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# Heating and Cooling Systems
# EEN-E4002 (5 cr)

## Introduction:

## Need of heating and cooling

## Theoretical principles and tools

# Learning objectives

Student will learn to

- know the significance of heating and cooling in terms of economics, indoor climate and the energy use of buildings
- know climatic factors behind heating and cooling demand
- understand the fundamentals of incompressible fluid mechanics and heat transfer in the context of building energy systems
- apply the aforementioned theories to calculate flow rates, pressure losses and heat transfer in the above context

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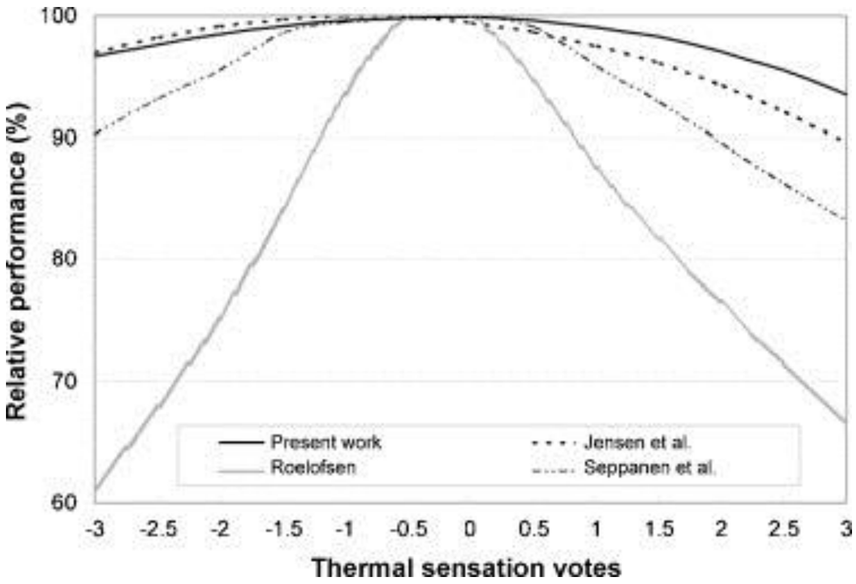
# Lesson outline

1. Significance of heating and cooling
2. Climatic factors behind heating and cooling demand
3. Fundamentals of fluid flow
4. Conservation laws
5. Energy and mass balances for open system
6. Bernoulli and continuity equations
7. Fundamentals of heat transfer

# Economic significance

- Buildings account for 70 % (400 G€) of the national wealth of Finland*
- 50 G€ turnover / year (greater than the value of Finnish forests)
- 20 % of the labour
- 500 000 person-years
- Costs of poor indoor environments: 3 G€/year

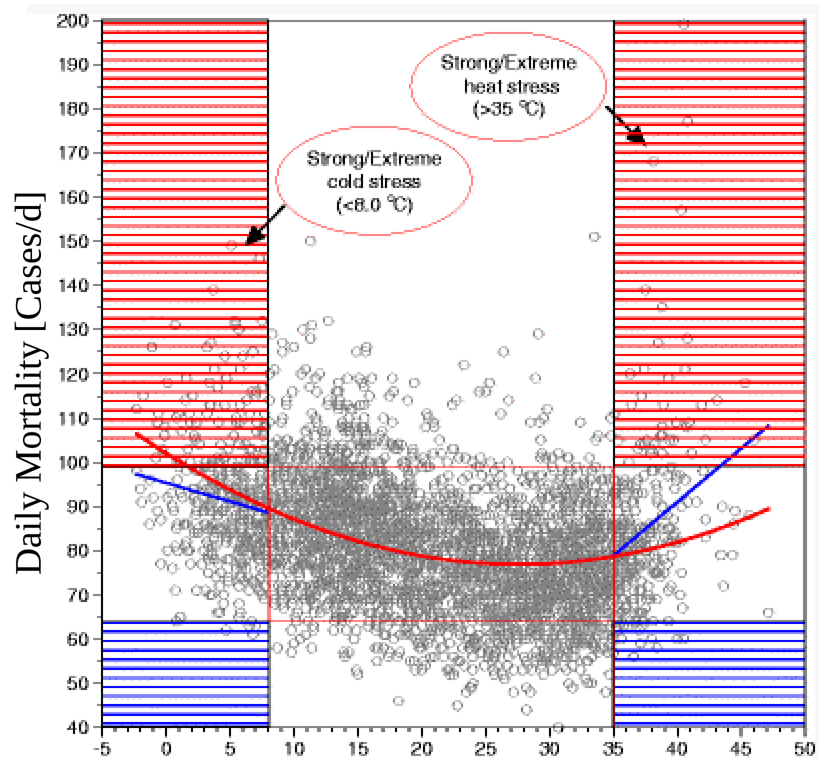



Lan et al. (2011)

*** Finland exemplifies a case of operational environment on this course. Data, regulations & computational methods vary by country.**

# Indoor environment (thermal comfort)

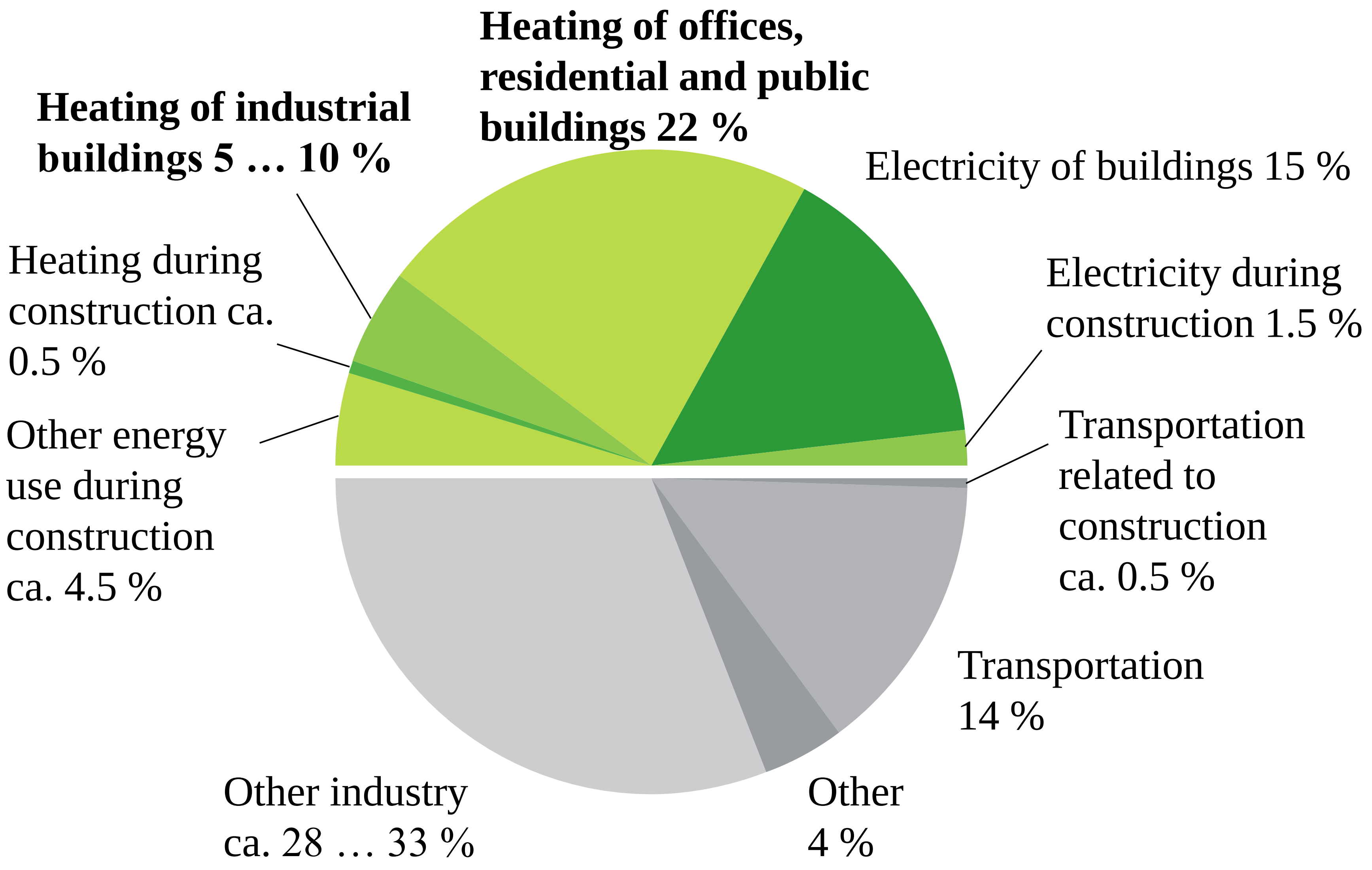
- We spend **90 %** of our time indoors…
- Optimal indoor temperature is around 21-22 °C.
- Thermal discomfort causes health impacts.




