Deriving the Lerner index for a monopoly

Let's assume a monopolist with a cost function $C(q)$ facing a market demand $q = D(p)$. To maximize profits, he sets an optimal price p^m (a monopoly can choose either price or quantity as decision variable).

$$\max_p pq - C(q) = pD(p) - C(D(p)$$

gives us the first order condition

$$1D(p^m) + p^m D^{'}(p^m) - C^{'}(D(p^m))D^{'}(p^m) = 0.$$

Dividing this by $p^m D^{'}(p^m)$ and arranging gives us

$\dfrac{p^m - C^{'}(D(p^m))}{p^m} = -\dfrac{D(p^m)1}{D^{'}(p^m)p^m}$. But the demand's price elasticity is $\eta = -\dfrac{\frac{dD(p)}{D(p)}}{\frac{dp}{p}} = -\dfrac{dD(p)p}{dpD(p)}$.

Noticing that $dp = 1$, we get $\dfrac{1}{\eta} = -\dfrac{D(p)}{D^{'}(p)p}$. So

$$\frac{p^m - C^{'}(D(p^m))}{p^m} = \frac{1}{\eta}$$