GIS-E3010 Least-Squares Methods in Geoscience

Lecture 7

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Learning objectives

- To understand
  - Bundle block adjustment (continue)

Bundle triangulation/adjustment, constraints

- Usually, some parameters or values of their functions are known so accurately that we can keep them as known constants
- This can be taken account by including constraints equations in bundle block adjustment. These constraints fix wanted parameters or their functions

3

Bundle triangulation/adjustment, constraints

- Constraints equation is $C\Delta = d$ and it establishes a bordered structure of a normal equation (LS condition $v^T Pv + k(C\Delta - d) = \min$)

$$\begin{bmatrix} N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k \end{bmatrix} = \begin{bmatrix} u \\ d \end{bmatrix}$$

(k includes Lagrange multipliers)

- Examples of constraints equations: a point lays on a line or on a circumference of a circle

4

Bundle triangulation/adjustment, constraints

- If basic assumptions are valid, a bordered normal equation is regular (non-singular), and therefore we get a unique solution of a system
- However, a coefficient matrix is not positively definite, and therefore we cannot use *Cholesky decomposition*.
- Instead we can use e.g. *LU decomposition* (*Gaussian elimination method*).

Bundle triangulation/adjustment, minimum constrained solution

- If we don't have (real) datum information, we can remove datum defect by defining the object coordinate system using (fictive) minimum constraints.
- As a result, we get the right shape of the model.
- It's crucially important to have the minimum amount of constraints equations that equal to datum defect, in which case constraints have no determinist effect to the shape of a reconstructed model

Bundle triangulation/adjustment, minimum constrained solution

- Minimum constraints can be selected in infinitely many ways

- A simple alternative is to fix 7 parameters or 7 functions of parameters by giving them (in principle) arbitrary values
- Such solution is called as a (minimum constrained) basic solution (outer constrained)

- A more complicated, but a numerically and statistically better alternative, method is so called minimum norm solution, which fulfills additional condition $\|\hat{x}\| = \min$
- That is found by using, e.g. singular value decomposition or by using inner constraints

7

Bundle triangulation/adjustment, minimum constrained solution

- All minimum constraints give identical
  - *Residual vector*
  - *Sum of squares of residuals*
  - *Standard deviation of unit weight*.
- However, the weight matrix of parameters change when minimum constraints change
- Therefore, the shape of reconstructed objects is dependent on the definition of datum.
- We can prove that the a minimum norm solution is also a minimum-variance solution, and therefore it is recommended

8

Bundle triangulation/adjustment, minimum constrained <u>basic solution</u>

- In a minimum constrained basic solution, we fix 7 parameters or functions of parameters by giving them arbitrary values
- The algorithm of such case is identical to the previously introduced case, in which part of the observations were known
- If we select 7 object coordinates to be fixed, our constrain equations become

$$\ddot{\Delta}_1 = 0 \quad \text{i.e.} \quad \begin{bmatrix} 0 & I & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \end{bmatrix} = 0$$

9

Bundle triangulation/adjustment, minimum constrained <u>basic solution</u>

$$\begin{bmatrix} 0 & I & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \end{bmatrix} = 0$$

- However, it appears that in this special case a constrained solution changes back to ordinary LS adjustment
- If we eliminate $\ddot{\Delta}_1$ from normal equations, we get error and normal equations

$$v' = \begin{bmatrix} \dot{A} & \ddot{A}_2 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_2 \end{bmatrix} - l \quad \text{therefore} \quad \begin{bmatrix} \dot{A}^T \dot{A} & \dot{A}^T \ddot{A}_2 \\ \ddot{A}_2^T \dot{A} & \ddot{A}_2^T \ddot{A}_2 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_2 \end{bmatrix} = \begin{bmatrix} \dot{A}^T l \\ \ddot{A}_2^T l \end{bmatrix}$$

i.e. we remove corresponding columns from error equation, and columns and rows from normal equations

10

Bundle triangulation/adjustment, minimum constrained basic solution

- We get similar type of equations if we fix, instead of object points, parameters of exterior orientation.
- Examples:
  - Two XYZ points and one Z point
  - The exterior orientation of one image and one coordinate of one projection center (relative orientation with successive images!)
  - Projection centers of two images and ω rotation of one image (relative orientation with rotating images!)
  - One XYZ point, azimuth angle, height angle and distance to another point, and furthermore height angle to the third point