Physiologically Equivalent Temperature [°C]

Nastos&Matzarakis (2012)

# Energy demand – I



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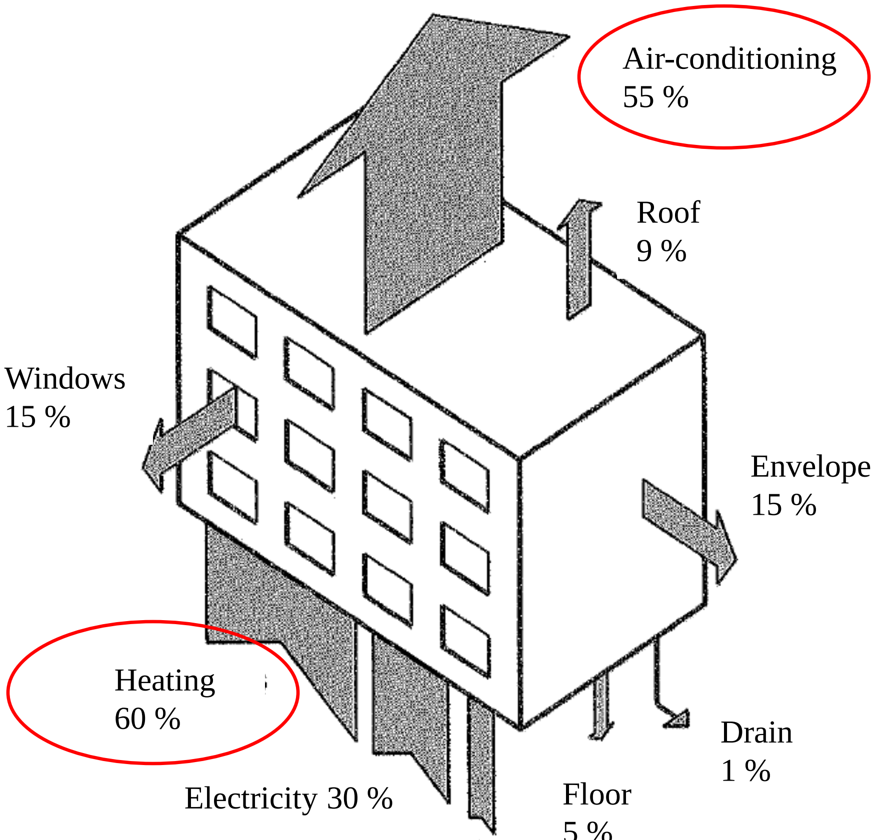
# Energy demand – II




Heat balance of an office building

# Need of heating and cooling

Weather (climatic factors)

- variation and duration of outdoor temperatures
- wind and precipitation
- snow cover and frost
- solar irradiation

Heating demand

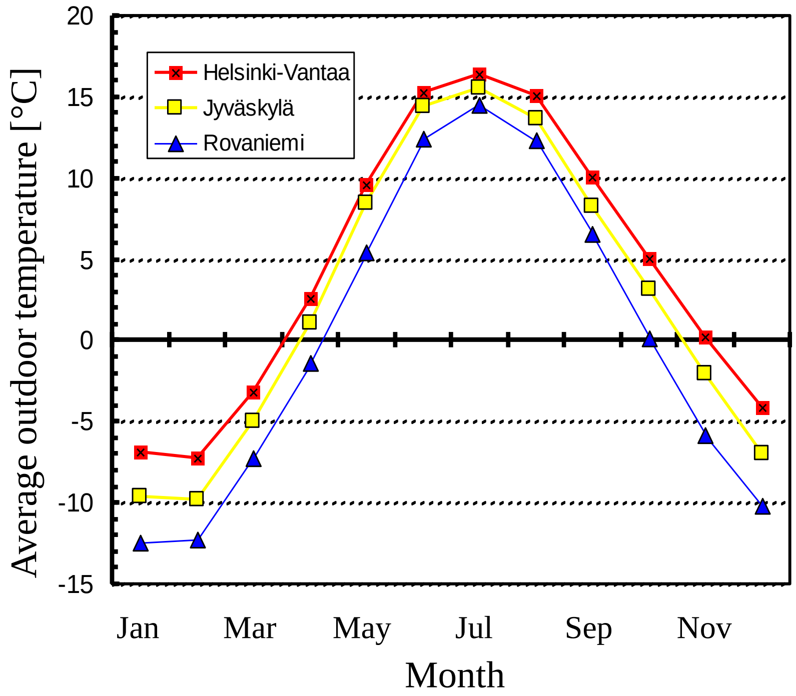
- conduction through envelope (outer walls, roof etc.) and floor
- ventilation, domestic hot water
- heat loads from solar, human and other sources
- degree days
- design temperature

Cooling demand

- heat loads from solar, human and other sources
- mass of the building
- overheating degree hours

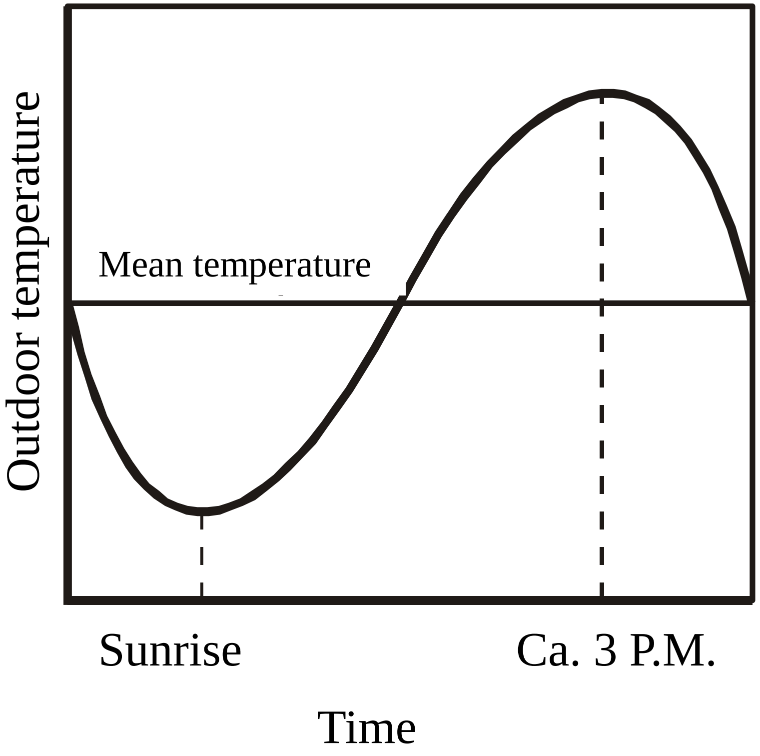
# Outdoor temperature

- Significant impact on heating demand
  - Remarkably affected by location: e.g. Annual mean temperature in Sodankylä: –0.4°C and in Helsinki +5.9°C
- Large fluctuations, from –30°C (winter minimum) to +30°C (summer maximum)



# Diurnal (daily) temperature variation

- The highest temperature occurs at some 2–3 hours after noon.
- The lowest temperature occurs approximately at the time of sunrise.
- In the darkest season (between November and January) the temperature variation does not really follow the altitude of the sun.

