{\rho_1}{\rho_o}\cdot\left(\frac{E_o}{E_1}\right)^{1/2} = 0.6$$

Here the subscript "o" refers to steel, the subscript "1" to aluminum. Inserting the data gives the ratio 0.6, meaning that the maximum possible weight saving is 40% rather than the factor of 3 that the ratio of the densities suggests.

Volume savings by material substitution

Example: Standard polystyrene foam is used as thermal insulation for a small refrigerator. The foam has a thermal conductivity $\lambda_o = 0.035$ W/m · °C. It is suggested that the same thermal performance can be had with thinner walls (increasing the useful volume) by using a polymethacrylimide foam instead, which has a thermal conductivity $\lambda_1 = 0.028$ W/m · °C. By what factor can the wall thickness be reduced by this substitution while still maintaining the same thermal performance as before?

Answer: From Table 9.6 the ratio of wall thicknesses that will give the same heat loss per unit area is

$$\frac{t_1}{t_o} = \frac{\lambda_1}{\lambda_o} = 0.8$$

Thus the substitution allows the walls to be made 20% thinner than before.

9.6 Resolving conflicting objectives: trade-off methods

Just as with cars, real-life materials selection almost always requires that a compromise be reached between conflicting objectives. Table 9.2 lists nine of them, and there are more. The choice of materials that best meets one objective will not usually be that which best meets the others; the lightest material, for instance, will generally not be the cheapest or the one with the lowest carbon footprint. To make any progress, the designer needs a way of trading mass against cost and both against carbon footprint. This section describes ways of resolving this and other conflicts of objective.

Such conflicts are not new; engineers have sought methods to overcome them for at least a century. The traditional approach is that of using experience and judgment to assign *weight factors* to each constraint and objective, using them to guide choice in the way summarized below.

Weight factors. Weight factors seek to quantify judgment. The method works like this. The key properties or indices are identified and their values, M_i, are tabulated for promising candidates. Since their absolute values can differ widely and depend on the units in which they are measured, each is first scaled by dividing it by the largest index of its group, $(M_i)_{max}$, so that the largest, after scaling, has the value 1. Each is then multiplied by a weight factor, w_i, with a value between 0 and 1, expressing its relative importance for the performance of the component. This gives a weighted index W_i:

$$W_i = w_i \frac{M_i}{(M_i)_{max}} \tag{9.5}$$

For properties that are to be minimized, like corrosion rate, the scaling uses the minimum value $(M_i)_{min}$, expressed in the form

$$W_i = w_i \frac{(M_i)_{min}}{M_i} \tag{9.6}$$

The weight factors w_i are chosen so that they add up to 1, that is: $w_i < 1$ and $\Sigma w_i = 1$. The most important property is given the largest w, the second most important, the second largest, and so on. The W_i's are calculated from equation (9.5) and (9.6) and summed. The best choice is the material with the largest value of the sum

$$W = \Sigma_i W_i \tag{9.7}$$

Sounds simple, but there are problems, some obvious, like that of subjectivity in assigning the weights, some more subtle. Experienced engineers can be good at assessing relative weights, but the method nonetheless relies on judgment, and judgments can differ. For this reason, the rest of this section will focus on systematic methods.

Systematic trade-off strategies. Consider the choice of material to minimize both mass (performance metric P_1) and cost (performance metric P_2) while also meeting a set of constraints such as a required strength or durability in a certain environment. Following the standard terminology of optimizations theory, we define a *solution* as a viable choice of material, meeting all the constraints but not necessarily optimal by either of the objectives. Figure 9.9 is a plot of P_1 against P_2 for alternative solutions, each bubble describing a solution. The solutions that minimize P_1 do not minimize P_2, and vice versa. Some solutions, such as that at **A**, are far from optimal—all the solutions in the box attached to it have lower values of both P_1 and P_2. Solutions like **A** are said to be *dominated* by others. Solutions like those at **B** have the characteristic that no other solutions exist with lower values of *both* P_1 and P_2. These are said to be *non-dominated* solutions. The line or surface on which they lie is called the non-dominated or optimal *trade-off line*. The values of P_1 and P_2 corresponding to the non-dominated set of solutions are called the *Pareto set*.

Just as with cars (see Figure 9.2), the solutions on or near the trade-off line offer the best compromise; the rest can be rejected. Often, this is enough to identify a short list, using intuition to rank them. When it is not, the strategy is to define a *penalty function*.

Penalty functions. Consider first the case in which one of the objectives to be minimized is cost, C (units: \$), and the other is mass, m (units: kg). We define a locally linear penalty function[1] Z:

$$Z = C + \alpha_m m \tag{9.8}$$

[1]Also called a *value function* or *utility function*. The method allows a local minimum to be found. When the search space is large, it is necessary to recognize that the values of the exchange constants, α_i, may themselves depend on the values of the performance metrics, P_i.

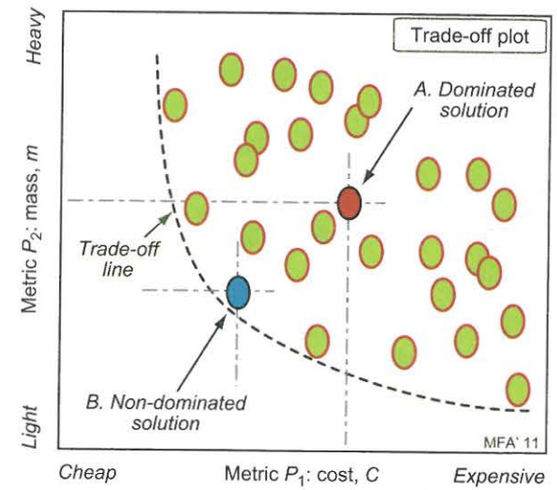


FIGURE 9.9 *Multiple objectives. Mass and cost for a component made from alternative material choices. The trade-off surface links non-dominated solutions.*