11

Bundle triangulation/adjustment, minimum constrained basic solution

- An example of outer constraints: how to fix one 3D object coordinate

Outer constraints:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dX_1 \\ dY_1 \\ dZ_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C\Delta = d$$

A minimum-constrained normal equation:

$$\begin{bmatrix} N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ \lambda \end{bmatrix} = \begin{bmatrix} u \\ d \end{bmatrix}$$

In which $N = A^T PA$ and $u = A^T Pl$

12

Bundle triangulation/adjustment, <u>minimum norm solution</u>

- Minimum norm condition $\|\hat{x}\| = \min$ can be written also

  $$\|\hat{x}\| = \min \quad \Leftrightarrow \quad \hat{x} \perp N(A) \quad \Leftrightarrow \quad E^T \hat{x} = 0$$

  in which N(A) is the kernel of *A*, and *E* is such a matrix whose columns establish the (some) base of kernel, i.e. $AE = 0$
- Minimum norm constraints $E^T \hat{x} = 0$ is also called as <u>inner constraints</u>.

13

Rank-deficient LS adjustment, <u>inner constraints</u>

- Many geodetic and photogrammetric observations are invariants to similarity transformation or to some special case of it. In such case, we have **datum defect**.
- Then, however, it is especially easy to find inner constraints starting from the (invariant) transformation

1D: $x' = \lambda(x - x_0)$ 2D: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \lambda R \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$ 3D: $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \lambda R \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$

14

Rank-deficient LS adjustment, inner constraints

- 1D: (translation & scale) $x' = \lambda(x - x_0)$

For small changes:

$$dx = dx_0 + x\,d\lambda = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} dx_0 \\ d\lambda \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

Rank-deficient LS adjustment, inner constraints

- 2D: (2 translations, rotation& scale) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \lambda R \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$

For small changes:

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} dx_0 \\ dy_0 \end{bmatrix} + \begin{bmatrix} 0 & d\alpha \\ -d\alpha & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + d\lambda \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & y & x \\ 0 & 1 & -x & y \end{bmatrix} \begin{bmatrix} dx_0 \\ dy_0 \\ d\alpha \\ d\lambda \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & y_1 & x_1 \\ 0 & 1 & -x_1 & y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & y_n & x_n \\ 0 & 1 & -x_n & y_n \end{bmatrix}$$

Rank-deficient LS adjustment, <u>inner constraints</u>

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \lambda R \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

- 3D: (3 translations, 3 rotations & scale)

For small changes:

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} dx_0 \\ dy_0 \\ dz_0 \end{bmatrix} + \begin{bmatrix} 0 & -d\chi & d\beta \\ d\chi & 0 & -d\alpha \\ -d\beta & d\alpha & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + d\lambda \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y & x \\ 0 & 1 & 0 & -z & 0 & x & y \\ 0 & 0 & 1 & y & -x & 0 & z \end{bmatrix} \begin{bmatrix} dx_0 \\ dy_0 \\ dz_0 \\ d\alpha \\ d\beta \\ d\chi \\ d\lambda \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & z_1 & -y_1 & x_1 \\ 0 & 1 & 0 & -z_1 & 0 & x_1 & y_1 \\ 0 & 0 & 1 & y_1 & -x_1 & 0 & z_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & z_n & -y_n & x_n \\ 0 & 1 & 0 & -z_n & 0 & x_n & y_n \\ 0 & 0 & 1 & y_n & -x_n & 0 & z_n \end{bmatrix}$$

17

Bundle triangulation/adjustment, <u>minimum norm solution</u>

- When the values of a 3D similarity transformation are small, the effect of the transformation to <u>the coordinates of an object point</u> j can be defined using differential equations

$$\ddot{\Delta} = \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_j = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y & X \\ 0 & 1 & 0 & -Z & 0 & X & Y \\ 0 & 0 & 1 & Y & -X & 0 & Z \end{bmatrix}_j \begin{bmatrix} dt_X \\ dt_Y \\ dt_Z \\ d\Omega \\ d\Phi \\ d\mathrm{X} \\ d\lambda \end{bmatrix} = \ddot{E}_j d$$