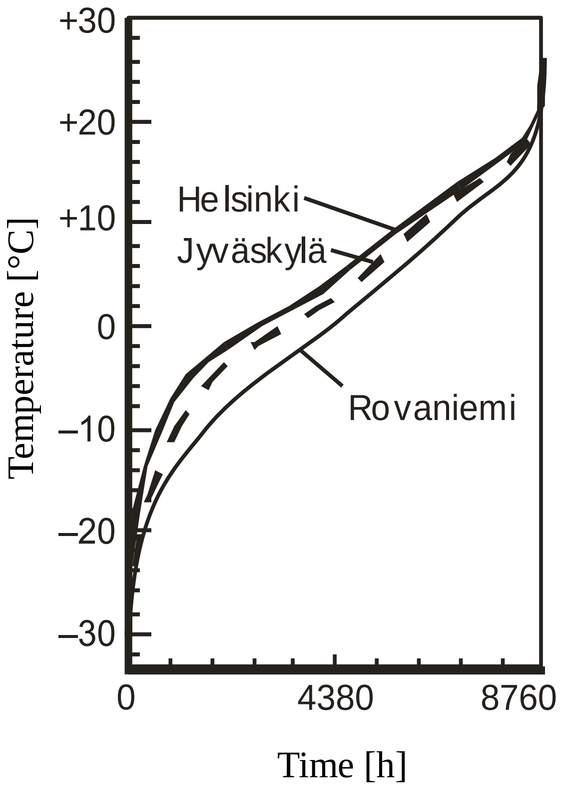


# Peak temperatures and design temperature

- Peak temperatures occur rarely
  - Maximum and minimum temperatures vary annually
  - Peak temperatures are momentary (~hours)
  - → Design on the basis of peak temperatures would result in overdimensioning of the heating system.
- *Design temperature* of a heating system is defined according to the duration of an outdoor temperature.
  - The indoor temperature (of the building stock) may not decrease ”disturbingly” during the peak temperatures.
  - Finnish Building Code (D1) represents the design temperatures by province, e.g. the ”Uusimaa” province (incl. Helsinki area): –26°C

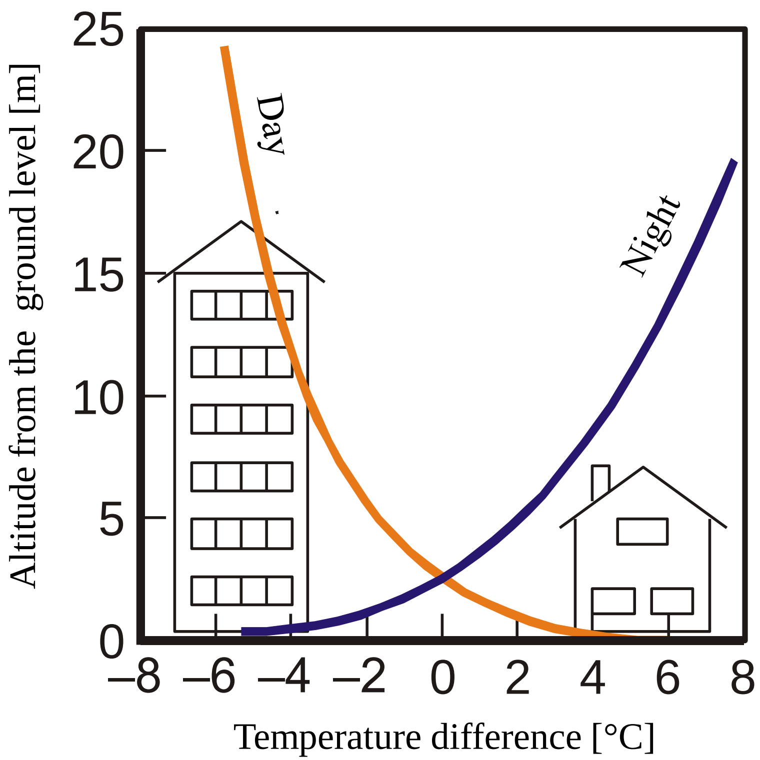
# Duration of temperature

- *Duration curve* is an expression of momentary temperatures sorted by the order of magnitude. The duration curve can be used to indicate the proportion of annual hours the outdoor temperature remains above/below a given threshold temperature.
  - Example: In Helsinki the outdoor temperature remains between +15°C and –15°C approximately 75% of the year.



# Impact of the altitude (on the temperature)

- Significant variations possible
  - affecting factors: ground heated by solar irradiation, thermal radiation from the ground to the space (ground temperature < air temperature)
  - Shape of terrain
- Official height of temperature measurement: 2 m



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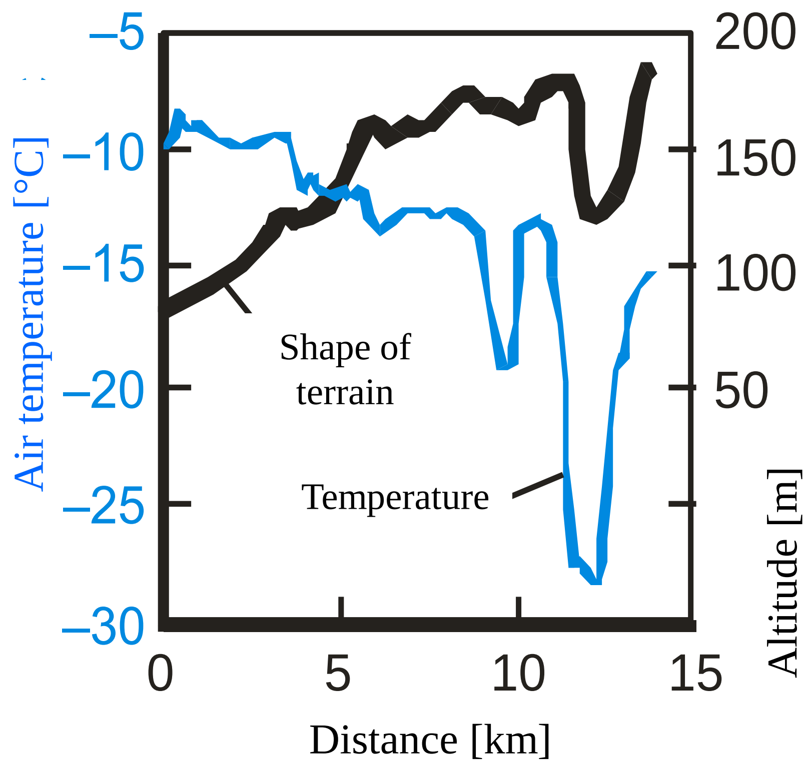
# Impact of shape of terrain (micro climate)

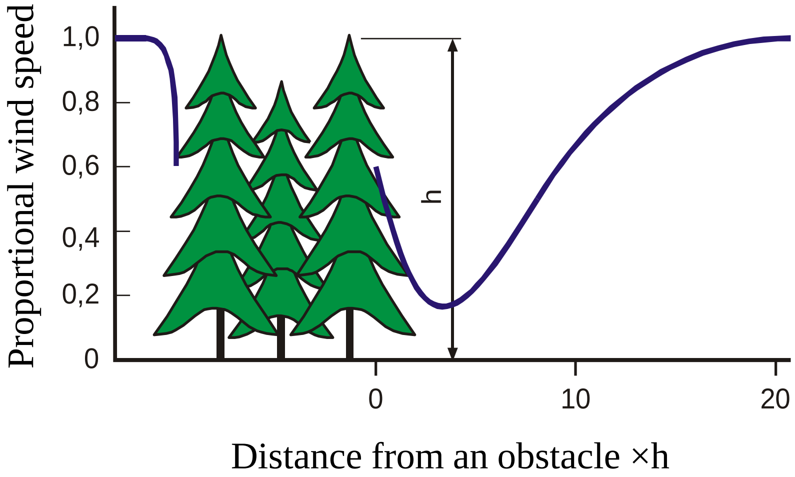
- Cold air accumulates into gorges, hollows, valleys and dingles
  - Air layer close to the ground cools down during clear nights (because of radiation)
  - Cool air flows towards valleys due to gravitation (”lakes of cold air”)
- Open waters (seas, lakes) significangly impact the micro climate



