Industrial organization-micro3-2019

INDIVIDUAL STUDY: BERTRAND AND COURNOT COMPETITION WITH DIFFERENTIATED MARKETS

Consider a duopolistic industry producing differentiated products. To simplify the exposition, we here assume production to be costless.

We assume the following inverse demand structure for the two products:

$$p_1 \quad = \quad \alpha - \beta q_1 - \gamma q_2 \qquad \textit{and} \qquad p_2 \quad = \quad \alpha - \beta q_2 - \gamma q_1 \,, \qquad (7)$$

where $\beta > 0 \quad and \quad \beta^2 > \gamma^2$.

$\beta^2 > \gamma^2$ is important as it captures that the own-price effect dominates relative to the cross-price effect.

The inverse demand structure (7) can be shown to be equivalent with the following system of direct demand functions:

$$q_1 \quad = \quad a - b\, p_1 + d\, p_2 \qquad \textit{and} \qquad q_2 \quad = \quad a - b\, p_2 + d\, p_1 \,, \qquad (8)$$

where $a = \dfrac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}$, $b = \dfrac{\beta}{\beta^2 - \gamma^2}$ *and* $d = \dfrac{\gamma}{\beta^2 - \gamma^2}$.

We can define $\delta = \dfrac{\gamma^2}{\beta^2}$ as a measure of the degree of differentiation between the brands. The brands are highly differentiated when $\delta \to 0$. The brands are almost homogenous when $\delta \to 1$ (and $\gamma > 0$).

COURNOT COMPETITION

$$\max_{q_i} \pi_i(q_i, q_j) \quad = \quad q_i \left(\alpha - \beta q_i - \gamma q_j\right) \quad , i, j = 1,2 , i \neq j . \qquad (9)$$

Reaction functions

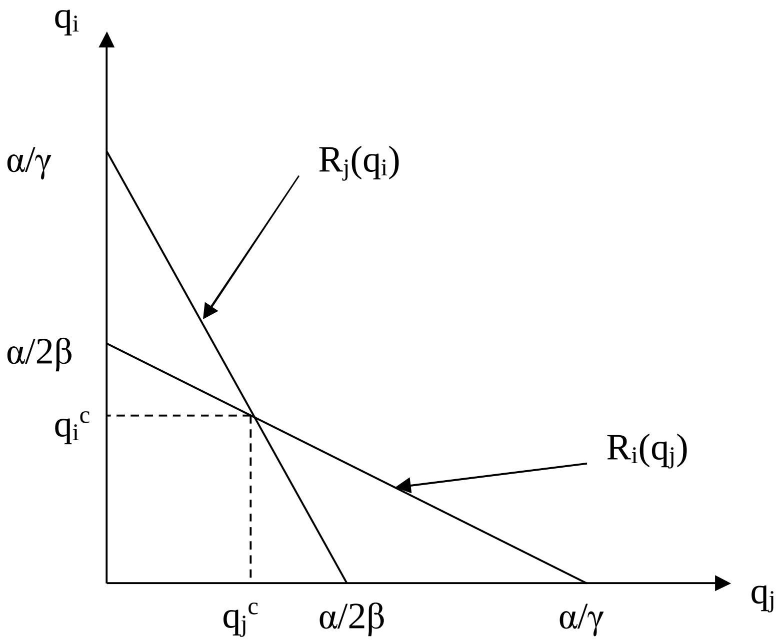
$$q_i \quad = \quad \frac{\alpha - \gamma q_j}{2\beta} \qquad , i, j = 1,2 , i \neq j \qquad (10)$$

Note that

$$\frac{\partial^2 \pi_i}{\partial q_i \, \partial q_j} \quad = \quad -\gamma \quad .$$

Thus, we can conclude that the production decisions are strategic substitutes if $\gamma > 0$.

Figure Reaction functions for quantity competition with differentiated products.



Note, the slope of the reaction function is $-\frac{\gamma}{2\beta}$.

Solving the system of equations defined by the reaction functions (10) we find that the Cournot equilibrium is given by

$$q_i^C \quad = \quad \frac{\alpha}{2\beta + \gamma} \quad . \tag{11}$$

The price and profit associated with (11) are given by

$$p_i^C \quad = \quad \frac{\alpha\,\beta}{2\beta + \gamma} \qquad \textit{and} \qquad \pi_i^C \quad = \quad \frac{\alpha^2\,\beta}{(2\beta + \gamma)^2} \quad . \tag{12}$$

We can conclude: Under Cournot competition with differentiated products, the profits of firms increase when the products become more differentiated.

