Static Games and Cournot Competition

Introduction

- In the majority of markets firms interact with *few competitors* – *oligopoly* market
- Each firm has to consider rival's actions
  - ***strategic interaction in prices, outputs, advertising …***
- This kind of interaction is analyzed using ***game theory***
  - assumes that "players" are rational
- Distinguish *cooperative* and *noncooperative* games
  - focus on noncooperative games
- Also consider *timing*
  - simultaneous versus sequential games

Oligopoly theory

- No single theory
  - employ game theoretic tools that are appropriate
  - outcome depends upon *information* available
- Need a concept of ***equilibrium***
  - players (firms?) choose *strategies*, one for each player
  - combination of strategies determines *outcome*
  - outcome determines *pay-offs* (profits?)
- Equilibrium first formalized by Nash: ***No firm wants to change its current strategy given that no other firm changes its current strategy***

Nash equilibrium

- Equilibrium need not be "nice"
  - firms might do better by coordinating but such coordination may not be possible (or legal)
- Some strategies can be eliminated on occasions
  - they are never good strategies *no matter what the rivals do*
- These are ***dominated strategies***
  - they are never employed and so can be eliminated
  - elimination of a dominated strategy may result in another being dominated: it also can be eliminated
- One strategy might always be chosen no matter what the rivals do: ***dominant strategy***

An example

- Two airlines
- Prices set: compete in departure times
- 70% of consumers prefer evening departure, 30% prefer morning departure
- If the airlines choose the same departure times they share the market equally
- Pay-offs to the airlines are determined by market shares
- Represent the pay-offs in a ***pay-off matrix***

The example 2

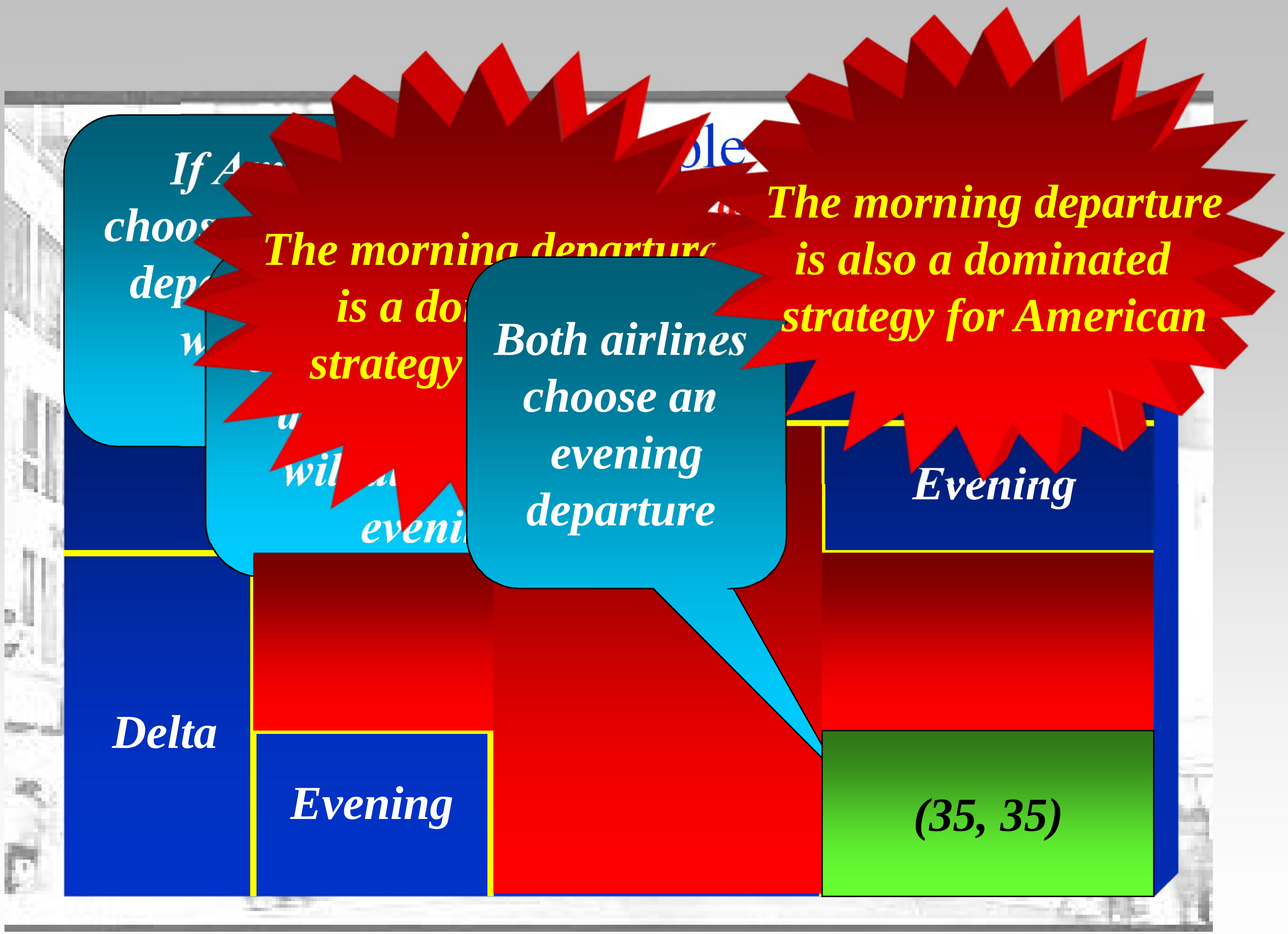
The Pay-Off Matri

What is the equilibrium for this game?

		American	
		Morning	***Evening***
Delta	***Morning***	*(15, 15)*	*(30, 70)*
	Evening	*(70, 30)*	

The left-hand number is the pay-off to Delta

The right-hand number is the pay-off to American


The morning departure is also a dominated strategy for American
Both airlines choose an evening departure
Evening
Delta
Evening
(35, 35)

The example 4

- **Now suppose that Delta has a frequent flier program**
- **When both airline choose the same departure times Delta gets 60% of the travelers**
- **This changes the pay-off matrix**

American has no dominated strategy

However, a morning departure is still a dominated strategy for Delta

American knows this and so chooses a morning departure

Delta	Evening	(70, 30)	(42, 28)

Nash equilibrium

- What if there are no dominated or dominant strategies?
- Then we need to use the ***Nash equilibrium*** concept.
- Change the airline game to a pricing game:
  - 60 potential passengers with a reservation price of $500
  - 120 additional passengers with a reservation price of $220
  - price discrimination is not possible (perhaps for regulatory reasons or because the airlines don’t know the passenger types)
  - costs are $200 per passenger no matter when the plane leaves
  - airlines must choose between a price of $500 and a price of $220
  - if equal prices are charged the passengers are evenly shared
  - the low-price airline gets all the passengers
- The pay-off matrix is now:

If both price hi

If Delta prices high and American low then American gets 180 passengers.

If Delta prices low and American
then Delta g
all 180 passen
Profit per pass
is $20

If both price low they each get 90 passengers. Profit per passenger is $20

		erican
		$P_L = \$220$
		($0, $3600)
Delta $P_L = \$220$	($3600, $0)	($1800, $1800)

There is no simple way to choose between these equilibria

“Regret” might cause both to price low

pricing
neither wants
to change

high th
also price high

		P_H … 500	
Delta		($9000, $9000)	($0, $3600)
	P_L = $220	($3600, $0)	($1800, $1800)

Oligopoly models

- **There are three dominant oligopoly models**
  - **Cournot**
  - **Bertrand**
  - **Stackelberg**
- **They are distinguished by**
  - **the decision variable that firms choose**
  - **the timing of the underlying game**
- **Concentrate on the Cournot model in this section**

The Cournot model

- Start with a duopoly
- Two firms making an identical product (Cournot supposed this was spring water)
- Demand for this product is

  $P = A - BQ = A - B(q_1 + q_2)$

  where q_1 is output of firm 1 and q_2 is output of firm 2
- Marginal cost for each firm is constant at c per unit
- To get the demand curve for one of the firms we treat the output of the other firm as constant
- So for firm 2, demand is $P = (A - Bq_1) - Bq_2$

The Cournot model

$P = (A - Bq_1) - Bq_2$

The profit-maximizing choice of output by firm 2 depends upon the output of firm 1

Marginal reven[illegible] firm 2 is

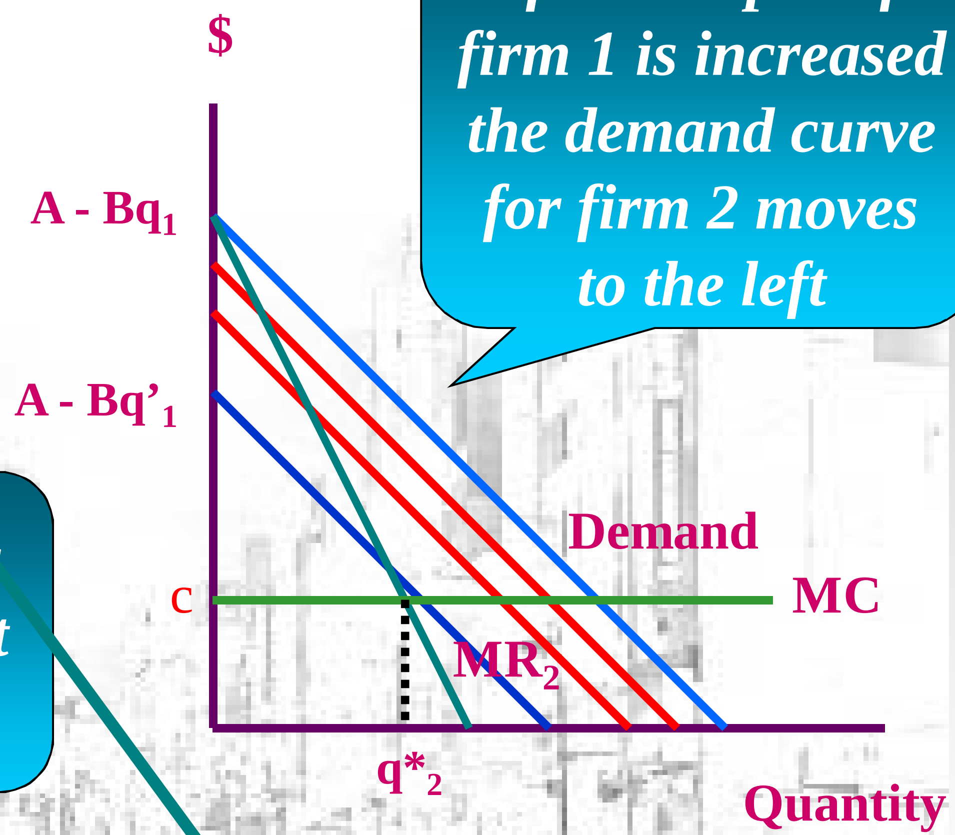
$MR_2 = (A - Bq$ [illegible]