# Impact of wind

- There is no accurate method to evaluate the impact of wind, because
  - Wind speed is measured from open areas (e.g. Airport) at the height of 10 m
  - Wind speed varies significantly in the built environment and because of the shape of terrain, vegetation and proximity of open waters

- The average wind speed in Finland is ca. 4 m/s

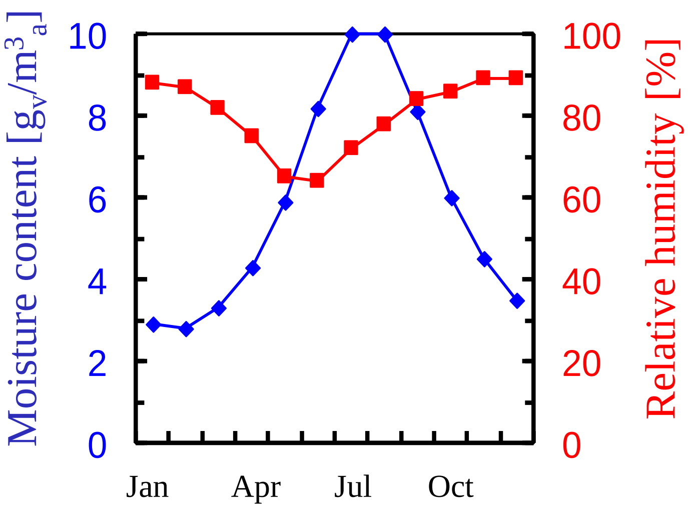


# Humidity of air

- The ***moisture content*** (aka ***specific humidity*** or ***humidity ratio***) is the mass of water vapour (v) in a kilogram of dry air (a).
- At a given temperature, only a limited moisture content is allowed without condensation taking place. The ***relative humidity*** (RH) indicates the used proportion of the capacity of the air to keep moisture at a given temperature.

→ The moisture content of outdoor air is low during cold weather (winter), but the relative humidity is high.

Montly average
Helsinki – Vantaa



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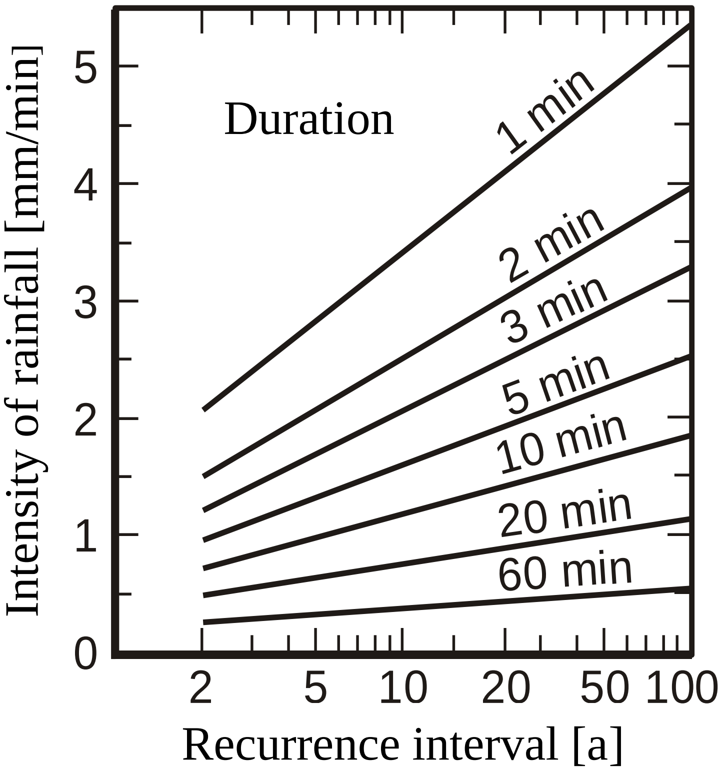
# Precipitation (rainfall)

- In Finland: evenly distributed precipitation throughout a year
- Annual precipitation: 400–700 mm
- Uniform frequency and intensity throughout the country
- Showers / flurries typically short



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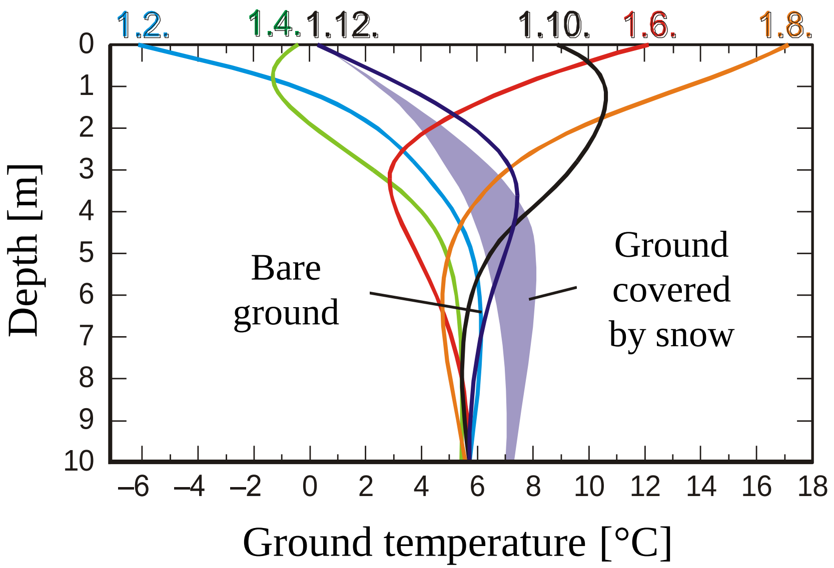
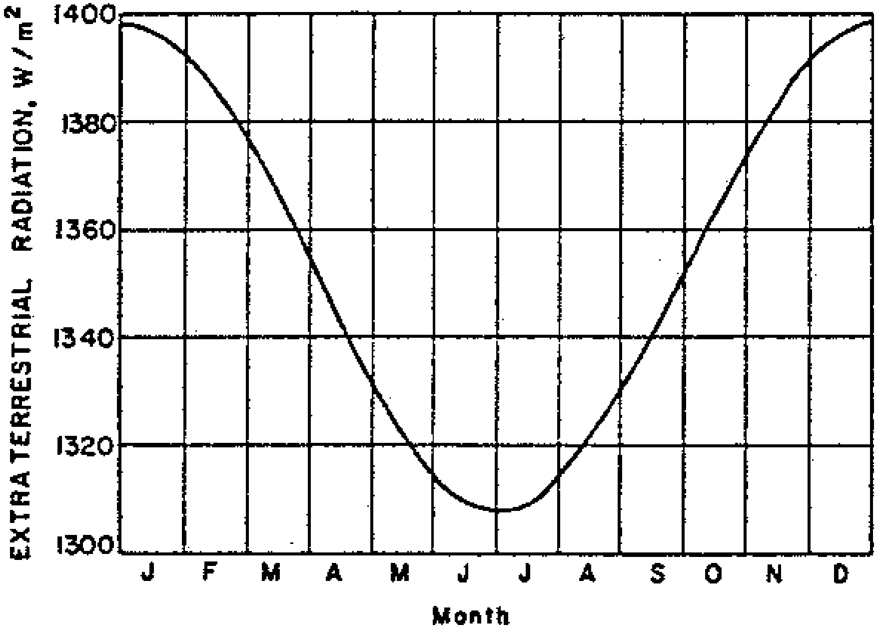
# Ground temperature, frost and blanket of snow