In other words, the firms have incentives to differentiate themselves.

BERTRAND COMPETITION

$$\max_{P_i} \pi_i(p_i, p_j) \quad = \quad p_i\left(a - b\,p_i + d\,p_j\right) \quad , i, j = 1,2\,, i \neq j \tag{13}$$

Reaction functions:

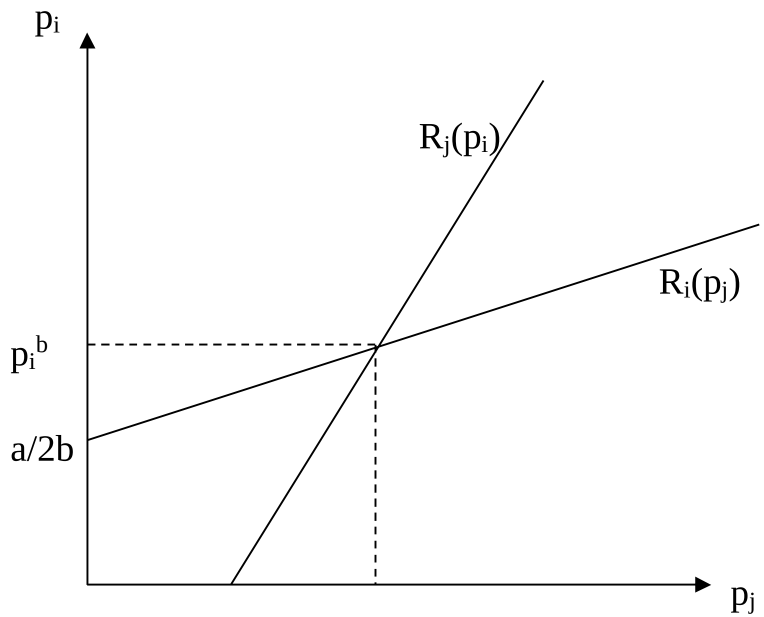
$$p_i = \frac{a + d\, p_j}{2b} \qquad , i, j = 1,2 , i \neq j \qquad (14)$$

Note that

$$\frac{\partial^2 \pi_i}{\partial p_i\, \partial p_j} = d \quad .$$

Thus, we can conclude that the price decisions are strategic complements if $d > 0$.

Figure Reaction functions for price competition with differentiated products.



a/2b P_j^b

Note, the slope of the reaction function is $\dfrac{d}{2b}$.

Solving the system of equations defined by the reaction functions (14) we find that the Bertrand equilibrium is given by

$$p_i^B = \frac{a}{2b-d} = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma} . \qquad (15)$$

The price and profit associated with (15) are given by

$$q_i^B = \frac{ab}{2b-d} \quad and \quad \pi_i^B = \frac{a^2 b}{(2b-d)^2} = \frac{\beta\alpha^2(\beta-\gamma)^2}{(2\beta-\gamma)^2} . \qquad (16)$$

We can conclude the profits of firms are also increasing as a function of the degree of differentiation when firms are engaged in Bertrand competition.

COURNOT VERSUS BERTRAND

Comparing the prices implied by Cournot and Bertrand competition we find that

$$p_i^C - p_i^B \quad = \quad \frac{\alpha\,\beta}{2\beta+\gamma} \quad - \quad \frac{a}{2b-d} \quad = \quad \frac{\alpha}{4\dfrac{\beta^2}{\gamma^2}-1} \quad > \quad 0 \qquad .$$

Consequently, Bertrand competition is more intense than Cournot competition. More precisely,

(a) The market price under Cournot competition is higher than that under Bertrand competition.

(b) The more differentiated the products are, the smaller is the difference between the Cournot and Bertrand prices.

Intuition: Under Cournot competition each firm expects the rival to hold its output constant. Hence, each firm would maintain a low output level, since it is aware that a unilateral output expansion would result in a drop in the market price. In contrast, under Bertrand competition each firm assumes that the rival holds its price constant. Hence, an output expansion will not result in a price reduction. Therefore, the production is higher under Bertrand than under Cournot competition.

Bertrand Competition with Asymmetric Firms

Let us now consider a duopoly where firms 1 and 2 have asymmetric costs. Assume that firm 1 and 2 have marginal costs c_1 and c_2 with $c_1 < c_2$.