$MR_2 = MC$

Solve this for output q_2

$A - Bq_1 - 2Bq_2 = c \quad \therefore \quad q^*_2 = (A - c)/2B - q_1/2$

If the output of firm 1 is increased the demand curve for firm 2 moves to the left



The Cournot model 3

$q^*_2 = (A - c)/2B - q_1/2$

This is the ***reaction function*** for firm 2

It gives firm 2's profit-maximizing choice of output for any choice of output by firm 1

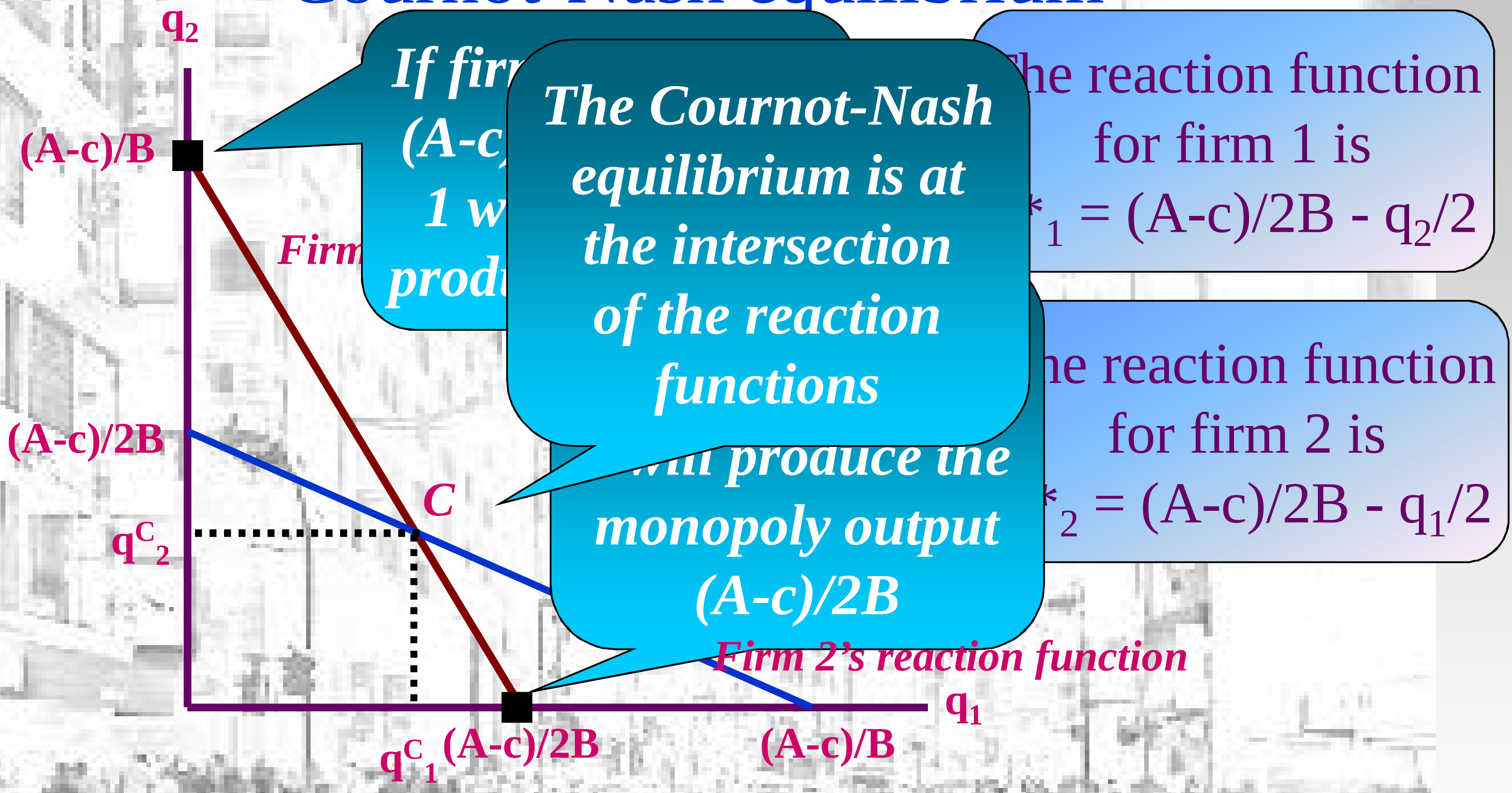
There is also a reaction function for firm 1

By exactly the same argument it can be written:

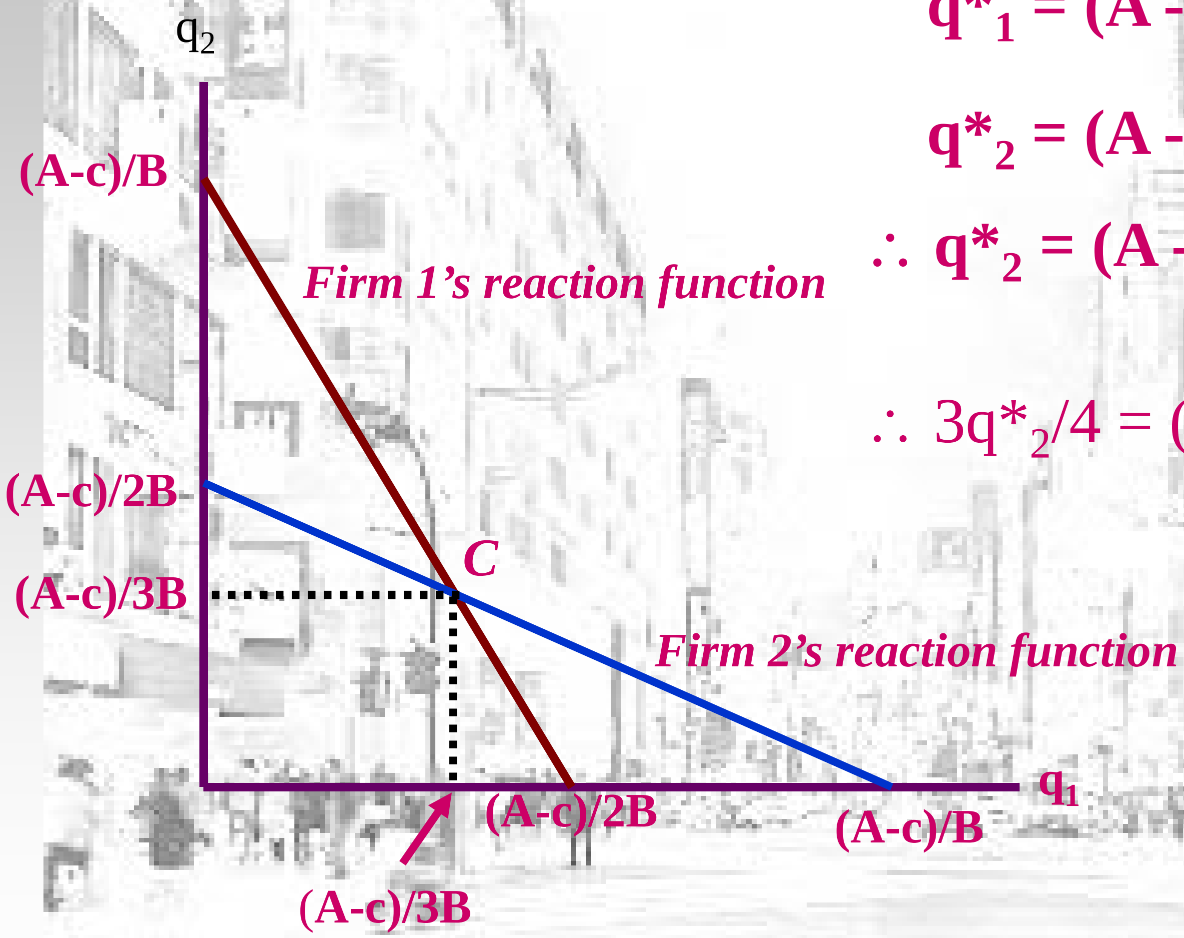
$$q^*_1 = (A - c)/2B - q_2/2$$

Cournot-Nash equilibrium requires that both firms be on their reaction functions.