- Ground temperaure is
  - A couple of degrees higher than the mean outdoor temperature
  - Follows the outdoor temperature with a long delay  (months)
- Snow blanket significantly affects ground frost:
  - Ground covered by snow only freezes close to the surface (depth 30–50 cm)
  - Ground frost depends on the soil: gravel freezes more deeply than clay

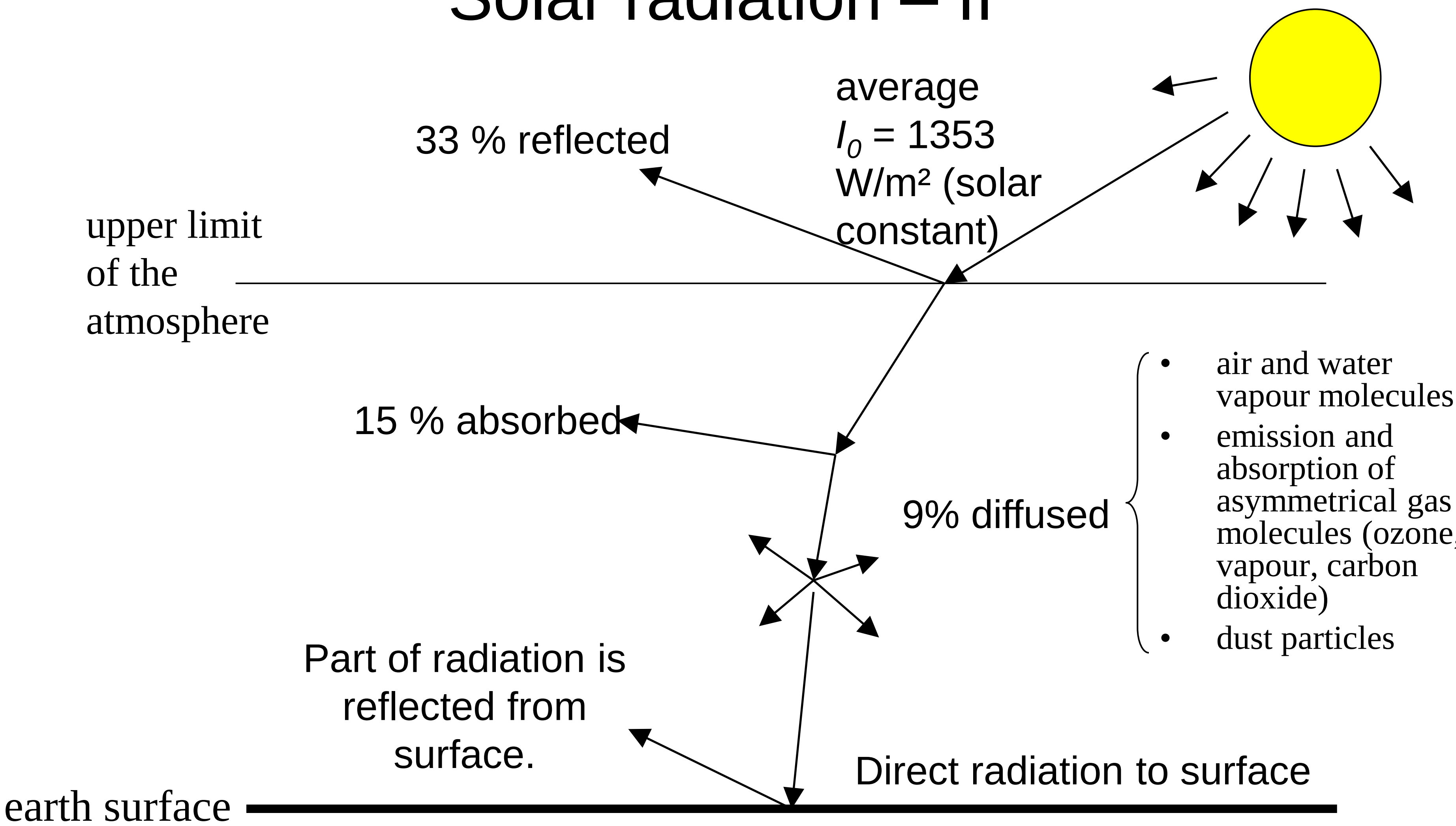


# Solar radiation – I

- Physical interpretation:
  - idealized radiating object ("black body")
- Major part of irradiance consists of wavelengths 380…780 nm
- Magnitude of solar radiation:
  - upper limit of the atmosphere: 1353 W/m$^2$ (*solar constant*)
  - earth surface (the sun vertically overhead, cloudless sky): 1025 W/m$^2$



# Solar radiation – II

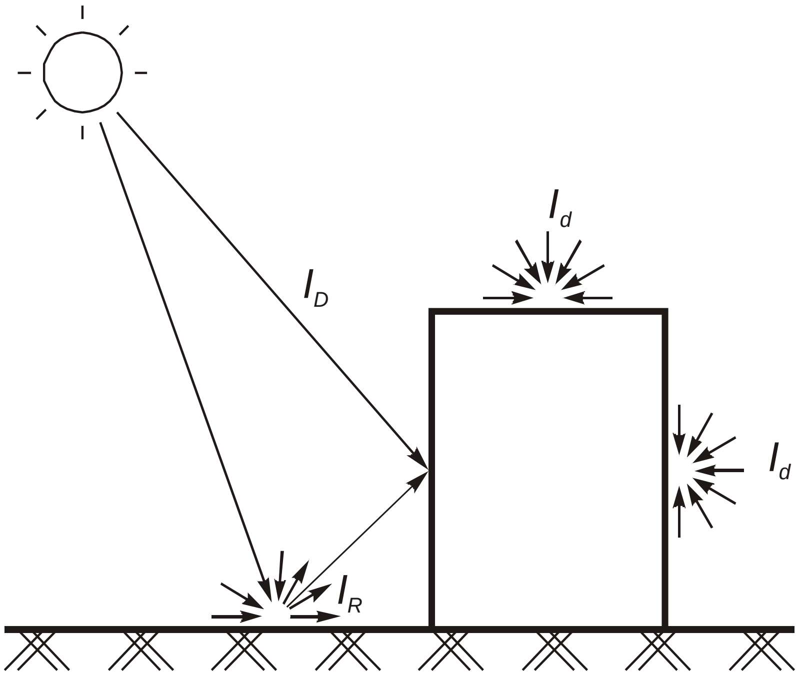


# Forms of solar radiation

1. *Direct (beam) radiation* $I_D$
   – definite direction

2. *Diffuse (sky) radiation* $I_d$
   – scattered + reflected from clouds

3. *Reflected radiation* $I_R$
   – direct and diffuse radiation reflected from environment



# Annual solar irradiation

Horizontal plane: ca. 900 kWh/m$^2$,a

- November to January: 19 kWh/m$^2$ in total (diffuse radiation incl.)
- Summer: ca. 150 kWh/m$^2$,kk

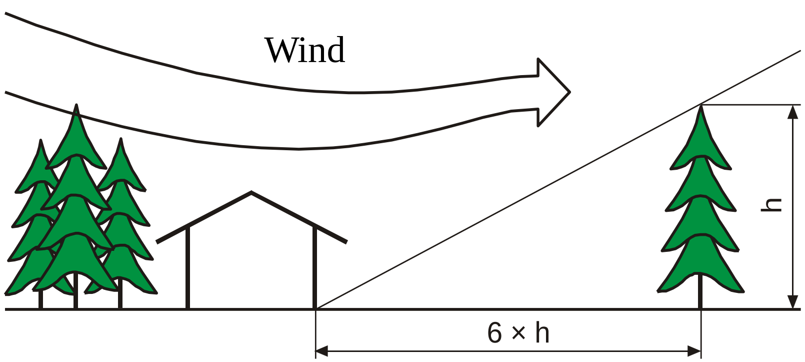
Vertical surfaces (rooms, windows):