Here α_m is the change in Z(\$) associated with unit increase in m (kg) and has the units of \$/kg. It is called an *exchange constant*. Rearranging gives:

$$m = -\frac{1}{\alpha_m} C + \frac{1}{\alpha_m} Z \tag{9.9}$$

This defines a linear relationship between m and C that plots as a family of parallel penalty lines, each for a given value of Z, as shown in Figure 9.10. The slope of the lines is the negative reciprocal of the exchange constant, $-1/\alpha_m$. The value of Z decreases toward the bottom left: the best choices lie there. The optimum solution is the one nearest the point at which a penalty line is tangential to the trade-off line, since it is the one with the smallest value of Z.

If instead the two objectives were cost (\$) and carbon footprint CO_2 (kg), the penalty function becomes

$$Z = C + \alpha_c \, CO_2 \tag{9.10}$$

Here the exchange constant, α_c, is the change in Z for unit increase in CO_2, and thus again has the units of \$/kg. Contours of Z can be plotted on a chart of mass versus carbon footprint, just as in the previous example.

When all three objectives are active, the penalty function becomes

$$Z = C + \alpha_m \cdot m + \alpha_c \, CO_2 \tag{9.11}$$

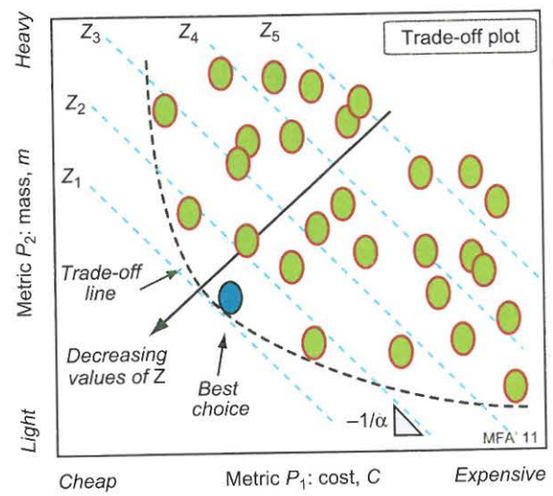


FIGURE 9.10 *The penalty function Z superimposed on the trade-off plot. The contours of Z have a slope of $-1/\alpha$. The contour that is tangent to the trade-off surface identifies the optimum solution.*

It can no longer be plotted as contours on a two-dimensional chart, but it can be evaluated for candidate materials, choosing the one that minimizes Z. To do this we need values for α_m and α_c.

Values for the exchange constants α_m and α_c. An exchange constant is the value or "utility" of a unit change in a performance metric. In the example we have just seen, α_m is the utility ($) of saving 1 kg of weight. Its magnitude depends on the application. Thus the utility of weight savings in a family car is small, though significant; in aerospace it is much larger. The utility of heat transfer in house insulation is directly related to the cost of the energy used to heat the house; that in a heat-exchanger for electronics can be much higher because high heat transfer allows faster data processing, something worth far more. The utility can be real, meaning that it measures a true saving of cost. But it can also, sometimes, be perceived, meaning that the consumer, influenced by scarcity, advertising, or fashion, will pay more or less than the true value of the performance metric.

In many engineering applications the exchange constants can be derived approximately from technical models for the life-cost of a system. Thus the utility of weight savings in transportation systems is derived from the value of the fuel saved or that of the increased payload, evaluated over the life of the system. Table 9.7 gives approximate values for α for various modes of transportation. The most striking thing about them is the enormous range: the exchange constant depends in a dramatic way on the application in which the material will be used. It

Table 9.7 Exchange constants α_m for the mass–cost trade-off for transport systems

Sector: transport systems	Basis of estimate	Exchange constant, α_m (US$/kg)
Family car	Fuel saving	1–2
Truck	Payload	5–20
Civil aircraft	Payload	100–500
Military aircraft	Payload, performance	500–1,000
Space vehicle	Payload	3,000–10,000

is this that lies behind the difficulty in adopting aluminum alloys for cars despite their universal use in aircraft, it explains the much greater use of titanium alloys in military than in civil aircraft, and it underlies the restriction of beryllium (a very expensive metal) to use in space vehicles.

Exchange constants can be estimated approximately in various ways. The cost of launching a payload into space lies in the range $3,000 to $10,000/kg; a reduction of 1 kg in the weight of the launch structure would allow a corresponding increase in payload, giving the ranges of α_m shown in the table. Similar arguments based on increased payload or decreased fuel consumption give the values shown for civil aircraft, commercial trucks, and automobiles. The values change with time, reflecting changes in fuel costs, legislation to increase fuel economy, and the like.

These values for the exchange constant are based on engineering criteria. More difficult to assess are those based on perceived value. That for the performance/cost trade-off for cars is an example. To the enthusiast, a car that is able to accelerate rapidly is alluring. He (or she) is prepared to pay more to go from 0 to 60 mph in 5 seconds than to wait around for 10, as we will see in Chapter 10.

There are other circumstances in which establishing the exchange constant can be more difficult. An example is that for *environmental impact*—the damage to the environment caused by manufacture, or use, or disposal of a given product. Minimizing environmental impact has now become an important objective, almost as important as minimizing cost. Ingenious design can reduce the first without driving the second up too much. But how much is a unit decrease in impact worth?