18

Bundle triangulation/adjustment, <u>minimum norm solution</u>

- The effect to the exterior orientation parameters of an image *i* is

$$\dot{\Delta} = \begin{bmatrix} dX_0 \\ dY_0 \\ dZ_0 \\ d\omega \\ d\varphi \\ d\kappa \end{bmatrix}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_0 & -Y_0 & X_0 \\ 0 & 1 & 0 & -Z_0 & 0 & X_0 & Y_0 \\ 0 & 0 & 1 & Y_0 & -X_0 & 0 & Z_0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_i \begin{bmatrix} dt_X \\ dt_Y \\ dt_Z \\ d\Omega \\ d\Phi \\ d\mathrm{X} \\ d\lambda \end{bmatrix} = \dot{E}_i d$$

Bundle triangulation/adjustment, minimum norm solution

- We can add constraints equations to the adjustment
  - the effect to orientation parameters of an image

$$E = \begin{bmatrix} \dot{E} \\ 0 \end{bmatrix}$$

  - the effect to object coordinates $E = \begin{bmatrix} 0 \\ \ddot{E} \end{bmatrix}$
  - Both $E = \begin{bmatrix} \dot{E} \\ \ddot{E} \end{bmatrix}$
- Therefore, minimum norm constraints are $E^T \Delta = 0$ and normal equation is $\begin{bmatrix} N & E \\ E^T & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}$

Bundle triangulation/adjustment, minimum norm solution

- For example if only the constraints of object coordinates are included in the normal equation

$$\begin{bmatrix} N & E \\ E^T & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}$$

we actually get

$$\begin{bmatrix} \dot{N} & \overline{N} & 0 \\ \overline{N}^T & \ddot{N} & \ddot{E} \\ 0 & \ddot{E}^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \ddot{u} \\ 0 \end{bmatrix}$$

21

Bundle triangulation/adjustment, minimum norm solution

$$E^T \Delta = 0$$

- An example of inner constraints matrix $\ddot{E}$ when we have 3 object points

$$\ddot{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_1 & -Y_1 & X_1 \\ 0 & 1 & 0 & -Z_1 & 0 & X_1 & Y_1 \\ 0 & 0 & 1 & Y_1 & -X_1 & 0 & Z_1 \\ 1 & 0 & 0 & 0 & Z_2 & -Y_2 & X_2 \\ 0 & 1 & 0 & -Z_2 & 0 & X_2 & Y_2 \\ 0 & 0 & 1 & Y_2 & -X_2 & 0 & Z_2 \\ 1 & 0 & 0 & 0 & Z_3 & -Y_3 & X_3 \\ 0 & 1 & 0 & -Z_3 & 0 & X_3 & Y_3 \\ 0 & 0 & 1 & Y_3 & -X_3 & 0 & Z_3 \end{bmatrix} \qquad \ddot{\Delta} = \begin{bmatrix} dX_1 \\ dY_1 \\ dZ_1 \\ dX_2 \\ dY_2 \\ dZ_2 \\ dX_3 \\ dY_3 \\ dZ_3 \end{bmatrix}$$

22

Bundle triangulation/adjustment, minimum norm solution

- Constraints can be applied also to the certain part of object points

$$C\ddot{\Delta}_1 = d \qquad \text{i.e.} \qquad \begin{bmatrix} 0 & C & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \end{bmatrix} = d$$

- Object points $\ddot{\Delta}_1$ are constrained but $\ddot{\Delta}_2$ are not

Bundle triangulation/adjustment, minimum norm solution

- Bordered normal equation becomes

$$\begin{bmatrix} \dot{N} & \overline{N}_1 & \overline{N}_2 & 0 \\ \overline{N}_1^T & \ddot{N}_1 & 0 & C^T \\ \overline{N}_2^T & 0 & \ddot{N}_2 & 0 \\ 0 & C & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \ddot{u}_1 \\ \ddot{u}_2 \\ d \end{bmatrix}$$

- If we solve the third equation block $\ddot{\Delta}_2 = \ddot{N}_2^{-1}(\ddot{u}_2 - \overline{N}_2^T \dot{\Delta})$ and place it to the first equation block, we get

$$\begin{bmatrix} \dot{N} - \overline{N}_2 \ddot{N}_2^{-1} \overline{N}_2^T & \overline{N}_1 & 0 \\ \overline{N}_1^T & \ddot{N}_1 & C^T \\ 0 & C & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \lambda \end{bmatrix} = \begin{bmatrix} \dot{u} - \overline{N}_2 \ddot{N}_2^{-1} \ddot{u}_2 \\ \ddot{u}_1 \\ d \end{bmatrix}$$