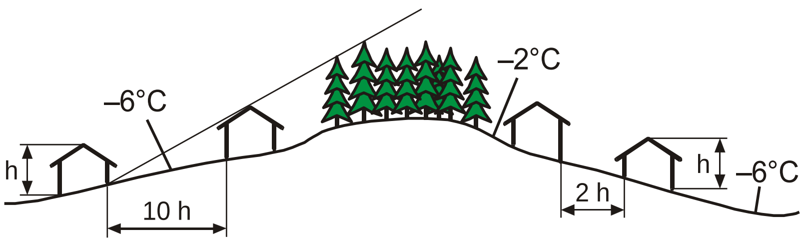
- Southern wall (ca. 550 kWh/m$^2$,a)
- Northern wall (ca. 230 kWh/m$^2$a)
- Eastern and western wall: in winter the irradiation is shared evenly with the northern wall, in summer they receive more irradiation than the southern wall

| | Latitude | Annual irradiation [kWh/m$^2$] |
|---|---|---|
| *El Paso, USA* | *31°48' N* | *2309* |
| *Jerusalem* | *31°47' N* | *2089* |
| *New Delhi* | *28°35' N* | *1987* |
| *Lissabon* | *38°43' N* | *1689* |
| *Ateena* | *37°58' N* | *1541* |
| *Rooma* | *41°48' N* | *1435* |
| *New York* | *40°47' N* | *1405* |
| *Belgrad* | *44°47' N* | *1384* |
| *Budapest* | *47°26' N* | *1210* |
| *Wien* | *48°15' N* | *1070* |
| *Pariisi* | *48°49' N* | *1032* |
| *Lontoo* | *51°31' N* | *1023* |
| *Moskova* | *55°45' N* | *1015* |
| *Tukholma* | *59°21' N* | *993* |
| *Kööpenhamina* | *55°40' N* | *976* |
| *Hampuri* | *53°38' N* | *938* |
| ***Helsinki*** | *60°12' N* | *938* |
| *Verhojansk* | *67°33' N* | *938* |
| *Leningrad* | *59°58' N* | *908* |
| ***Jokioinen*** | *60°49' N* | *887* |
| ***Sodankylä*** | *67°22' N* | *807* |
| *Reykjavik* | *64°08' N* | *798* |

# Perspectives on climate-conscious building design

- Solar irradiation
  - Orientation: south
  - Open space towards the south
- Wind protection
  - Forest close to the building
  - In Finland: the dominant wind direction is the south-west
- Rainwater protection
  - Permeable soil
  - Sloped ground
- Outdoor air quality
  - Proximity of factories, road infrastructure etc.



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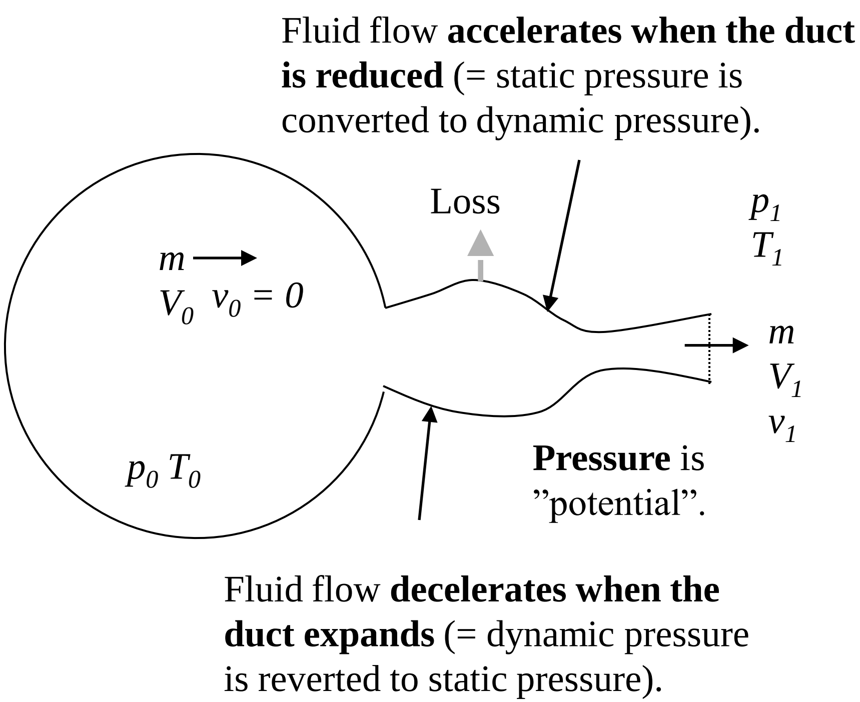
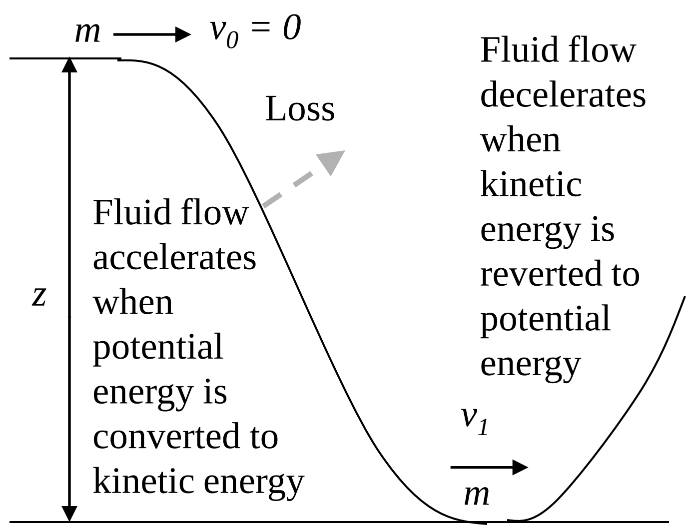
# Fundamentals of fluid flow

- The origin of fluid flow: pressure differences tend to balance.
- The direction of fluid flow is from higher to lower pressure.
- For mass particle *m*:

Gas discharging from a reservoir    Water fall from a lake




$$mgz = \frac{1}{2}mv_1^2 + \text{loss}$$

**Height** is ”potential”.

# Definitions

- *Static pressure* is the force per unit area normal to a surface at any point on any plane in a fluid at rest (=$p$)
- *Dynamic pressure* (aka *velocity pressure*) is the kinetic energy per unit volume of a fluid particle in a flowing fluid (= ½ $\rho v^2$).
- *Hydrostatic pressure* is the pressure exerted by a fluid at equilibrium at a given point within the fluid, due to the force of gravity (= $\rho g z$).
- *Total pressure* is the sum of static, dynamic and hydrostatic pressures (see: the left side of the Bernoulli equation).
- *Absolute pressure* is the pressure zero-referenced against a perfect vacuum.
- *Gauge pressure* is the pressure zero-referenced against the local atmospheric pressure.
- *Datum* (zero-referenced elevation) can be established case by case, e.g. at the lowest elevation of the target system or at the sea level.
- *Flow rate* is the mass or volume of fluid which passes through a pre-defined cross-section per given time.