Exchange constants for eco-design. One outcome of the Kyoto Protocol (see Chapter 5) was the creation of a market in carbon permits. A carbon permit to emit 1 metric ton of CO_2 per year sells today (March 2011) for €17 ($24, or £15). Not all emitters of carbon have to hold carbon permits—ordinary households, for example, are exempt—but those that do not are now threatened with a carbon tax, at present aiming for €20 per metric ton). The permit price or tax set a value for the carbon-to-currency exchange constant, α_c, important because the penalty function can now be evaluated, allowing a properly balanced trade-off.

Using penalty functions

Example (1): The exchange constant for weight savings in a light goods vehicle is $\alpha_m = \$12/\text{kg}$, meaning that the value of weight reduction over the life of the vehicle is \$12 for each kilogram saved. A maker of such vehicles offers three models. The first uses steel panels for the body work. The second uses aluminum, costs \$2,500 more, but weighs 300 kg less. The third offers carbon-fiber paneling, costs \$8,000 more and weighs 500 kg less. Which is the best buy?

Answer: The penalty functions for the steel (1) and aluminum (2) vehicles are

$$Z_1 = C_1 + \alpha_m m_1$$

and

$$Z_2 = C_2 + \alpha_m m_2$$

The aluminum vehicle is attractive only if its value of Z is lower than that of the steel one. Writing

$$\Delta Z = Z_2 - Z_1 = C_2 - C_1 + \alpha_m(m_2 - m_1)$$
$$= 2{,}500 - 12 \times 300 = -\$\,1{,}100$$

The aluminum-paneled vehicle offers a life saving of \$1,100—it is a good buy. Repeating the comparison for the composite-paneled vehicle gives a value of $\Delta Z = +\$2{,}000$. It is not a good buy.

Example (2): A European-wide carbon tax is planned. It will be set initially at around \$20/metric ton (\$0.02/kg) of carbon emitted. If this is applied to material production, will it result in a significant rise in material prices? Express the result as a % increase above the current untaxed price.

Answer: Form a penalty function using $\alpha_c = 0.02$ \$/kg as the carbon-to-price exchange constant:

$$Z = C_m + 0.02 \times CO_2$$

where C_m is the material price in \$/kg and CO_2 is its carbon footprint in kg/kg. Rearranging to express Z as a % increase in C_m gives

$$\left(\frac{Z - C_m}{C_m}\right) \times 100 = \frac{2 \times CO_2}{C_m}$$

This quantity is plotted in the chart in Figure 9.11. The tax results in a 20% increase in the price of cement, a 10% increase in the price of aluminum and magnesium and smaller, but still significant increases in the price of the other materials.

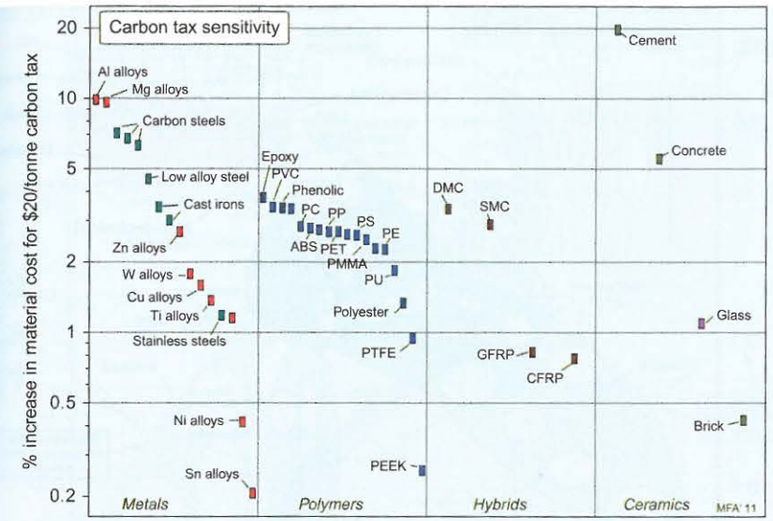


FIGURE 9.11 *Carbon tax sensitivity*

9.7 Seven useful charts

Seven material property charts guide materials selection to minimize mass, material embodied energy, material carbon footprint, and thermal losses using the indices of Tables 9.3 and 9.4. They are five of a much larger collection that can be found in the texts listed under Further reading at the end of this chapter.[2]

The Modulus–Density chart (Figure 9.12). The modulus E of engineering materials spans seven decades,[3] from 0.0001 GPa to nearly 1000 GPa; the density ρ spans a factor of 2,000, from less than 0.01 to 20 Mg/m^3. Members of each family cluster together and can be enclosed in envelopes, each of which occupies a characteristic part of the chart. The members of the ceramics and metals families have high moduli and densities; none has a modulus less than 10 GPa or a density less than 1.7 Mg/m^3. Polymers, by contrast, all have moduli below 10 GPa and densities that are lower than those of any metal or ceramic—most are close to 1 Mg/

[2]Some of charts can be downloaded, free, from grantadesign.com/education.

[3]Very low-density foams and gels (which can be thought of as molecular-scale, fluid-filled foams) can have lower moduli than this. As an example, gelatin (as in Jello) has a modulus of about 10^{-5} GPa.