Bundle triangulation/adjustment, minimum norm solution

$$\begin{bmatrix} \dot{N} - \overline{N}_2 \ddot{N}_2^{-1} \overline{N}_2^T & \overline{N}_1 & 0 \\ \overline{N}_1^T & \ddot{N}_1 & C^T \\ 0 & C & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta}_1 \\ \lambda \end{bmatrix} = \begin{bmatrix} \dot{u} - \overline{N}_2 \ddot{N}_2^{-1} \ddot{u}_2 \\ \ddot{u}_1 \\ d \end{bmatrix}$$

- Now we notice that this result is a reduced normal equation that has been bordered with constraints equations

25

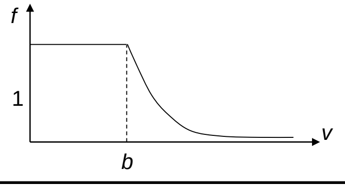
Robustified LS adjustment (weight iteration)

- The principle is simple: we repeat adjustments in such a way that new weights (k=1,2,...) are computed from residuals of previous iteration round using the equation

$$p_i^{k+1} = p_i^k \, f(v_i^k)$$

in which f is a properly selected non-growing function, e.g.

$$f(v) = \begin{cases} 1 & \text{if } \; |v| \le b \\ e^{-a(|v|-b)} & \text{if } \; |v| > b \end{cases}$$



26

Robustified LS adjustment (weight iteration)

- If the residual of an observation is large, such observation will get a small weight in the next iteration round
- Because of that, the residual of such observation will increase further
- As a result, we (hopefully) get a situation in which the weights of a gross error observations are close to zero, and residuals are good estimates to gross errors
- A threshold *b* is typically proportional to the standard deviation of observations $b = 2\sigma_0$ or $b = 3\sigma_0$

Robustified LS adjustment (weight iteration)

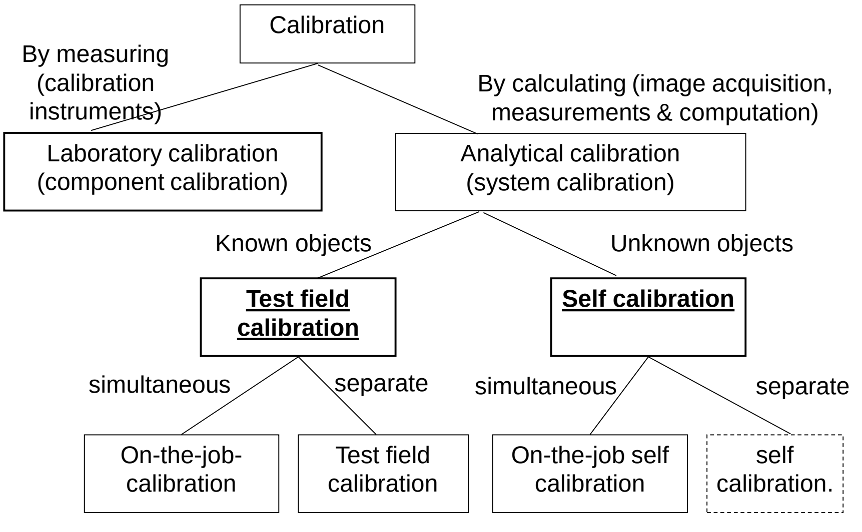
- The Robustified LS adjustment method is a simple alternative to automatize the searching of gross errors
- The method functions well when we have a lot of redundancy
- In a non-linear case, we might face problems, if initial values of parameters are not close enough to the correct ones
- In such case, the changing of weights is recommended, and can be applied after couple of adjustment iterations

Camera calibration

- Camera calibration includes
  - Geometric calibration
  - Evaluation of image quality and
  - Radiometric calibration (perhaps)
- Here we examine only geometric calibration
- We want to solve
  - Interior orientation (principle point and camera constant)
  - Systematic errors (deformations of image plane, lens distortions, refraction of atmosphere)
- We need <u>extended collinearity</u> equations

29

Camera calibration



30

Test field /range calibration

- Test field calibration is an analytical calibration method in which interior orientation and systematic errors are defined from images by using a known test field
- In principle, a test field can be any target with known geometry
- Usually, a test field includes a set of points, whose coordinates are known with a great accuracy
- Instead of points, we can also use lines and planes for calibration

Test field /range calibration

- One problem with field calibration is that even if the measurements are accurate, the results do not necessarily be valid in operative conditions
- This problem can be avoided only, if we solve calibration and object points simultaneously
- This is possible only when we measure relatively small objects
- If an object to be measured is placed inside of a test field, or a test field is constructed around a target (e.g. a car measurement system).
- Such ”local” calibration is called as *on-the-job calibration*.