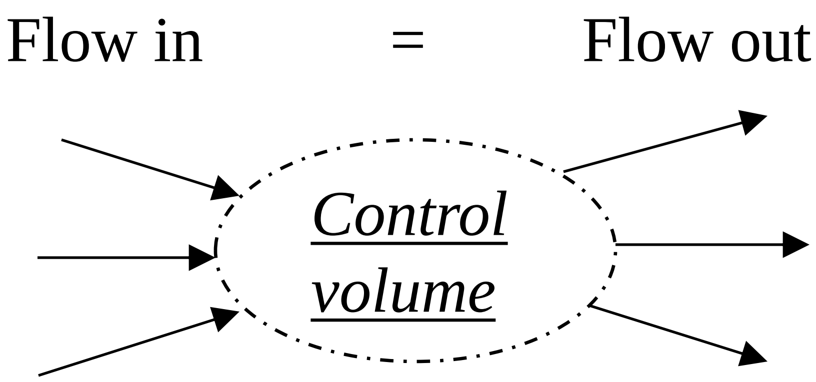
# Pressures

- Atmospheric pressure:
  - Sea level: 101325 Pa
- Selected pressures in heating and piping technology:
  - District heating (primary): +1.5 Mpa (15 bar) (max.)
  - City water: +300 kPa
  - Cold water distribution system: +200...250 kPa
  - Pressure difference over a radiator valve: 2...4 kPa
  - Pressure drop of a heat distribution network: > 10 kPa

# Conservation laws

- ”...cannot be created or destroyed”

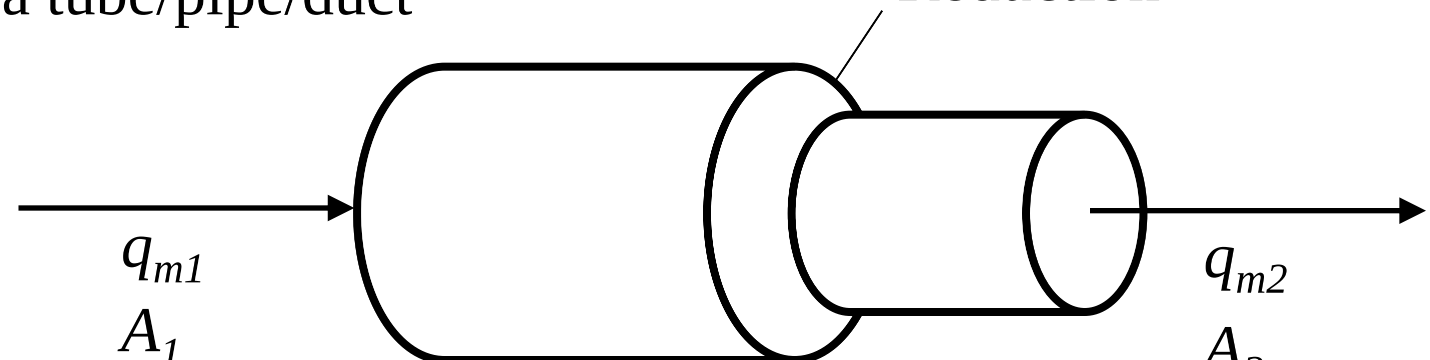
1. Conservation of mass
2. Conservation of energy
3. Conservation of momentum




**Self-studying: Learn the governing principle for the conservation of momentum and apply it to determine the force caused by the change of direction of a fluid flow in a tube.**

# Conservation of mass

Cross sections 1 and 2
of a tube/pipe/duct

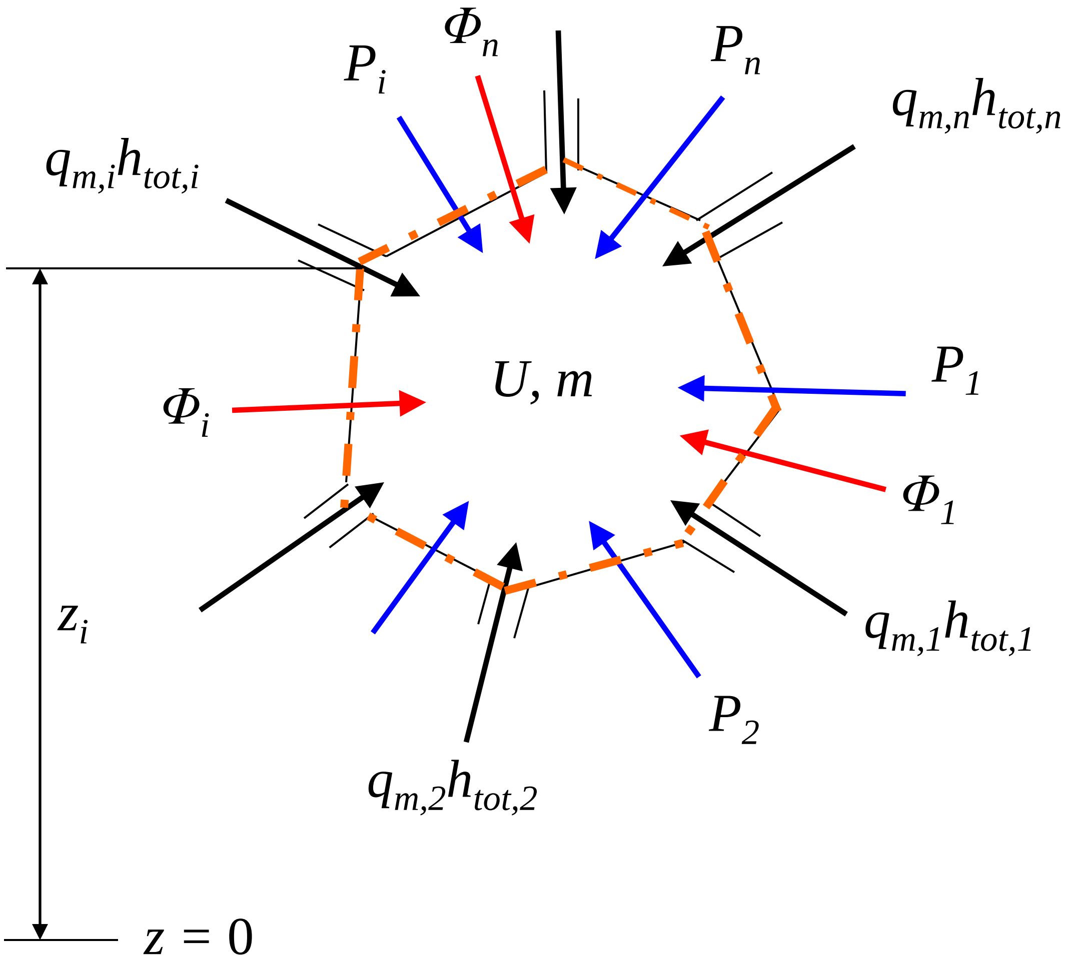



$$q_{m1} = q_{m2} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Continuity equation
$[q_m] = \text{kg/s}$

# Conservation of energy (1st Law of Thermodynamics)

Open flow system:




Enthalpy flow

Mechanical power

Heat flow

System boundary

Energy balance:

$$\sum_i P_i + \sum_i \Phi_i + \sum_i q_{m,i} h_{tot,i} = \frac{dU}{dt}$$

Mass balance:

$$\sum_i q_{m,i} = \frac{dm}{dt}$$

# Energy content of fluid flow (enthalpy)

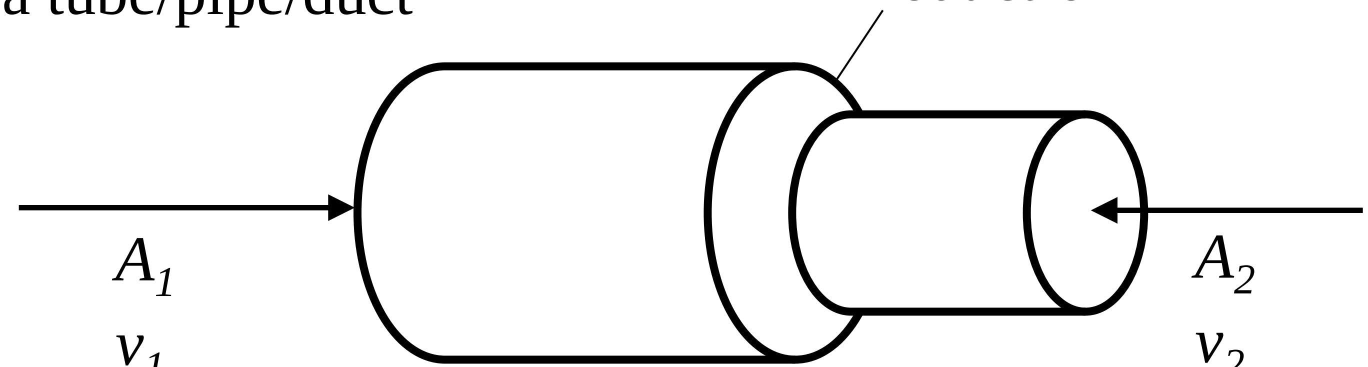
- Total specific enthalpy $$h_{tot} = c_p t + h_m + \frac{1}{2}v^2 + gz = c_V t + \frac{p}{\rho} + h_m + \frac{1}{2}v^2 + gz$$