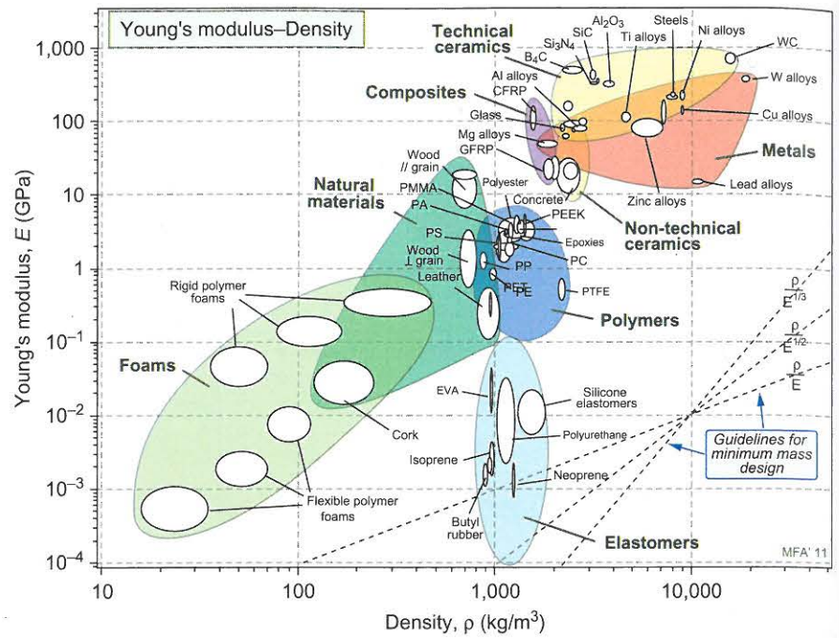


FIGURE 9.12 *The Modulus–Density chart: the one for stiffness at minimum weight*

m^3. Elastomers have roughly the same density as other polymers but their moduli are lower by a further factor of 100 or more. Materials with a lower density than polymers are porous: man-made foams and natural cellular structures like wood and cork.

This chart lets you select materials to minimize the mass of stiffness-limited structures. To do that, you need the three indices for the lightweight, stiffness-limited design in Table 9.3. Guidelines showing the slope of each of these are plotted on the chart. You might think that most structures are strength-, not stiffness-, limited, but that is wrong. Stiffness determines not only elastic deflection under load, but also vibration frequencies and resistance to buckling. When a hard-top vehicle line is augmented with an open-top model, its structure is beefed up to maintain stiffness, not strength.

The Strength–Density chart (Figure 9.13). The range of the yield strength σ_y or elastic limit σ_{el} of engineering materials, like that of the modulus, spans about six decades: from less than 0.01 MPa for foams, used in packaging and energy-absorbing systems, to 10^4 MPa for diamond, exploited in diamond tooling for machining and polishing. Members of each family again cluster together and can be enclosed in envelopes.

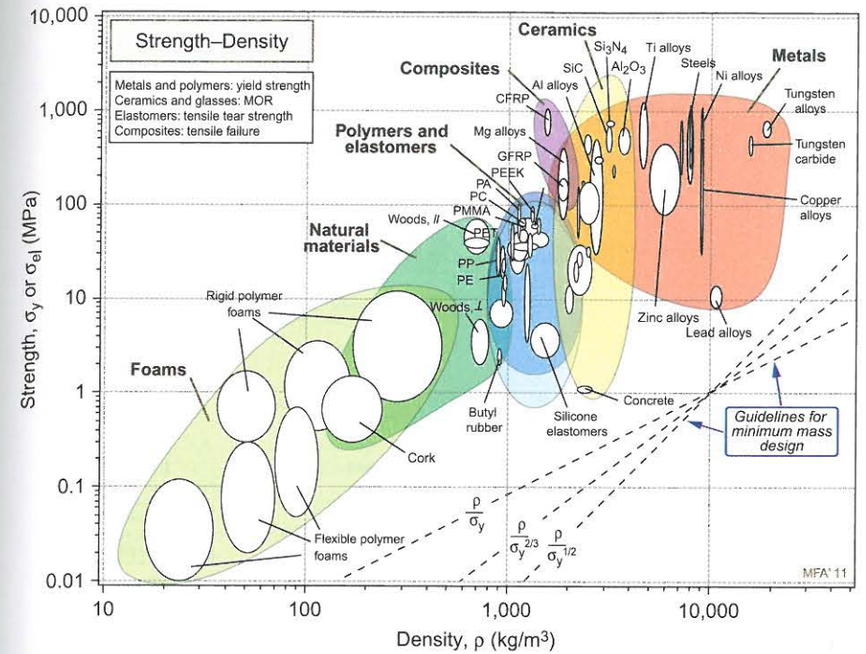


FIGURE 9.13 *The Strength–Density chart: the one for strength at minimum weight.*

Comparison with the Modulus-Density chart (Figure 9.12) reveals some marked differences. The modulus of a solid is a well-defined quantity with a narrow range of values. The yield strength is not. The strength range for a given class of metals, such as stainless steels, can span a factor of 10 or more, depending on its state of work hardening and heat treatment—it is this that leads to the elongated strength bubbles for metals. Polymers cluster together with strengths between 10 and 100 MPa. The composites CFRP and GFRP have strengths that lie between those of polymers and ceramics, as one might expect since they are mixtures of the two.

This chart is the one to select materials to minimize the mass of strength-limited structures. To do that you need the three indices for lightweight, strength-limited design in Table 9.3. Guidelines showing the slope of each of these are plotted on the chart.

The Modulus–Embodied energy and Strength–Embodied energy charts (Figures 9.14 and 9.15). The two charts just described guide design to minimize mass. If the objective becomes to minimize the energy embodied in the material of the product, we need equivalent charts for these.