Basic method of test field calibration

- In the basic method of test field calibration, interior orientation and systematic errors are defined by taking images from a test field that includes known points, and by solving simultaneously
  - Exterior orientation parameters and
  - Calibration parameters of extended collinearity equations (additional parameters)
- In principle, one image is enough, however, including more images we can increase the accuracy (by improving the point distribution in the image plane and by improving imaging geometry)
- If we have a lot of points in our test field, we have better possibilities to solve all deformations reliably

33

Basic method of test field calibration

- A functional model of a test field calibration is the extended collinearity equations

$$\begin{cases} x - x_0 + dx = -c\dfrac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \\ y - y_0 + dy = -c\dfrac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \end{cases}$$

- Alternatives to extend (*dx* & *dy*) the model:
  - Physical approach
  - Mathematical approach

34

Basic method of test field calibration

- Typical physical model is e.g. Brown's model

$$dx = x_0 - \frac{\bar{x}}{c}dc + \bar{x}a_1 + \bar{y}a_2 + \bar{x}r^2K_1 + \bar{x}r^4K_2 + \bar{x}r^6K_3 + (2\bar{x}^2 + r^2)P_1 + 2\bar{x}\bar{y}$$

$$dy = y_0 - \frac{\bar{y}}{c}dc + \bar{y}r^2K_1 + \bar{y}r^4K_2 + \bar{y}r^6K_3 + 2\bar{x}\bar{y}P_1 + (2\bar{y}^2 + r^2)P_2$$

in which $\bar{x} = x - x_0$, $\bar{y} = y - y_0$ and $r = \sqrt{\bar{x}^2 + \bar{y}^2}$

Basic method of test field calibration

- In order to solve unknown parameters we get an error equation

$$v = \dot{A}\dot{\Delta} + \ddot{A}\ddot{\Delta} - f \qquad \text{i.e.} \qquad v = \begin{bmatrix} \dot{A} & \ddot{A} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \dddot{\Delta} \end{bmatrix} - f$$

  In which $\dot{\Delta}$ contains <u>improvements of exterior orientations</u> and $\ddot{\Delta}$ contains <u>additional parameters</u>. Notice that $\ddot{\Delta} = 0$, because the object coordinates are known
- An explicit non-linear model => linearization and LS adjustment with observation equations

Basic method of test field calibration

- When the model is extended with additional parameters, there is a danger that we are not able to solve all parameters reliably (overparametrization problem)
- It is important to distinguish two different cases:
  - We cannot solve a single parameter
  - It's not possible to define a linear combination of two or more parameters. Such parameters has perfect or high correlation. If the number of correlating parameters is *k*, we have to remove at least one, but maximum of *k*-1 parameters, from the adjustment

Basic method of test field calibration

- Examples of parameters that typically has strong correlations:
  - From interior and exterior orientation: x_0 & X_0 , y_0 & Y_0 and c & Z_0
  - Principle point and tangential lens distortion: (x_0, y_0) and (P_1, P_2). This is understandable, because tangential lens distortion parameters corresponds to a case that we place a thin prism in the front of objective. Correlation increases when camera constant grows
  - Parameters of radial lens distortion K_i

Basic method of test field calibration

- High correlation between interior and exterior orientation parameters can be avoided by ensuring sufficient variation of scale within image-object transformation. This happens if we
  - Use a 3D test field (the variation of points should be large in all directions) and/or
  - Using convergent imaging geometry (large variation of image rotation angles)

39

Basic method of test field calibration

- Rotation around optical axis (κ rotation) is enough to significantly reduce correlations between interior and exterior orientation parameters (x_0 & X_0 and y_0 & Y_0). Instead, they have no significant effect on correlations between principle point and tangential lens distortion ((x_0, y_0) & (P_1, P_2))

- Correlation problems can be reduced during the computation, e.g., by following acts:
  - Use orthogonalized functional model
  - *Use weights for additional parameters* (let the expected values of fictive observations to equal to zero)

40

Self calibration

- If we define *"self calibration"* strictly, it means such calibration, in which we do not use any known external information, i.e., calibration is completely based on information from measured corresponding points (image measurements)
- Only condition is that we have two or more (partially) overlapping images