Cournot-Nash equilibrium



Cournot-Nash equilibrium 2



$$q^*_1 = (A - c)/2B - q^*_2/2$$

$$q^*_2 = (A - c)/2B - q^*_1/2$$

$$\therefore\ q^*_2 = (A - c)/2B - (A - c)/4B + q^*_2/4$$

$$\therefore\ 3q^*_2/4 = (A - c)/4B$$

$$\therefore\ q^*_2 = (A - c)/3B$$

$$\therefore\ q^*_1 = (A - c)/3B$$

Cournot-Nash equilibrium 3

- **In equilibrium each firm produces $q^C_1 = q^C_2 = (A - c)/3B$**
- **Total output is, therefore, $Q^* = 2(A - c)/3B$**
- **Recall that demand is $P = A - BQ$**
- **So the equilibrium price is $P^* = A - 2(A - c)/3 = (A + 2c)/3$**
- **Profit of firm 1 is $(P^* - c)q^C_1 = (A - c)^2/9B$**
- **Profit of firm 2 is the same**
- **A monopolist would produce $Q^M = (A - c)/2B$**
- **Competition between the firms causes them to overproduce. Price is lower than the monopoly price**
- **But output is less than the competitive output $(A - c)/B$ where price equals marginal cost**

Cournot-Nash equilibrium: many firms

- **What if there are more than two firms?**
- **Much the same approach.**
- **Say that there are N identical firms producing identical products**
- **Total output $Q = q_1 + q_2 + .$**
- **Demand is $P = A - BQ = A$ $)$**
- **Consider firm 1. It's dema ten:**

  $$P = A - B(q_2 + \ldots + q_N) - Bq_1$$

- **Use a simplifying notation: $Q_{-1} = q_2 + q_3 + \ldots + q_N$**
- **So demand for firm 1 is $P = (A - BQ_{-1}) - Bq_1$**

*This denotes output of every firm **other than** firm 1*

The Cournot model: many

$P = (A - BQ_{-1}) - Bq_1$

The profit-maximizing choice of output by firm 1 depends upon the output of the other firms

Marginal reve... f... firm 1 is

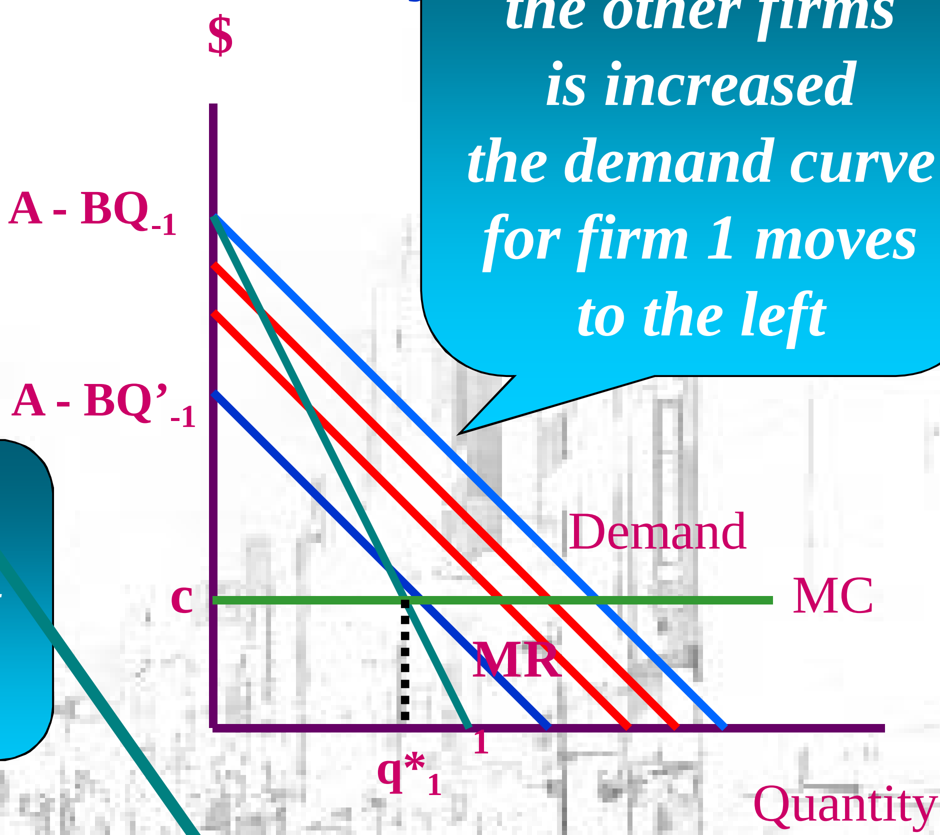
Solve this for output q_1

$MR_1 = (A - B$...

$MR_1 = MC$

$A - BQ_{-1} - 2Bq_1 = c \;\therefore\; q^*_1 = (A - c)/2B - Q_{-1}/2$

If the output of the other firms is increased the demand curve for firm 1 moves to the left



Cournot-Nash equilibrium: many firms

$q^*_1 = (A - c)/2B - Q_{-1}/2$

$\therefore\ Q^*_{-1} = (N - 1)q^*_1$

$\therefore\ q^*_1 = (A - c)/2B - (N$

$\therefore\ (1 + (N - 1)/2)q^*_1 = (A -$

$\therefore\ q^*_1(N + 1)/2 = (A - c)/2B$

$\therefore\ q^*_1 = (A - c)/(N + 1)B$

$\therefore\ Q^* = N(A - c)/(N + 1)B$

$\therefore\ P^* = A - BQ^* = (A + Nc)/(N + 1)$

Profit of firm 1 is $P^*_1 = (P^* - c)q^*_1 = (A - c)^2/(N + 1)^2B$

As the number of firms increases profit of each firm falls

Cournot-Nash equilibrium: different costs

- **What if the firms do not have identical costs?**
- **Much the same analysis can be used**
- **Marginal costs of firm 1 are c_1 and of fi**
- **Demand is $P = A - BQ = A - B(q_1 + q_2)$**
- **We have marginal revenue for firm 1 a**
- **$MR_1 = (A - Bq_2) - 2Bq_1$**
- **Equate to marginal cost: (**

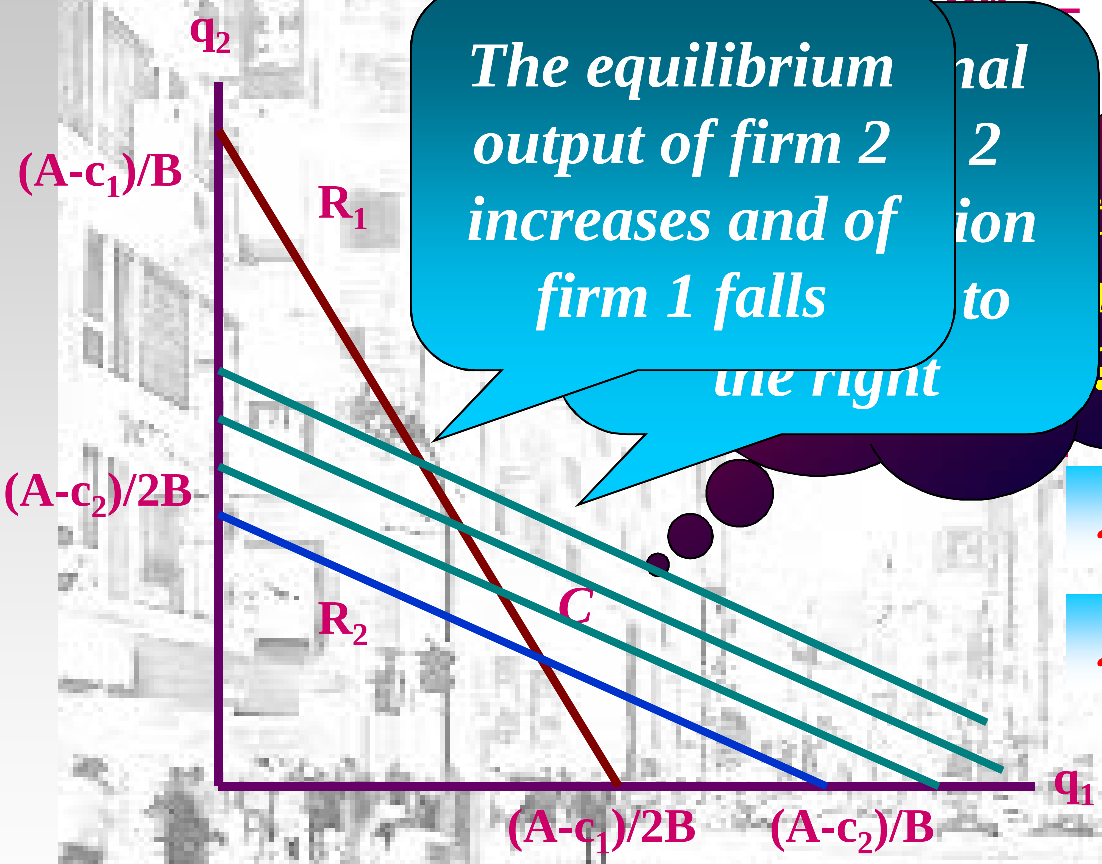
$$\therefore\ q^*_1 = (A - c_1)/2B - q_2/2$$

$$\therefore\ q^*_2 = (A - c_2)/2B - q_1/2$$

Solve this for output q_1

A symmetric result holds for output of firm 2

Cournot-Nash equilibrium: different costs 2



The equilibrium output of firm 2 increases and of firm 1 falls

nal 2 ion to the right

o this en ?