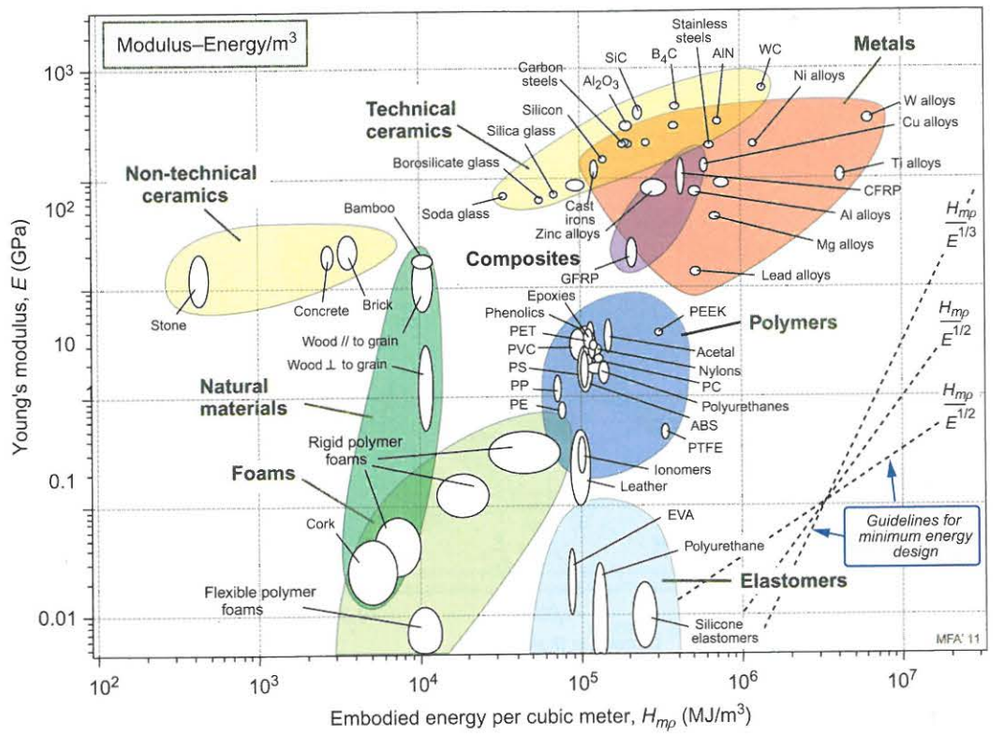


FIGURE 9.14 *The Modulus–Embodied energy chart: the one for stiffness at minimum embodied energy*

Figure 9.14 shows modulus E plotted against $H_m\rho$; the guidelines give the slopes for three of the commonest performance indices for stiffness-limited design at minimum embodied energy. Figure 9.15 shows strength σ_y plotted against $H_m\rho$. Guidelines give the slopes for strength-limited design at minimum embodied energy. They are used in exactly the same way as the $E-\rho$ and $\sigma_y-\rho$ charts for minimum mass design.

The Modulus–Carbon footprint and Strength–Carbon footprint charts (Figures 9.16 and 9.17). Two further charts allow optimized choice of when the objective is to minimize the carbon footprint of the material. The first, Figure 9.16, shows modulus plotted against carbon footprint per unit volume, $CO_2.\rho$, where CO_2 is the carbon footprint per kg of the material. The second, Figure 9.17, does the same for strength. Guidelines show the slopes associated with the indices of Table 9.3.

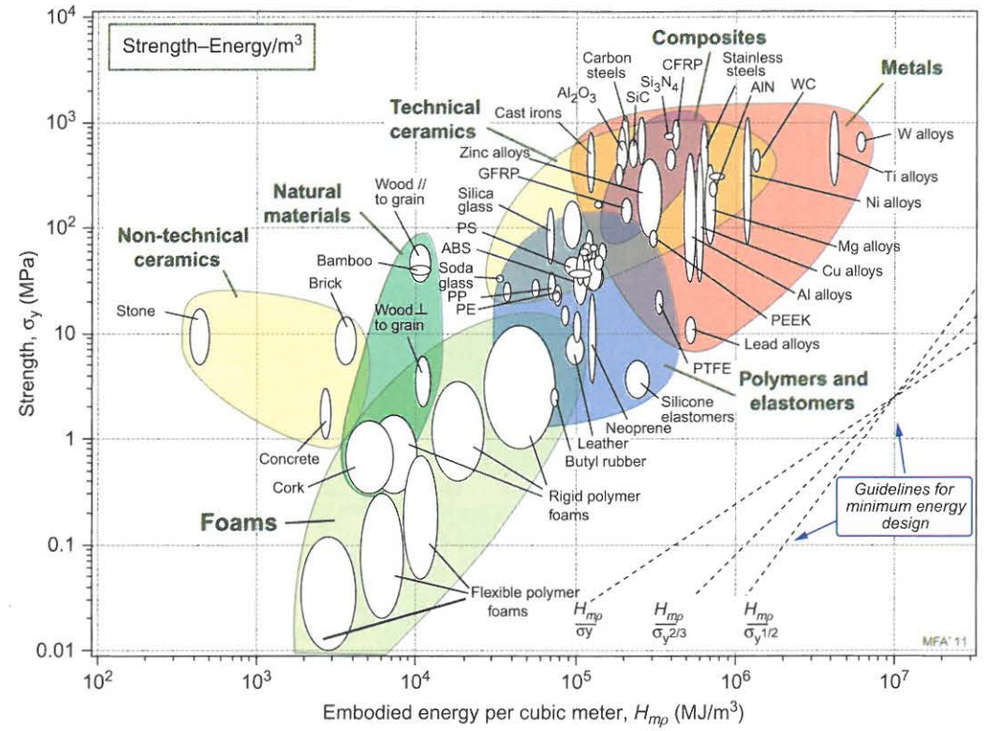


FIGURE 9.15 *The Strength–Embodied energy chart: the one for strength at minimum embodied energy*

The Thermal conductivity–Thermal diffusivity chart (Figure 9.18). The thermal conductivity, λ, is the material property that governs the flow of heat, q (W/m^2), in a steady temperature gradient dT/dx:

$$q = -\lambda \frac{dT}{dx} \tag{9.12}$$

The thermal diffusivity, a (m^2/s), is the property that determines how quickly a thermal front diffuses into a material. It is related to the conductivity

$$a = \frac{\lambda}{\rho C_p} \tag{9.13}$$

where ρC_p is the specific heat per unit mass (J/kg · K). The contours show the volumetric specific heat, ρC_p, equal to the ratio of the two, λ/α. The data span almost five decades in λ and a. Solid materials are strung out along the line