Self calibration

- If we have three images, we are able to establish two 3D models that are identical, if no errors are present
- Systematic image errors cause differences/conflicts that can be used when solving calibration parameters
- It's obvious that the more we have overlapping images, the better possibilities we have in order to make a successful self-calibration
- Even if a self-calibration is possible by using only image information, it is advantageous to use all available information (e.g. known coordinates or distances)
- Additional information always improves accuracy and reliability of results. In addition, we can remove or reduce the effect of datum defect

Self calibration

- Such adjustment of an image block, in which interior orientation and systematic errors are solved simultaneously with other unknown parameters, is called *self calibrating bundle block adjustment*
- The functional model of adjustment is the same as in the test field calibration, i.e.

$$\begin{cases} x - x_0 + dx = -c\dfrac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \\ y - y_0 + dy = -c\dfrac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \end{cases}$$

Self calibration

- Error equation is

$$v = \dot{A}\dot{\Delta} + \ddot{A}\ddot{\Delta} + \dddot{A}\dddot{\Delta} - f \qquad \text{i.e.} \qquad v = \begin{bmatrix} \dot{A} & \ddot{A} & \dddot{A} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \\ \dddot{\Delta} \end{bmatrix} - f$$

in which $\dot{\Delta}$ include improvements of interior orientation parameter approximations, $\ddot{\Delta}$ contain improvements of additional parameter approximations and $\dddot{\Delta}$ include improvements of object coordinate approximations

Self calibration

- LS condition gives a normal equation

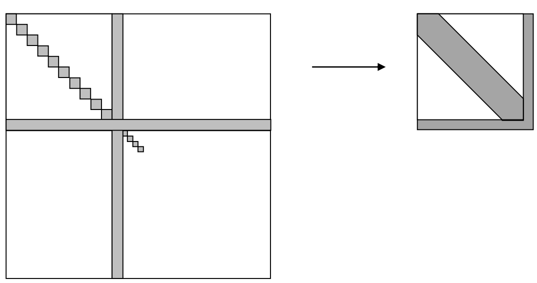
$$\begin{bmatrix} \dot{A}^T P\dot{A} & \dot{A}^T P\ddot{A} & \dot{A}^T P\dddot{A} \\ \ddot{A}^T P\dot{A} & \ddot{A}^T P\ddot{A} & \ddot{A}^T P\dddot{A} \\ \dddot{A}^T P\dot{A} & \dddot{A}^T P\ddot{A} & \dddot{A}^T P\dddot{A} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \\ \dddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{A}^T Pf \\ \ddot{A}^T Pf \\ \dddot{A}^T Pf \end{bmatrix}$$

- If we eliminate object coordinates ($\dddot{\Delta}$) we get reduced normal equation (solution for image orientations and additional parameters)

$$\left(\begin{bmatrix} \dot{A}^T P\dot{A} & \dot{A}^T P\ddot{A} \\ \ddot{A}^T P\dot{A} & \ddot{A}^T P\ddot{A} \end{bmatrix} - \begin{bmatrix} \dot{A}^T P\dddot{A} \\ \ddot{A}^T P\dddot{A} \end{bmatrix} (\dddot{A}^T P\dddot{A})^{-1} \begin{bmatrix} \dddot{A}^T P\dot{A} & \dddot{A}^T P\ddot{A} \end{bmatrix} \right) \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{A}^T Pf \\ \ddot{A}^T Pf \end{bmatrix} - \begin{bmatrix} \dot{A}^T P\dddot{A} \\ \ddot{A}^T P\dddot{A} \end{bmatrix} (\dddot{A}^T P\dddot{A})^{-1} \dddot{A}^T Pf$$

Self calibration

- The coefficient matrix of a reduced normal equation is a bordered band matrix, in which the thickness of borders equals to the number of additional parameters



Appendix. The accuracy of bundle block adjustment (this is not presented in the lecture)

Accuracy of bundle block adjustment

- Alternatives to evaluate accuracy:
  - Theoretically
    - Law of random error propagation (inverse method)
    - Simulation
  - Empirically (we use known control points or test fields)
- Here we focus on theoretical alternative, by applying law of random error propagation

Accuracy of bundle block adjustment

- <u>*Variance covariance matrix of parameters*</u> is defined as

$$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$$

- σ_0^2 is the variance of unit weight (reference variance) and $Q_{\Delta\Delta}$ is a cofactor matrix of parameters
- <u>Unbiased estimate of reference variance</u> is computed from adjustment by using the equation

$$\hat{\sigma}_0^2 = \frac{\hat{v}^T P \hat{v}}{r} \qquad r = n - u + s$$

(s is the number of constraints)