$q^*_1 = (A - c_1)/2B - q^*_2/2$

$c_2)/2B - q^*_1/2$

$/2B - (A - c_1)/4B$

$q^*_2/4$

$(A - 2c_2 + c_1)/4B$

$\therefore q^*_2 = (A - 2c_2 + c_1)/3B$

$\therefore q^*_1 = (A - 2c_1 + c_2)/3B$

Cournot-Nash equilibrium: different costs 3

- **In equilibrium the firms produce $q^C_1 = (A - 2c_1 + c_2)/3B$; $q^C_2 = (A - 2c_2 + c_1)/3B$**
- **Total output is, therefore, $Q^* = (2A - c_1 - c_2)/3B$**
- **Recall that demand is $P = A - B.Q$**
- **So price is $P^* = A - (2A - c_1 - c_2)/3 = (A + c_1 + c_2)/3$**
- **Profit of firm 1 is $(P^* - c_1)q^C_1 = (A - 2c_1 + c_2)^2/9$**
- **Profit of firm 2 is $(P^* - c_2)q^C_2 = (A - 2c_2 + c_1)^2/9$**
- **Equilibrium output is less than the competitive level**
- **Output is produced inefficiently: the low-cost firm should produce all the output**

Concentration and profitability

- **Assume there are N firms with different marginal costs**
- **We can use the N-firm analysis with a simple change**
- **Recall that demand for firm 1 is $P = (A - BQ_{-1}) - Bq_1$**
- **But then demand for firm *i* is $P = (A - BQ_{-i}) - Bq_i$**
- **Equate this to marginal cost c_i**

$$A - BQ_{-i} - 2Bq_i = c_i$$

This can be reorganized to give the e

But $Q^*_{-i} + q^*_i = Q^*$ and $A - BQ^* = P^*$

$$A - B(Q^*_{-i} + q^*_i) - Bq^*_i - c_i = 0$$

$$\therefore P^* - Bq^*_i - c_i = 0 \quad \therefore P^* - c_i = Bq^*_i$$

Concentration and pro

$P^* - c_i = Bq^*_i$

Divide by P* and multiply

$$\frac{P^* - c_i}{P^*} = \frac{BQ^*}{P^*}\frac{q^*_i}{Q^*}$$

But BQ*/P* = 1/η and q

so: $$\frac{P^* - c_i}{P^*} = \frac{s_i}{\eta}$$

Extending this we have

$$\frac{P^* - c}{P^*} = \frac{H}{\eta}$$ (p. 155)

The price-cost margin for each firm is determin d b its

Average price-cost margin is determined by industry concentration

Price Competition: Introduction

- **In a wide variety of markets firms compete in prices**
  - **Internet access**
  - **Restaurants**
  - **Consultants**
  - **Financial services**
- **With monopoly setting price or quantity first makes no difference**
- **In oligopoly it matters a great deal**
  - **nature of price competition is much more aggressive the quantity competition**

Price Competition: Bertrand

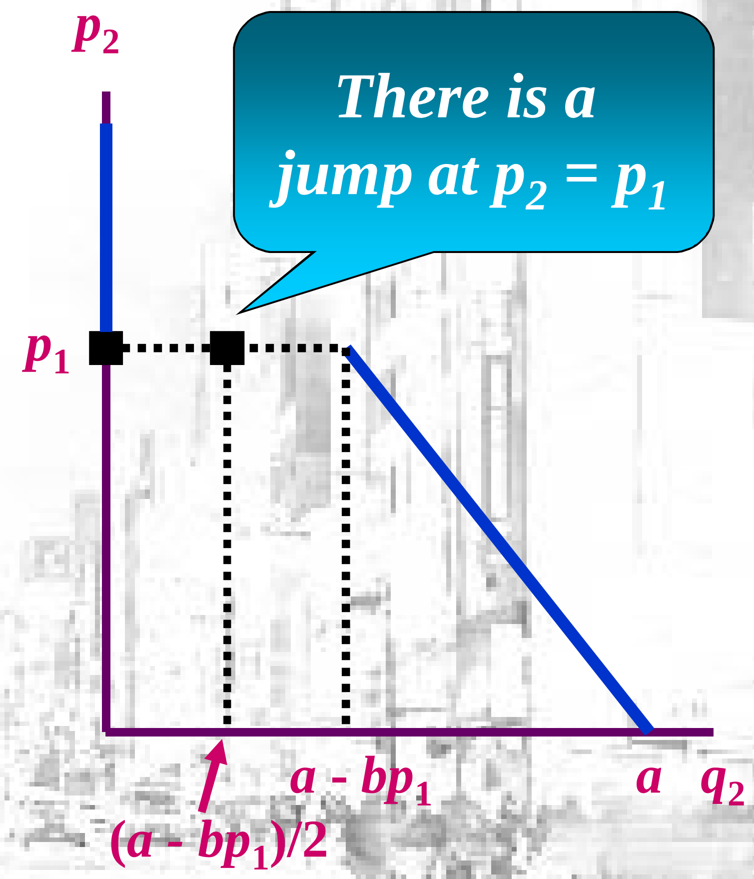
- **In the Cournot model price is set by some market clearing mechanism**
- **An alternative approach is to assume that firms compete in prices: this is the approach taken by Bertrand**
- **Leads to dramatically different results**
- **Take a simple example**
  - **two firms producing an identical product (spring water?)**
  - **firms choose the prices at which they sell their products**
  - **each firm has constant marginal cost of c**
  - **inverse demand is $P = A – B.Q$**
  - **direct demand is $Q = a – b.P$ with $a = A/B$ and $b = 1/B$**

Bertrand competition

- **We need the *derived demand* for each firm**
  - **demand conditional upon the price charged by the other firm**
- **Take firm 2. Assume that firm 1 has set a price of p_1**
  - **if firm 2 sets a price greater than p_1 she will sell nothing**
  - **if firm 2 sets a price less than p_1 she gets the whole market**
  - **if firm 2 sets a price of exactly p_1 consumers are indifferent between the two firms: the market is shared, presumably 50:50**
- **So we have the derived demand for firm 2**
  - $q_2 = 0$ if $p_2 > p_1$
  - $q_2 = (a - bp_2)/2$ if $p_2 = p_1$
  - $q_2 = a - bp_2$ if $p_2 < p_1$

Bertrand competition 2

- **This can be illustrated as follows:**
- **Demand is discontinuous**
- **The discontinuity in demand carries over to profit**



Bertrand competition 3

Firm 2's profit is:

$\pi_2(p_1, p_2) = 0$ if $p_2 > p_1$

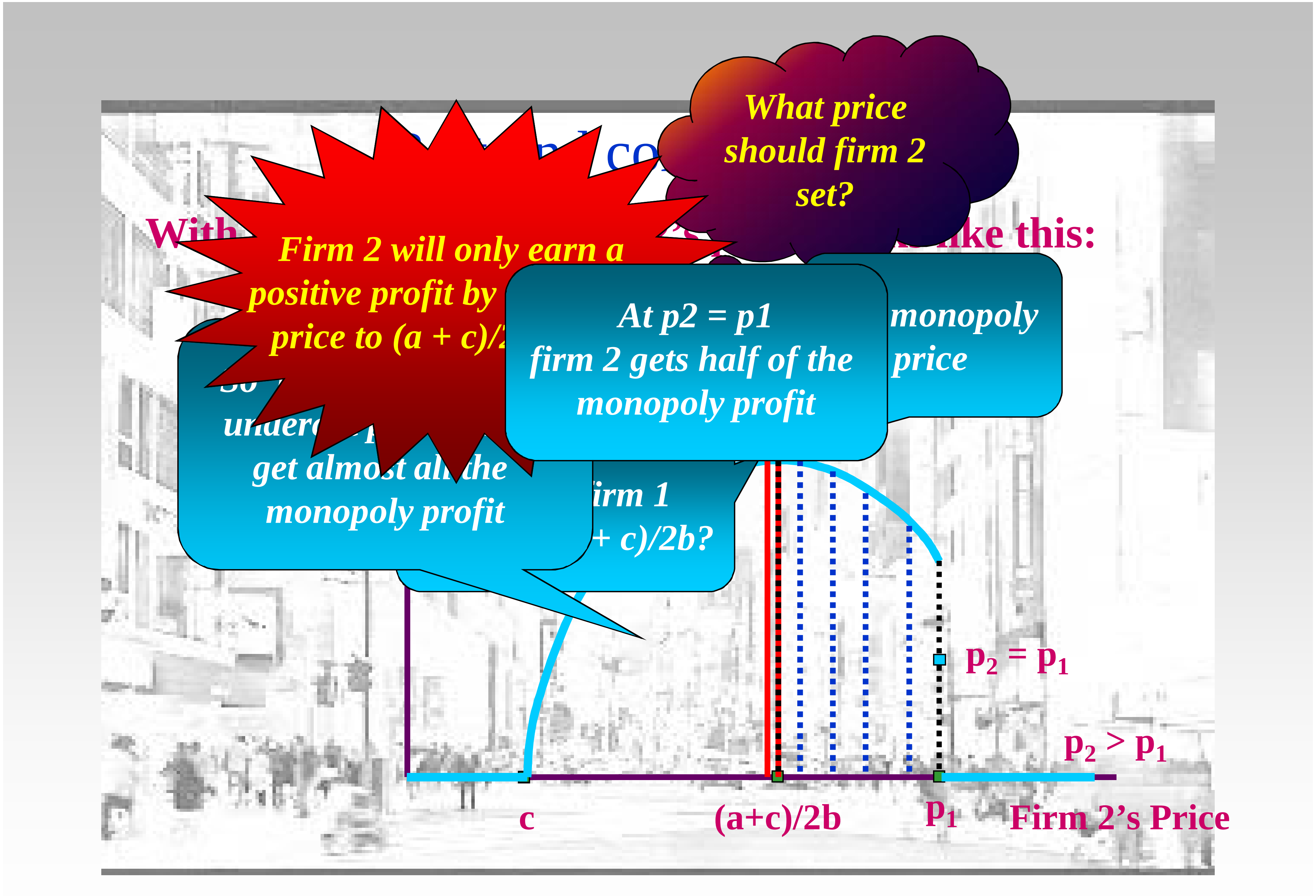
$\pi_2(p_1, p_2) = (p_2 - c)(a - bp_2)$ if $p_2 < p_1$

$\pi_2(p_1, p_2) = (p_2 - c)(a - bp_2)/2$ if $p_2 = p_1$

For whatever reason!