To keep the analysis simple, let us focus on the case where firm i's demand function is given by

$$
\begin{aligned}
D_i &= 1 - p_i \,, \quad if \quad p_i < p_j \,, \\
D_i &= 0 \,, \quad if \quad p_i > p_j \,, \\
D_i &= \frac{1}{2}(1 - p_i) \,, \quad if \quad p_i = p_j \,.
\end{aligned}
$$

We have to separate two cases: Large asymmetries and small asymmetries.

Large Asymmetries:

If $c_2 > \frac{1}{2}(1+c_1)$, at equilibrium firm 1 will set $p_1^m = \frac{1}{2}(1+c_1)$, and firm 2 a price $p_2 = c_2 > p_1^m$. Firm 1 will get monopoly profits $\frac{1}{4}(1-c_1{}^2)$, and firm 2 zero profit.

In the study of innovations, this case corresponds to a *drastic innovation*, that is an innovation that makes firm 1 so much more competitive than its rival that it can behave as a monopolist.

Small Asymmetries:

Consider now the case where the firms' costs are close enough meaning that $c_1 \quad < \quad c_2 \quad < \quad \frac{1}{2}(1+c_1)$.

The following price combination is now a Nash equilibrium:

$$(p_1^*, p_2^*) \quad = \quad (c_2 - \varepsilon, c_2) \, .$$

Firm 2 charges its marginal cost, and firm 1 a price slightly below that.
At this equilibrium, firm 1 makes profits $(c_2 - c_1)(1-c_2)$, and firm 2 gains zero.

(Strictly speaking, there exist other Nash equilibria of this game, but these are less "reasonable". Consider a price $p \in]c_1, c_2[$. It can be verified that the pair $(p_1^{**}, p_2^{**}) = (p-\varepsilon, p)$ represents a Nash equilibrium of the game. It seems reasonable that the equilibrium $(p_1^*, p_2^*) \quad = \quad (c_2 - \varepsilon, c_2)$ would be reached, because it gives higher profits to firm 1 while keeping the same profits (zero) for firm 2 ("elimination of weakly dominated strategies").)

D. INDIVIDUAL STUDY: PRICE COMPETITION WITH SEQUENTIAL DECISIONS

Sequential decisions: Firms make their decisions sequentially.

Distribution of roles: Leader (first mover) and follower (second mover).

The decision of the leader serves as a commitment and the follower must make its decision taking the leader's commitment as a restriction.

The follower (firm 2): Sets it price according to its reaction function given by (14), i.e.

$$p_2 \quad = \quad R_2(p_1) \quad = \quad \frac{a + d\, p_1}{2b} \qquad .$$

Taking this reaction function into account the leader (firm 1) makes it price commitment in order to solve

$$\max_{p_1} \pi_1(p_1, p_2) \quad = \quad p_1 \left(a - b\, p_1 + d\, p_2 \right)$$

under the restriction that

$$p_2 \quad = \quad R_2(p_1) \quad = \quad \frac{a + d\, p_1}{2b} \qquad .$$

Numerical illustration

Let the demand functions be given by

$$q_1 \quad = \quad 168 - 2\,p_1 + \; p_2 \qquad and \qquad q_2 \quad = \quad 168 - 2\,p_2 + \; p_1 .$$

Applying (15) and (16) we find the Bertrand equilibrium (with simultaneous decisions) to be $p_i^B = 56 \quad and \quad \pi_i^B = 6272$.

Now the follower's reaction function is given by

$$p_2 \quad = \quad R_2(p_1) \quad = \quad \frac{168 + \; p_1}{4}$$

and the leader's problem is that of solving

$$\max_{p_1} \pi_1(p_1, R_2(p_1)) \quad = \quad p_1 \left(168 - 2q_1 + \frac{168 + \; p_1}{4} \right).$$

Solving this problem yields leader's optimal price $p_1^S = 60$, which when substituted back into the follower's reaction function shows that the follower's price would be $p_2^S = 57$. Substituting these prices back into the profit function we find

the leader's profit: $\pi_1^S = 6300$

the follower's profit: $\pi_2^S = 6498$.

From this numerical example we can see that $\pi_2^S > \pi_1^S > \pi_i^B$!

This numerical example illustrates the following general insight.

Under price competition with sequential decisions

(a)Both firms collect a higher profit under a sequential-moves game than under the simultaneous Bertrand game.

(b) The firm that sets its price first (the leader) makes a lower profit than the firm that sets it price second (the follower).

This result exemplifies the following even more general principle: There are second-mover advantages in all games where the decisions are strategic complements.

For price competition there are second-mover advantages, because the follower can undercut the leader.

In contrast, under Cournot competition with sequential decisions there are first-mover advantages (compare to the analysis of Stackelberg competition).