49

Accuracy of bundle block adjustment

- From the adjustment, we know (the law of error propagation) that the cofactor matrix is an inverse of coefficient matrix of normal equations (or reduced normal matrix), i.e.

$$Q_{\Delta\Delta} = N^{-1} = (A^T P A)^{-1}$$

- It is important to realize that we are able to compute $Q_{\Delta\Delta}$ even before the solution of adjustment, because the weight and design matrices are known

50

Accuracy of bundle block adjustment

- From the mixed adjustment of image and other observations

$$\begin{bmatrix} \dot{N}+\dot{P} & \overline{N} \\ \overline{N}^T & \ddot{N}+\ddot{P} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{u}+\dot{P}\dot{f} \\ \ddot{u}+\ddot{P}\ddot{f} \end{bmatrix}$$

we get the cofactor matrix of parameters

$$Q_{\Delta\Delta} = \begin{bmatrix} Q_{\dot{\Delta}\dot{\Delta}} & Q_{\dot{\Delta}\ddot{\Delta}} \\ Q^T_{\dot{\Delta}\ddot{\Delta}} & Q_{\ddot{\Delta}\ddot{\Delta}} \end{bmatrix} = \begin{bmatrix} \dot{N}+\dot{P} & \overline{N} \\ \overline{N}^T & \ddot{N}+\ddot{P} \end{bmatrix}^{-1}$$

Accuracy of bundle block adjustment

$$Q_{\Delta\Delta} = \begin{bmatrix} Q_{\dot{\Delta}\dot{\Delta}} & Q_{\dot{\Delta}\ddot{\Delta}} \\ Q^T_{\dot{\Delta}\ddot{\Delta}} & Q_{\ddot{\Delta}\ddot{\Delta}} \end{bmatrix} = \begin{bmatrix} \dot{N}+\dot{P} & \overline{N} \\ \overline{N}^T & \ddot{N}+\ddot{P} \end{bmatrix}^{-1}$$

- For sub-matrices $Q_{\dot{\Delta}\dot{\Delta}}$ and $Q_{\ddot{\Delta}\ddot{\Delta}}$ we get

$$Q_{\dot{\Delta}\dot{\Delta}} = \{(\dot{N}+\dot{P}) - \overline{N}(\ddot{N}+\ddot{P})^{-1}\overline{N}^T\}^{-1}$$

$$Q_{\ddot{\Delta}\ddot{\Delta}} = (\ddot{N}+\ddot{P})^{-1} + (\ddot{N}+\ddot{P})^{-1}\overline{N}^T Q_{\dot{\Delta}\dot{\Delta}}\overline{N}(\ddot{N}+\ddot{P})^{-1}$$

(the 1st term is a weight matrix that corresponds to the exterior orientation and the 2nd term is an addition caused by inaccuracies of exterior orientations)

Accuracy of bundle block adjustment

Cofactor matrix of minimum norm solution

- Here we examine a general case, in which only object points are included in inner constraints, i.e. bordered normal equation is

$$\begin{bmatrix} \dot{N} & \overline{N} & 0 \\ \overline{N}^T & \ddot{N} & \ddot{G} \\ 0 & \ddot{G}^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \ddot{u} \\ 0 \end{bmatrix}$$

- Usually, the primary focus is on $Q_{\ddot{\Delta}\ddot{\Delta}}$
- If we change the order of rows and columns, we get a new version of normal equations

$$\begin{bmatrix} \dot{N} & 0 & \overline{N} \\ 0 & 0 & \ddot{G}^T \\ \overline{N}^T & \ddot{G} & \ddot{N} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{k} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ 0 \\ \ddot{u} \end{bmatrix}$$

Accuracy of bundle block adjustment

$$\begin{bmatrix} \dot{N} & 0 & \overline{N} \\ 0 & 0 & \ddot{G}^T \\ \overline{N}^T & \ddot{G} & \ddot{N} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{k} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ 0 \\ \ddot{u} \end{bmatrix}$$