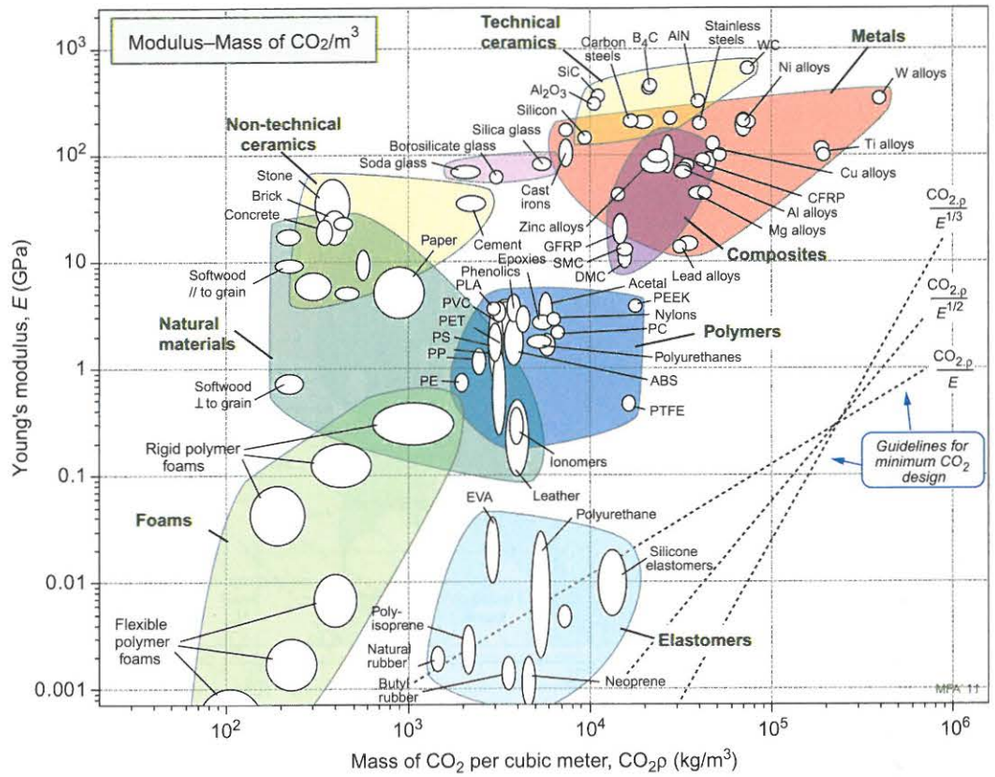


FIGURE 9.16 *The Modulus–Carbon footprint chart: the one for stiffness at minimum carbon release*

$$\rho C_p \approx 3 \times 10^6 \, J/m^3 \cdot K \tag{9.14}$$

meaning that the heat capacity per unit volume, ρC_p, is almost constant for all solids, something to remember for later. As a general rule, then,

$$\lambda = 3 \times 10^6 \, a$$

(λ in W/m·K and a in m²/s). Some materials deviate from this rule: they have lower-than-average volumetric heat capacity. The largest deviations are shown by porous solids: foams, low-density firebrick, woods, and the like. Because of their low density they contain fewer atoms per unit volume and, averaged over the volume of the structure, ρC_p is low. The result is that, although foams have low *conductivities* (and are widely used for insulation because of this), their *thermal diffusivities* are not necessarily low. This means that they don't transmit much heat, but they do heat up or cool down quickly.

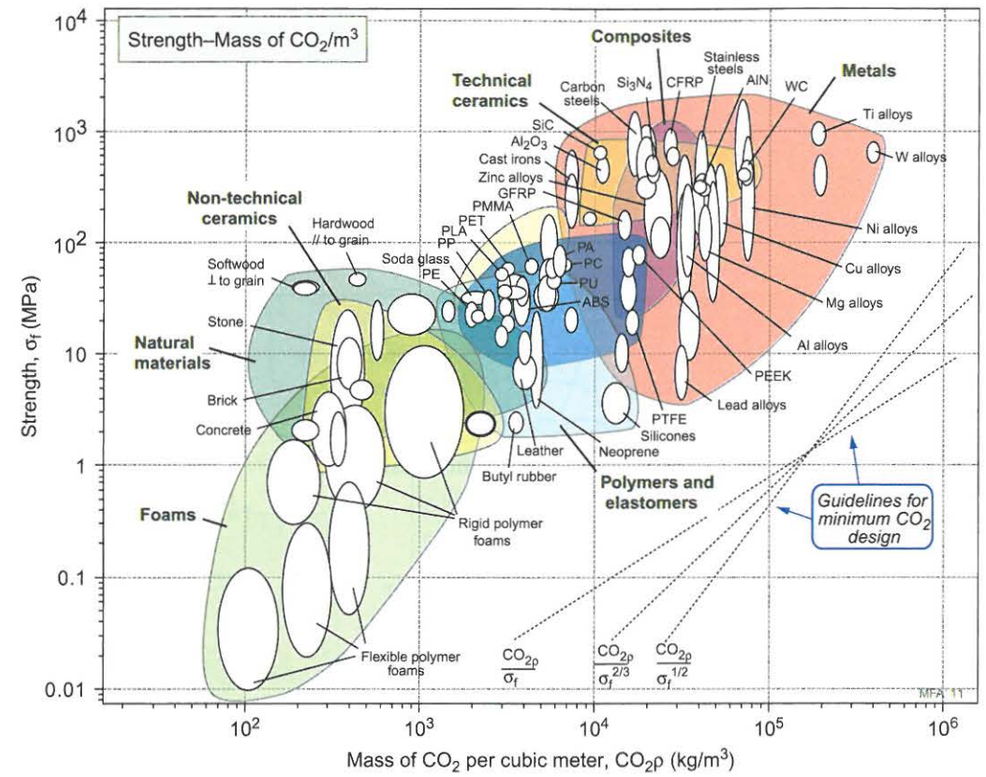


FIGURE 9.17 *The Strength–Carbon footprint chart: the one for strength at minimum carbon release*

Use of the eco-charts

Example: Which polymer, in sheet form, offers the lowest carbon footprint per unit of bending stiffness?