Clearly this depends on p_1.

Suppose first that firm 1 sets a "very high" price: greater than the monopoly price of $p^M = (a + c)/2b$

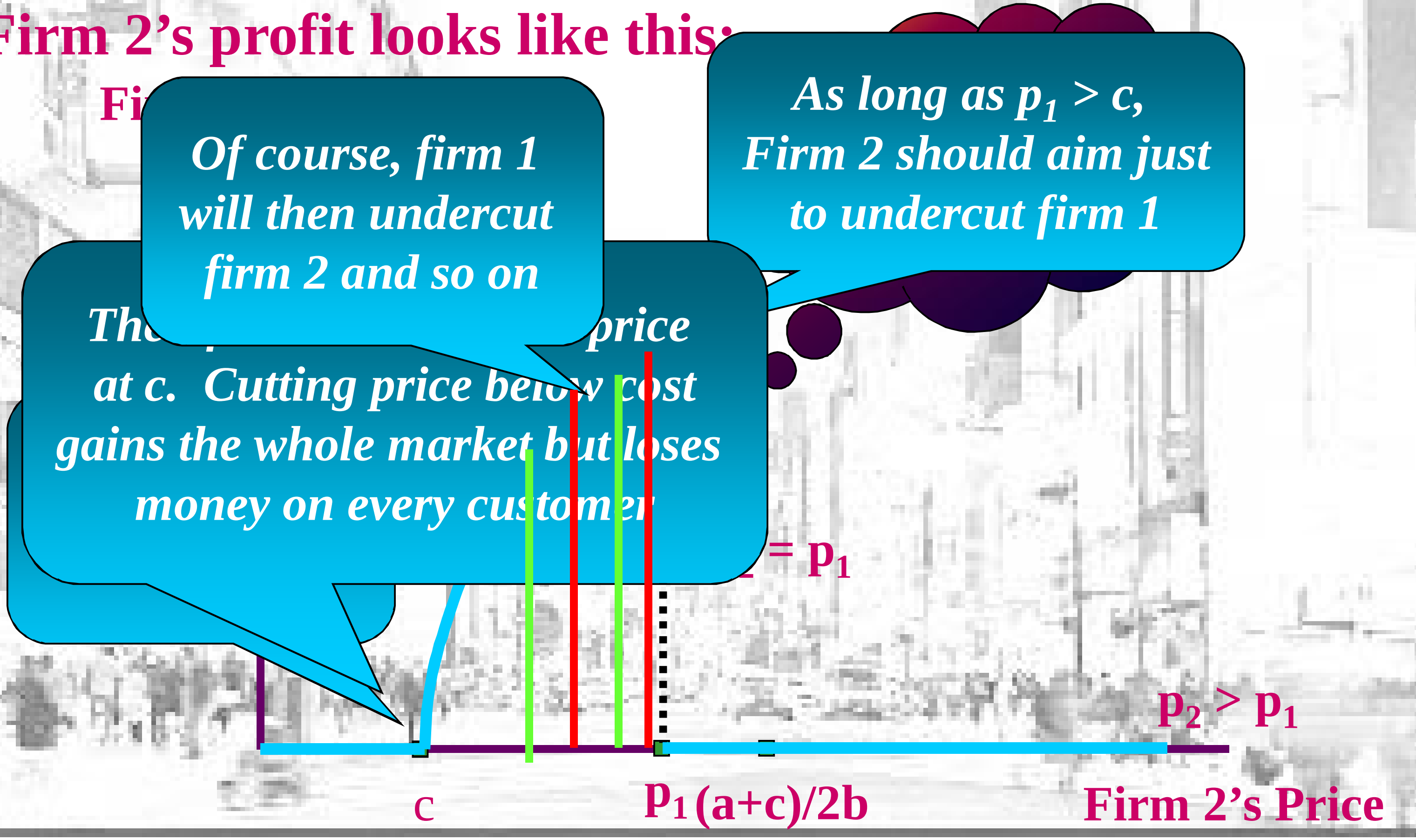


What price should firm 2 set?
Firm 2 will only earn a
positive profit by
At p2 = p1
firm 2 gets half of the
monopoly profit
monopoly
price
get almost all the
monopoly profit
p2 = p1
p2 > p1
c
(a+c)/2b
p1
Firm 2's Price

Bertrand competition 5

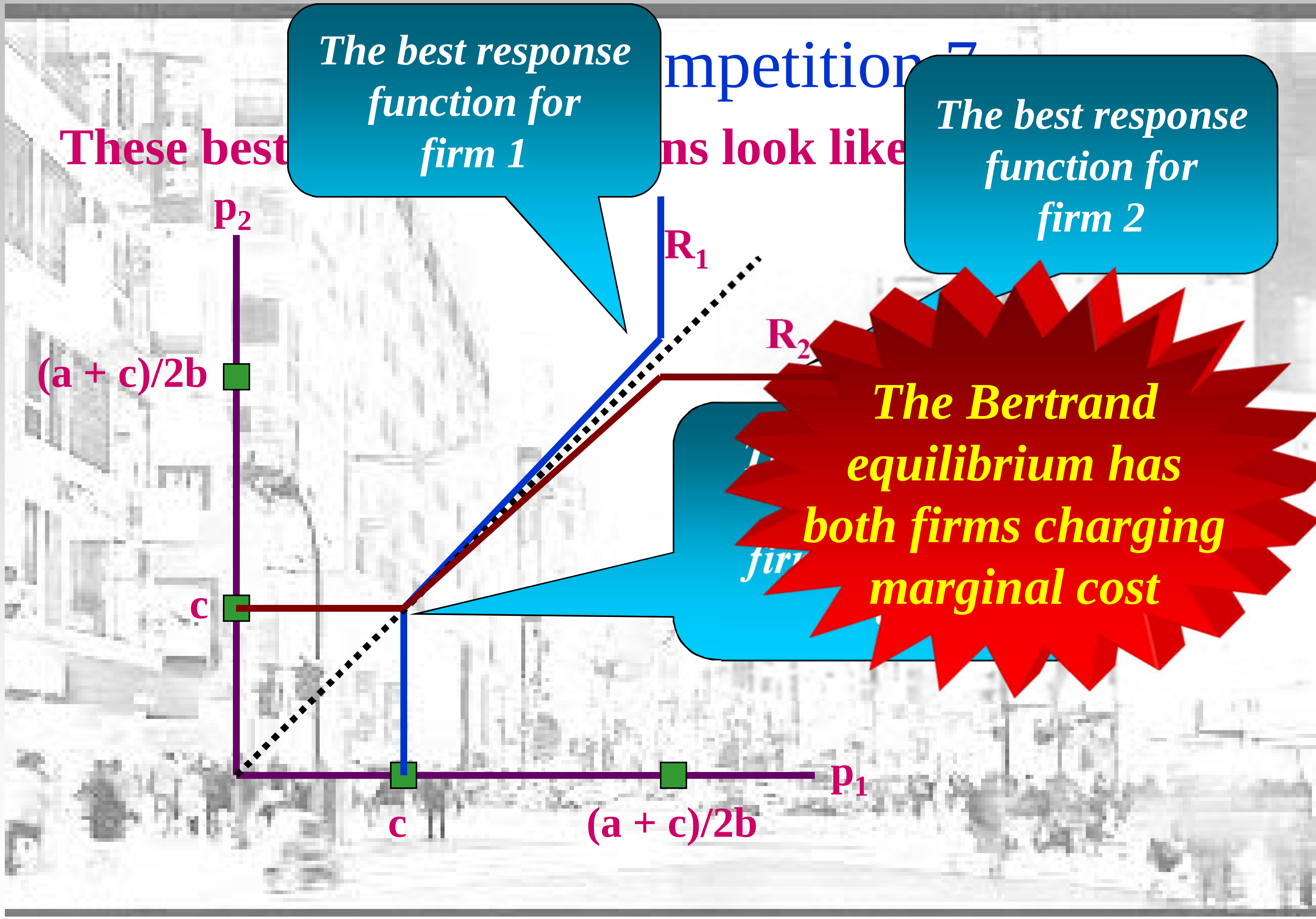
Now suppose that firm 1 sets a price less than (a + c)/2b

Firm 2's profit looks like this:



Bertrand competition 6

- **We now have Firm 2's best response to any price set by firm 1:**
  - **$p^*_2 = (a + c)/2b$ if $p_1 > (a + c)/2b$**
  - **$p^*_2 = p_1$ - "something small" if $c < p_1 \leq (a + c)/2b$**
  - **$p^*_2 = c$ if $p_1 \leq c$**
- **We have a symmetric best response for firm 1**
  - **$p^*_1 = (a + c)/2b$ if $p_2 > (a + c)/2b$**
  - **$p^*_1 = p_2$ - "something small" if $c < p_2 \leq (a + c)/2b$**
  - **$p^*_1 = c$ if $p_2 \leq c$**



mpetition 7
These best
ns look like
The best response function for firm 1
The best response function for firm 2
The Bertrand equilibrium has both firms charging marginal cost
p2
R1
R2
(a + c)/2b
c
c
(a + c)/2b
p1

Bertrand Competition

Why the wildly different result from Cournot?