- If we name

$$Q'_{\dot{\Delta}\dot{\Delta}} = \left[\begin{bmatrix} \dot{N} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \overline{N} \\ \ddot{G}^T \end{bmatrix} \ddot{N}^{-1} \begin{bmatrix} \overline{N}^T & \ddot{G} \end{bmatrix} \right]^{-1}$$

we get

$$Q_{\ddot{\Delta}\ddot{\Delta}} = \ddot{N}^{-1} + \ddot{N}^{-1} \begin{bmatrix} \overline{N}^T & \ddot{G} \end{bmatrix} Q'_{\dot{\Delta}\dot{\Delta}} \begin{bmatrix} \overline{N} \\ \ddot{G}^T \end{bmatrix} \ddot{N}^{-1}$$

Accuracy of bundle block adjustment, approximate evaluation

- Calculation of variance-covariance matrix requires a lot of calculation if image block is large
- Therefore, there are methods how to approximately evaluate accuracy
- This requires pre-knowledge of factors that affect to accuracy
- Accuracy is depended on
  - The accuracy of measurements (standard deviation of weight unit σ_0 and weight matrix P) and
  - The geometry of image block (design matrix A)

Accuracy of bundle block adjustment, approximate evaluation

- The factors that affect to measuring accuracy are usually easy to evaluate
- However, it is more difficult to evaluate the geometric structure of an image block
- In following, we examine the most important factors of accuracy

Accuracy of bundle block adjustment, approximate evaluation

- *Imaging scale*
  - Accuracy is linearly dependent on imaging scale
- *Imaging geometry*
  - Intersecting angle of observation rays has a significant effect to accuracy
  - Convergent imaging has better geometry than the normal case of stereo imaging
  - In the case of the normal case of stereo imaging, the ratio between base and imaging distance is significant
- *The number of image locations*
  - Adding more intersecting rays increases strongly the accuracy, at first (2→3→4 images).
  - True effect is difficult to separate, because if the number of image locations increases, also imaging geometry changes

Accuracy of bundle block adjustment, approximate evaluation

- *The number of object points*
  - Effect of this is (surprisingly) small. Relatively sparse , but evenly distributed, set of control points is sufficient, at least in the design phase
  - Self calibration?
- *Camera constant*
  - If a camera constant shortens, angles of intersecting rays grow (in the normal case of stereo imaging)
  - When a camera constant grows (and imaging scale remains) imaging geometry becomes more homogeneous, which makes also accuracy more homogeneous

Accuracy of bundle block adjustment, approximate evaluation

- In large image blocks, solving of cofactor matrix $Q_{\ddot{\Delta}\ddot{\Delta}}$ using equations

$$Q_{\ddot{\Delta}\ddot{\Delta}} = (\ddot{N} + \ddot{P})^{-1} + (\ddot{N} + \ddot{P})^{-1} \overline{N}^T Q_{\dot{\Delta}\dot{\Delta}} \overline{N} (\ddot{N} + \ddot{P})^{-1}$$

requires a lot of computation!

- We can make computation more efficient if we leave out correlations between points, i.e. calculate only 3x3 sub-matrices

$$Q_{\ddot{\Delta}_j\ddot{\Delta}_j} = (\ddot{N}_j + \ddot{P}_j)^{-1} + (\ddot{N}_j + \ddot{P}_j)^{-1} \overline{N}_j^T Q_{\dot{\Delta}\dot{\Delta}} \overline{N}_j (\ddot{N}_j + \ddot{P}_j)^{-1}$$

59

Accuracy of bundle block adjustment, approximate evaluation

- Because image blocks are typically very regular, the structure of accuracies is simple and can be evaluated reliably by using results from theoretical research of accuracies
- In theory, the accuracy usually is claimed to be related with following project parameters:
  - Measurement accuracy
  - Camera constant
  - Image scale
  - Side and forward overlap
  - The size of a block
  - The number and distribution of ground control points

60

Accuracy of bundle block adjustment, approximate evaluation

- We can examine separately planimetric and height accuracies
- Planimetric accuracy of aerial triangulation
  - Ground control points are needed only in the borders of image block
  - The effect of image block size to the accuracy is small
  - Increase of side overlap (20%→60%) improves accuracy, especially when we have relatively few ground control points (improvement ratio is 1.5 – 2)
  - Camera constant has no effect on planimetric accuracy

61

Accuracy of bundle block adjustment, approximate evaluation

- The height accuracy of aerial triangulation
  - We need height ground control points also in the middle of an image block
  - Height accuracy is linearly dependent on the spacing between ground control points
  - 60% side overlap provides ca. two times better accuracy
  - Change of camera constant affects to the height accuracy by simply scaling with the ratio of values of camera constants

62