Answer: Table 9.3 gives the index for stiffness at a minimum carbon footprint for a panel in bending

$$M = \frac{CO_2 \cdot \rho}{E^{1/3}}$$

Selecting the corresponding guideline and displacing it to the left until only one polymer remains above it identifies PLA (polylactide—a biopolymer) as the best choice.

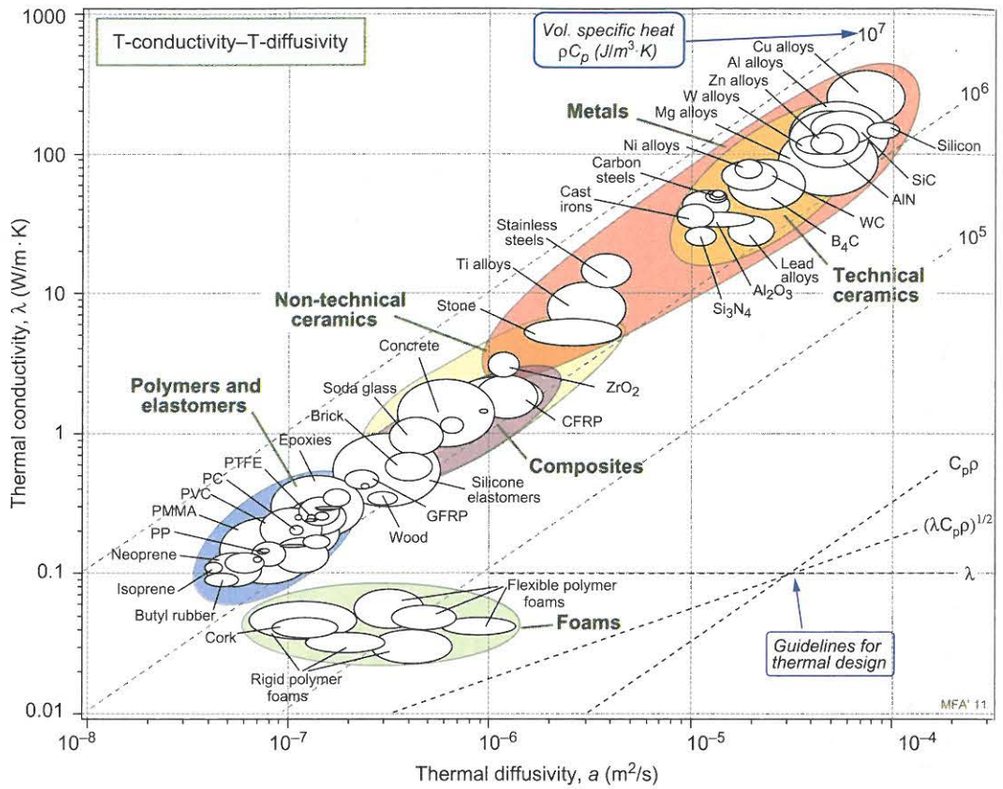


FIGURE 9.18 *The Thermal conductivity–Thermal diffusivity chart with contours of volumetric specific heat: the one for minimum thermal loss*

9.8 Computer-aided selection

The charts give an overview, but the number of materials that can be shown on any one of them is obviously limited. Selection using them is practical when there are very few constraints, but when there are many—as there usually are—checking that a given material meets them all is cumbersome. Both problems are overcome by a computer implementation of the method.

The *CES* material selection software is one such implementation (Figure 9.19). Its databases contain records for materials. It has a search and selection engine. It has plotting facilities to make material property charts and the ability to plot material indices on the charts and use them to optimize material selection, implementing the strategy of this chapter. More information can be found at grantadesign.com/education.

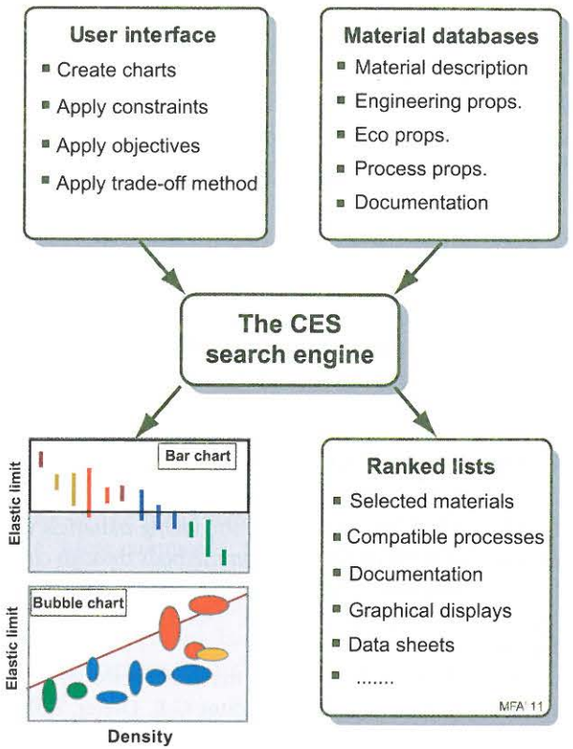


FIGURE 9.19 *The operation and outputs of typical selector software*

9.9 Summary and conclusions

A broad strategy exists that works for selecting almost anything anything—products, services, ... and materials. First decide on the attributes the material must (or must not) have, defining a set of constraints. Then apply the constraints, leaving a list of candidate-materials that meet them. Decide on an objective—a measure of excellence. It could be price (the cheaper the better), or weight (the lighter the better), or eco-impact (the lower the better), or some other measure of performance. Use this to rank the list of surviving candidates. Then get to work researching the top three or four on the list, gathering as much information as possible to make a well-informed final choice.