-Homogenous goods – no difference

-One-shot game – no difference

-Demand – no difference

-In Bertrand, the firm supplies all demand – Key difference

-How realistic?

Bertrand Equilibrium: modifications

- The Bertrand model makes clear that competition in prices is very different from competition in quantities
- Since many firms seem to set prices (and not quantities) this is a challenge to the Cournot approach
- But the extreme version of the difference seems somewhat forced
- Two extensions can be considered
  - impact of capacity constraints
  - product differentiation

Capacity Constraints

- **For the *p* = *c* equilibrium to arise, both firms need enough capacity to fill all demand at *p* = *c***
- **But when *p* = *c* they each get only half the market**
- **So, at the *p* = *c* equilibrium, there is huge excess capacity**
- **So *capacity constraints* may affect the equilibrium**
- **Consider an example**
  - **daily demand for skiing on Mount Norman $Q = 6{,}000 - 60P$**
  - **Q is number of lift tickets and P is price of a lift ticket**
  - **two resorts: Pepall with daily capacity 1,000 and Richards with daily capacity 1,400, both fixed**
  - **marginal cost of lift services for both is $10**

The Example

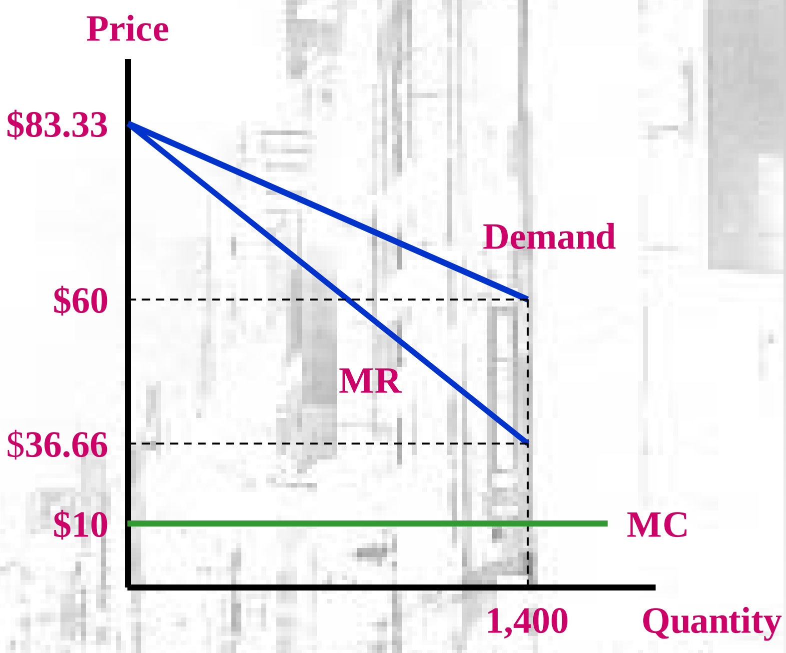
- **Is a price *P* = *c* = $10 an equilibrium?**
  - **total demand is then 5,400, well in excess of capacity**
- **Suppose both resorts set *P* = $10: both then have demand of 2,700**
- **Consider Pepall:**
  - **raising price loses some demand**
  - **but where can they go? Richards is already above capacity**
  - **so some skiers will not switch from Pepall at the higher price**
  - **but then Pepall is pricing above MC and making profit on the skiers who remain**
  - **so *P* = $10 cannot be an equilibrium**

The example 2

- **Assume that at any price where demand at a resort is greater than capacity there is *efficient rationing***
  - **serves skiers with the highest willingness to pay**
- **Then can derive residual demand**
- **Assume P = \$60**
  - **total demand = 2,400 = total capacity**
  - **so Pepall gets 1,000 skiers**
  - **residual demand to Richards with efficient rationing is $Q = 5000 - 60P$ or $P = 83.33 - Q/60$ in inverse form**
  - **marginal revenue is then $MR = 83.33 - Q/30$**

The example 3

- **Residual demand and MR:**
- **Suppose that Richards sets *P* = \$60. Does it want to change?**
  - **since MR > MC Richards does not want to raise price and lose skiers**
  - **since Q_R = 1,400 Richards is at capacity and does not want to reduce price**



- **Same logic applies to Pepall so *P* = \$60 is a Nash equilibrium for this game.**

Capacity constraints again

- **Logic is quite general**
  - **firms are unlikely to choose sufficient capacity to serve the whole market when price equals marginal cost**
    - ***since they get only a fraction in equilibrium***
  - **so capacity of each firm is less than needed to serve the whole market**
  - **but then there is no incentive to cut price to marginal cost**
- **So the efficiency property of Bertrand equilibrium breaks down when firms are capacity constrained**

Product differentiation

- **Original analysis also assumes that firms offer homogeneous products**
- **Creates incentives for firms to *differentiate* their products**
  - **to generate consumer loyalty**
  - **do not lose all demand when they price above their rivals**
    - ***keep the "most loyal"***

An example of product differentiation

Coke and Pepsi are similar but not identical. As a result, the lower priced product does not win the entire market.

Econometric estimation gives:

$$Q_C = 63.42 - 3.98P_C + 2.25P_P$$

$$MC_C = \$4.96$$

$$Q_P = 49.52 - 5.48P_P + 1.40P_C$$

$$MC_P = \$3.96$$

There are at least two methods for solving for P_C and P_P

Bertrand and product differentiation

Method 1: Calculus

Profit of Coke: $\pi_C = (P_C - 4.96)(63.42 - 3.98P_C + 2.25P_P)$

Profit of Pepsi: $\pi_P = (P_P - 3.96)(49.52 - 5.48P_P + 1.40P_C)$

Differentiate with respect to P_C and P_P respectively

Method 2: MR = MC

Reorganize the demand functions

$P_C = (15.93 + 0.57P_P) - 0.25Q_C$

$P_P = (9.04 + 0.26P_C) - 0.18Q_P$

Calculate marginal revenue, equate to marginal cost, solve for Q_C and Q_P and substitute in the demand functions

Bertrand and product differentiation 2

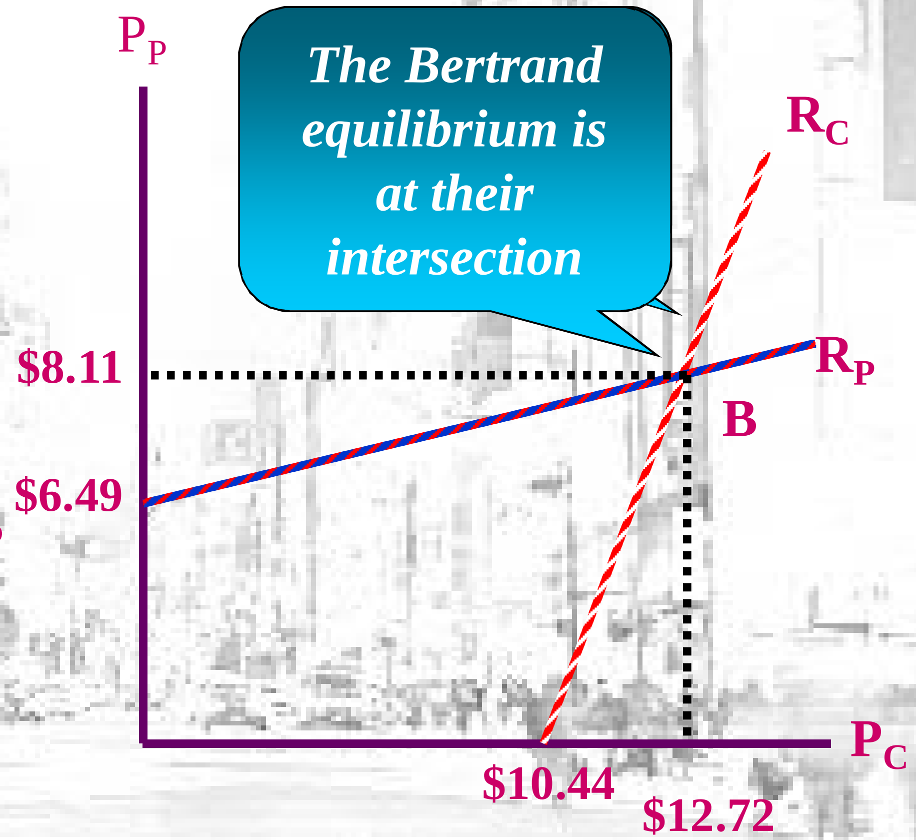
Both methods give the best response functions:

$$P_C = 10.44 + 0.2826P_P$$

$$P_P = 6.49 + 0.1277P_C$$

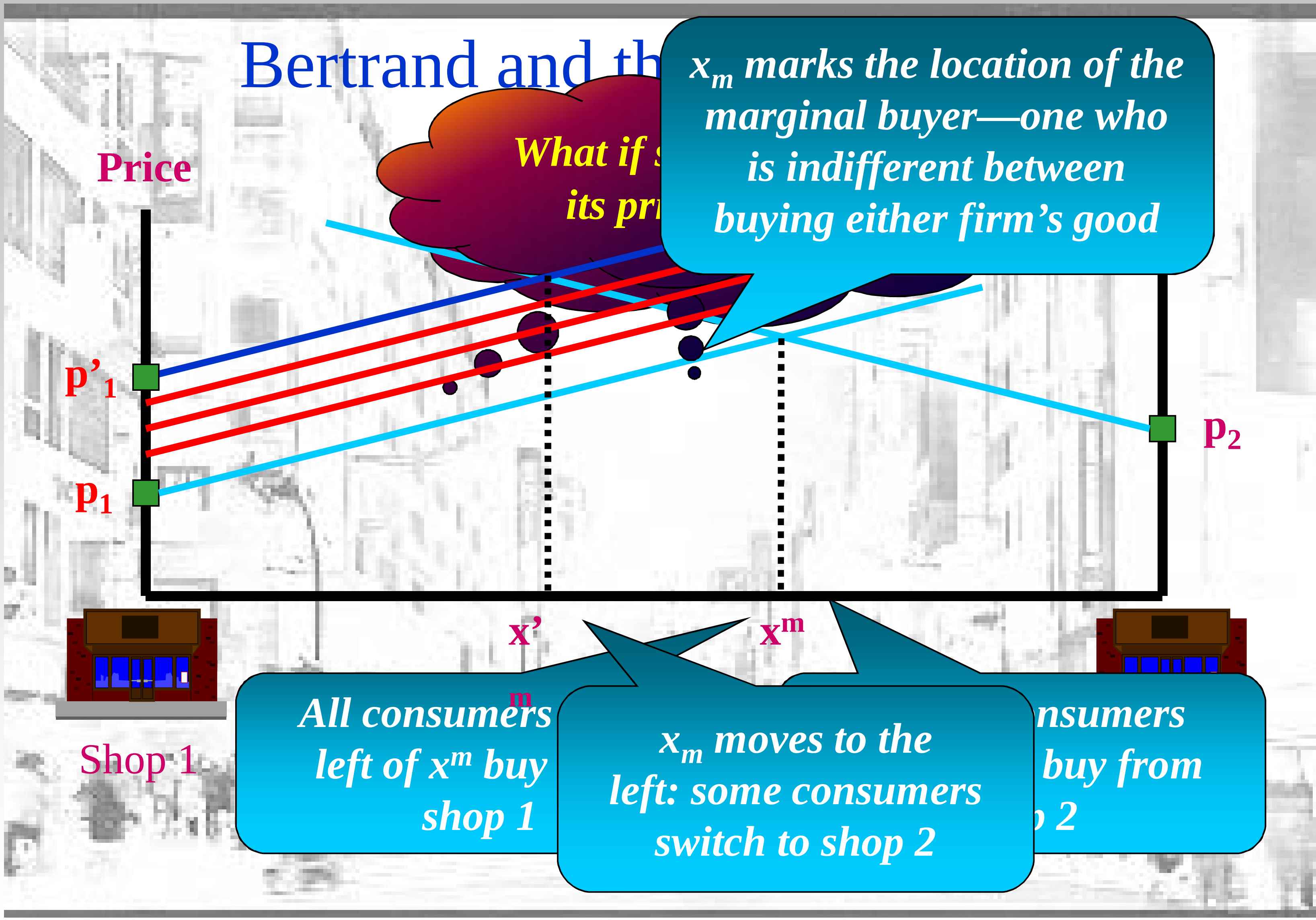
These can be solved for the equilibrium prices as indicated

The equilibrium prices are each greater than marginal cost



Bertrand competition and the spatial model

- **An alternative approach: spatial model of Hotelling**
  - **a Main Street over which consumers are distributed**
  - **supplied by two shops located at opposite ends of the street**
  - **but now the shops are competitors**
  - **each consumer buys exactly one unit of the good provided that its full price is less than V**
  - **a consumer buys from the shop offering the lower full price**
  - **consumers incur transport costs of t per unit distance in travelling to a shop**
- **Recall the broader interpretation**
- **What prices will the two shops charge?**



Bertrand and th
Price
What if
its pr
x_m marks the location of the marginal buyer—one who is indifferent between buying either firm's good
p'1
p1
p2
x'
m
x^m
Shop 1
All consumers left of x^m buy shop 1
x_m moves to the left: some consumers switch to shop 2
nsumers
buy from
2

Bertrand and the spatial model 2

$p_1 + tx^m = p_2 + t(1 - x^m)$ $\therefore 2tx^m = p_2 - p_1 + t$

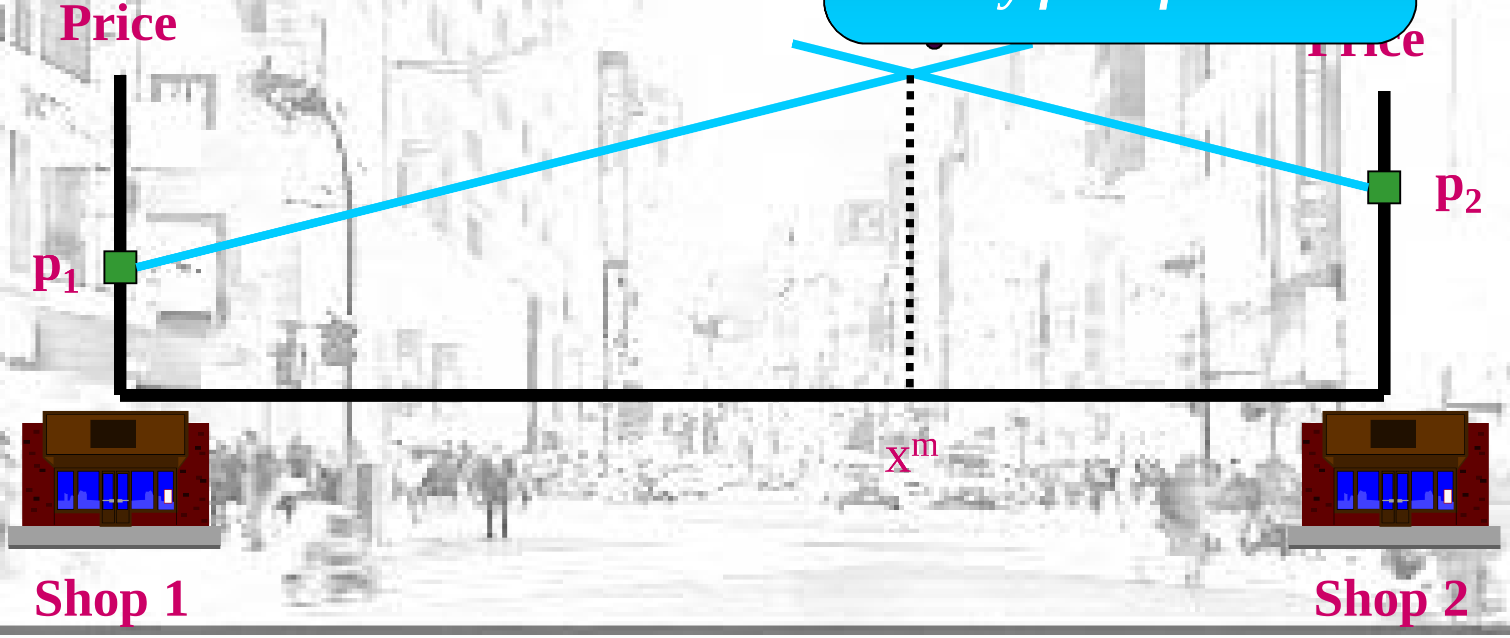
$\therefore x^m(p_1, p_2) = (p_2 - p_1 + t)/2t$

How is x^m

This is the fraction of consumers who buy from firm 1

There are N consumers in total

So demand to firm 1 is $D^1 = N(p_2 - p_1 + t)/2t$



Bertrand equilibrium

Profit to firm 1 is $\pi_1 = (p_1 - c)$ … $p_1 + t)/2t$

$\pi_1 = N(p_2 p_1 - p_1^2 + tp_1 + cp_1$ …

Differentiate with respect t…

$$\partial\pi_1 / \partial p_1 = \frac{N}{2t}(p_2 - 2p_1 + t + c) = 0$$

$p^*_1 = (p_2 + t + c)/2$

What about firm 2 … a similar best response … tion.