There are almost always many constraints, but this does not create a difficulty: simply apply them sequentially, retaining only those entities that meet them all. Often, too, there are two or more objectives and that does create a difficulty: the choice that best satisfies one is rarely the best choice for the other. Then trade-off

methods become useful, either graphical ones, which allow you to plot the alternatives, identify the trade-off line, and then use your judgment to select an entity on or near the line; or analytical ones, where you formulate a penalty function and seek the entities that carry the lowest penalty.

Now we have a set of tools. In Chapter 10 they are used to analyze and select materials for design for the environment.

9.10 Further reading

Ashby, M. F. (2011), *Materials selection in mechanical design*, 4th edition, Butterworth Heinemann, Oxford, UK. ISBN 978-1-85617-663-7. *(A text that develops the ideas presented here in more depth, including the derivation of material indices, a discussion of shape factors, and a catalog of simple solutions to standard problems)*

Ashby, M. F., Shercliff, H. R., and Cebon, D. (2009), *Materials: engineering, science, processing and design*, 2nd edition, Butterworth Heinemann, Oxford, UK. ISBN 978-1-85617-895-2. *(An elementary text introducing materials through material property charts, and developing the selection methods through case studies)*

Bader, M. G. (1977), "Composites applications and design," Proc. of ICCM-11, Gold Coast, Australia, Vol. 1: ICCM, London. *(An example of trade-off methods applied to the choice of composite systems)*

Bourell, D. L. (1997), "Decision matrices in materials selection," *ASM Handbook Vol. 20, Materials Selection and Design*, editor G.E. Dieter, ASM International, Materials Park, Ohio, USA, pp 291–296. ISBN 0-87170-386-6. *(An introduction to the use of weight factors and decision matrices)*

Dieter, G. E. (2000), *Engineering design, a materials and processing approach*, 3rd edition, McGraw-Hill, New York, USA, pp 150–153, 255–257. ISBN 0-07-366136-8. *(A well-balanced and respected text, now in its 3rd edition, focusing on the role of materials and processing in technical design)*

Field, F. R. and de Neufville, R. (1988), "Material selection—maximizing overall utility," *Metals and Materials*, June, pp 378–382. *(A summary of utility analysis applied to material selection in the automobile industry)*

Goicoechea, A., Hansen, D. R., and Druckstein, L. (1982), *Multi-objective decision analysis with engineering and business applications*, Wiley, New York, NY, USA. *(A good starting point for the theory of multi-objective decision making)*

Keeney, R. L. and Raiffa, H. (1993), *Decisions with multiple objectives: preferences and value tradeoffs*, 2nd edition, Cambridge University Press, Cambridge, UK. ISBN 0-521-43883-7. *(A notably readable introduction to methods of decision making with multiple, competing objectives)*

9.11 Appendix: deriving material indices

This Appendix describes how material indices are derived. You can find out more about them and their use in the first two texts listed under Further reading. The Appendix has five sections:

(a) Material indices for stiffness and strength at minimum mass—simple section shapes
(b) Using shape to increase stiffness and strength at minimum mass
(c) Arches and shells
(d) Indices for stiffness and strength at minimum material embodied energy or carbon footprint
(e) Indices for stiffness and strength at minimum material cost

The performance of a component is characterized by a performance equation called the *objective function*. The performance equation contains a group of material properties. This group is the material indices of the problem. Sometimes the "group" is a single property. Thus if the performance of a uniform beam is measured by its stiffness, the performance equation contains only one property, the elastic modulus E. More commonly, the performance equation contains a group of two or more properties. Familiar examples are the *specific stiffness*, E/ρ (where E is Young's modulus, and ρ is the density), and the *specific strength*, σ_y/ρ (where σ_y is the yield strength or elastic limit), but there are many others. For reasons that will become apparent, we express the indices in a form for which a *minimum*, not a *maximum* is sought.

Recall that the life-energy and emissions for transport systems are dominated by the fuel consumed during use. The lighter the system is made, the less fuel it consumes and the less carbon it emits. So a good starting point is *minimum weight design*, subject, of course, to the other necessary constraints of which the most important, here, have to do with stiffness and strength. We consider the generic components shown in Figure 9.20: ties, panels, beams and shells, loaded as shown. The derivation when the objective is that of minimizing embodied energy, carbon footprint, or material cost follows in a similar way.

(a) Material indices for stiffness and strength at minimum mass—simple section shapes. **A light, stiff tie-rod.** A material is sought for a cylindrical tie rod that must be as light as possible (Figure 9.20(a)). Its length L_o is specified and it must carry a tensile force F without extending elastically by more than δ. Its stiffness must be at least $S^* = F/\delta$. We are free to choose the cross-section area A and, of course, the material. The design requirements, translated, are listed in Table 9.8.

We first seek an equation that describes the quantity to be minimized, here the mass m of the tie. This equation, the *objective function*, is

$$m = A\,L_o\,\rho \tag{9.15}$$