$p^*_2 = (p_1 + t + c)/2$

This is the best response function for firm 1

… is

This is the best response function for firm 2

Bertrand equilibrium 2

$p^*_1 = (p_2 + t + c)/2$

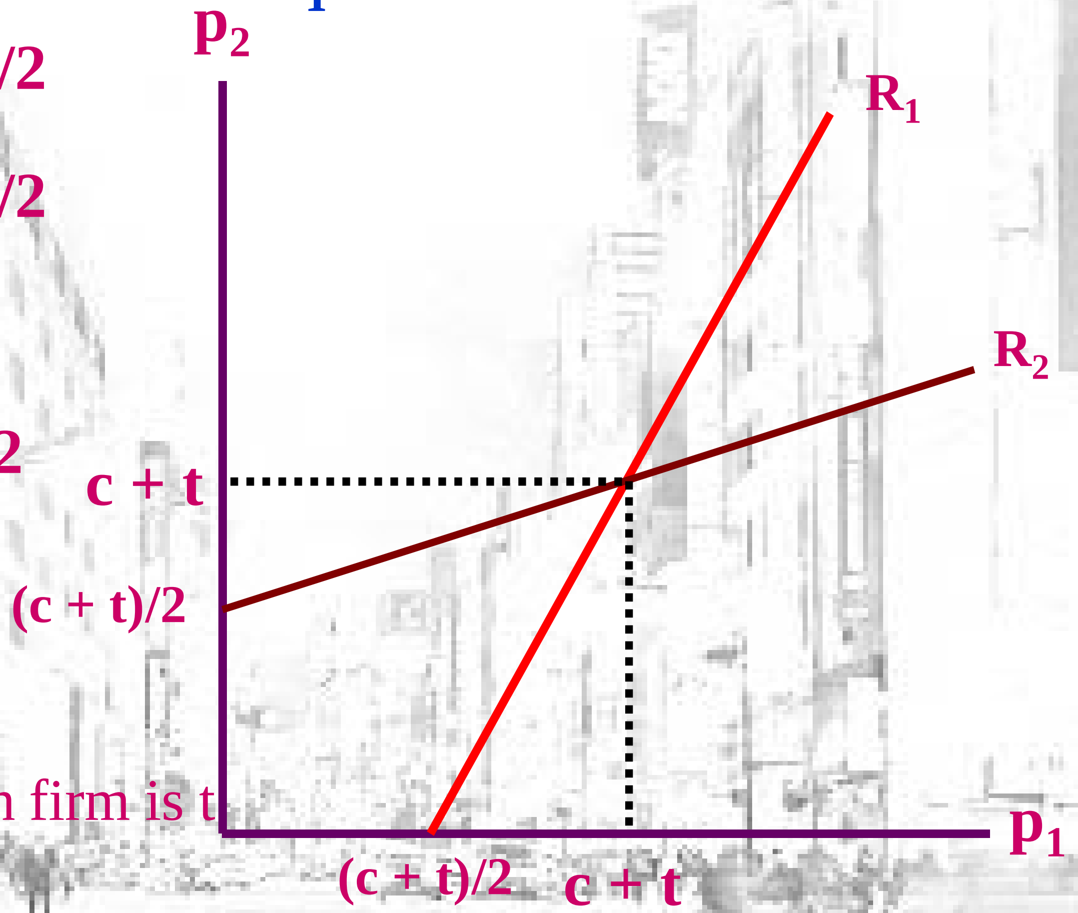
$p^*_2 = (p_1 + t + c)/2$

$2p^*_2 = p_1 + t + c$

$= p_2/2 + 3(t + c)/2$

$\therefore\ p^*_2 = t + c$

$\therefore\ p^*_1 = t + c$



Profit per unit to each firm is t

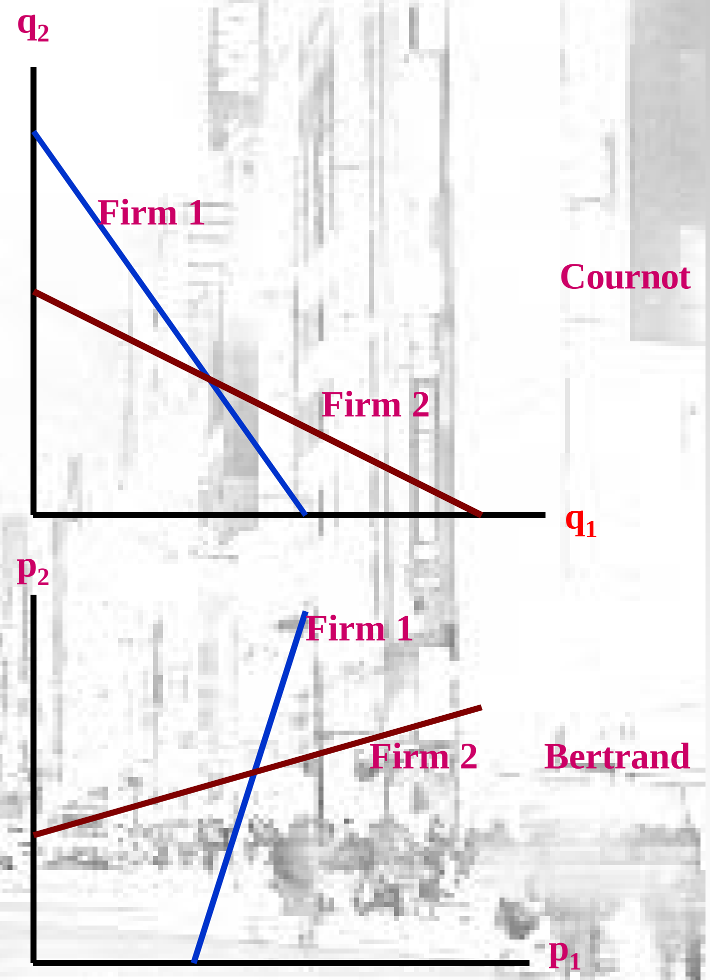
Aggregate profit to each firm is Nt/2

Bertrand competition 3

- **Two final points on this analysis**
- ***t* is a measure of transport costs**
  - **it is also a measure of the value consumers place on getting their most preferred variety**
  - **when *t* is large competition is softened**
    - ***and profit is increased***
  - **when *t* is small competition is tougher**
    - ***and profit is decreased***
- **Locations have been taken as fixed**
  - **suppose product design can be set by the firms**
    - ***balance "business stealing" temptation to be close***
    - ***against "competition softening" desire to be separate***

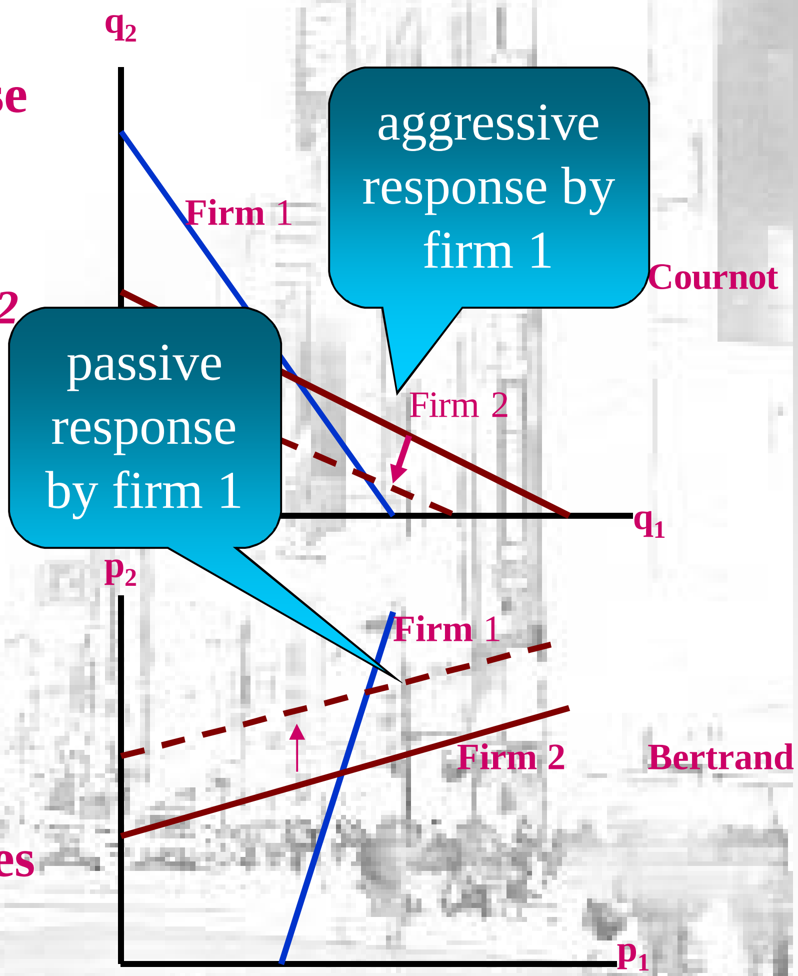
Strategic complements and substitutes

- **Best response functions are very different with Cournot and Bertrand**
  - **they have opposite slopes**
  - **reflects very different forms of competition**
  - **firms react differently e.g. to an increase in costs**



Strategic complements and substitutes

- suppose firm 2's costs increase
- this causes Firm 2's Cournot best response function to fall
  - *at any output for firm 1 firm 2 now wants to produce less*
- firm 1's output increases and firm 2's falls
- Firm 2's Bertrand best response function rises
  - *at any price for firm 1 firm 2 now wants to raise its price*
- firm 1's price increases as does firm 2's



Strategic complements and substitutes 2

- **When best response functions are upward sloping (e.g. Bertrand) we have *strategic complements***
  - **passive action induces passive response**
- **When best response functions are downward sloping (e.g. Cournot) we have *strategic substitutes***
  - **passive actions induces aggressive response**
- **Difficult to determine strategic choice variable: price or quantity**
  - **output in advance of sale – probably quantity**
  - **production schedules easily changed and intense competition for customers – probably price**

Assume payoff (ie. profit) u for strategies (ie. prices, quantities) s

The necessary first order condition (FOC) for player i is

$$\frac{\partial u_i(s_i, s_{-i})}{\partial s_i} = 0$$

The Nash equilibrium is typically calculated by solving the system of equations determined by the FOC:s for each player.

Consider a situation with two players (i and j). By totally differentiating the necessary FOC and noting that $s_i = r_i\left(s_{-i}\right)$

$$\frac{\partial^2 u_i(r_i(s_{-i}), s_j)}{\partial s_i^2} r_i'(s_{-i}) + \frac{\partial^2 u_i(s_i, s_j)}{\partial s_i\, \partial s_j} = 0$$

the slope of player i's reaction function can be found to be

- $$r_i'(s_{-i}) = -\frac{\dfrac{\partial^2 u_i(s_i, s_j)}{\partial s_i\, \partial s_j}}{\dfrac{\partial^2 u_i(s_i, s_j)}{\partial s_i^2}}$$

Because we have assumed concavity it follows from this that

$$sign\{r_i'\} = sign\left\{\frac{\partial^2 u_i(s_i, s_j)}{\partial s_i\, \partial s_j}\right\}$$

Consequently, the reaction function is upward (downward) sloping if and only if

$$\frac{\partial^2 u_i(s_i, s_j)}{\partial s_i\, \partial s_j} > 0 \quad (< 0)$$