where
$c_p$ = specific heat capacity of the fluid at constant pressure
$c_V$ = specific heat capacity of the fluid at constant volume
$p$ = static pressure of the fluid at the system boundary
$\rho$ = density of the fluid
$t$ = temperature of the fluid (zero-point of the temperature scale is an agreed reference temperature e.g. 0°C)
$h_m$ = chemical specific energy, ”enthalpy of formation” (e.g. LHV or HHV)
$v$ = fluid velocity
$g$ = gravitational acceleration
$z$ = altitude (zero-point optional case by case)

In the equation of total specific enthalpy [J/kg], the relevant energy forms are included. The equation above does not contain surface energy (in HVAC applications, the surface energy can be neglected since it does not significantly affect the accuracy of the results).

# Bernoulli equation – I

Cross sections 1 and 2
of a tube/pipe/duct




<u>Assumptions:</u>

- incompressible flow ($\rho_1 = \rho_2 = \rho$)
- isothermal system ($T_1 = T_2 = T$)
- no friction
- continuity: $q_{V1} = q_{V2} = A_1 v_1 = A_2 v_2$

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# Bernoulli equation – II

- Energy conservation: total specific enthalpy remains

$$c_V t + \frac{p_1}{\rho} + h_m + \frac{1}{2} v_1^{\ 2} + g z_1 = c_V t + \frac{p_2}{\rho} + h_m + \frac{1}{2} v_2^{\ 2} + g z_2$$

- By reducing (adiabatic fluid) the term $c_v t$, considering that $h_m = 0$ and multiplying both sides of the equation by density:

$$p_1 + \frac{1}{2}\rho v_1^{\ 2} + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^{\ 2} + \rho g z_2 = p_{tot}$$

**In words:**
**Total pressure ($p_{tot}$) =**
**static pressure**
**+ dynamic pressure**
**+ hydrostatic pressure**

**This is the Bernoulli equation, which is analogous to the conservation of energy (e.g. potential and kinetic energy).**

**Self-studying: Learn how to derive the Bernoulli equation through integrating Newton's Second Law of Motion (i.e. Euler Equation).**

# Example

Calculate the fluid velocity at state 2.

$$p_1 + \rho g z_1 + \rho \frac{v_1^2}{2} = \text{constant}$$

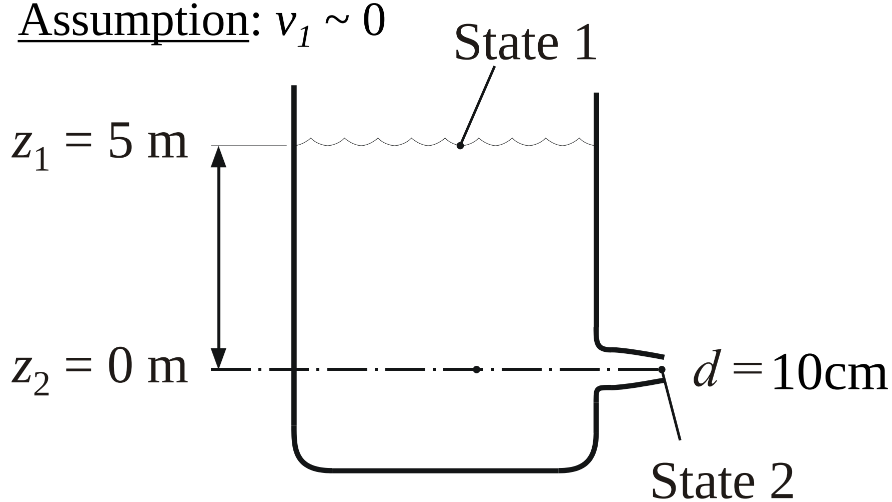
$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \text{constant}$$

$$\underbrace{z_1 + \frac{p_1}{\rho g} + 0}_{\text{State 1}} = \underbrace{0 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g}}_{\text{State 2}}$$

$$p_1 \approx p_2 \Rightarrow v_2 = \sqrt{2 g z_1} =$$

$$\sqrt{2 \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 5\,\text{m}} = \underline{\underline{9{,}9\ \frac{\text{m}}{\text{s}}}}$$

Assumption: $v_1 \sim 0$



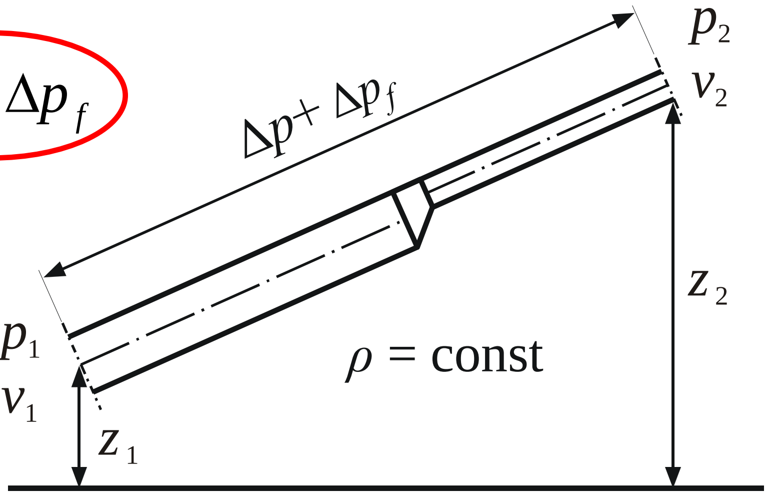

Assumption: no pressure loss

# Pressure loss (head)

When pressure loss is accounted, the law of energy conservation is applied as follows:

$$p_1 + \rho g z_1 + \frac{1}{2}\rho v_1^{\,2} = p_2 + \rho g z_2 + \frac{1}{2}\rho v_2^{\,2} + \Delta p + \Delta p_f$$

*loss terms*




For straight pipe:

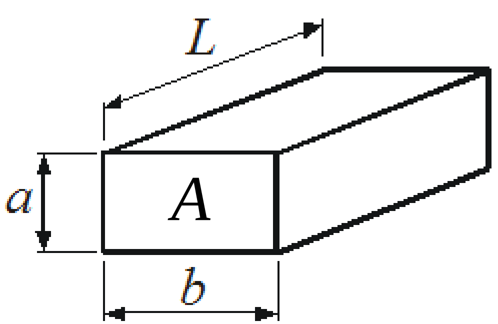
$$\Delta p_f = f\left(\frac{L}{D_h}\right)\cdot\frac{1}{2}\rho v^2$$ *Friction loss*

Pipe components (crosses, elbows, bends etc.):

$$\Delta p = K\frac{1}{2}\rho v^2$$ *Minor loss*

For rectangular tubes $D_h$ is calculated from

$$D_h = \frac{4A}{P} = \frac{4ab}{2(a+b)}$$ *Hydraulic diameter*



# Friction factor

- Friction factor *f*:

$$f = \frac{1.325}{\left\{ \ln\left[ \frac{(\varepsilon / D_h)}{3.7} + \frac{5.7}{\mathrm{Re}^{0.9}} \right] \right\}^2}$$

$$\varepsilon = \text{roughness of the pipe [m]}$$

- Reynolds number:

$$\mathrm{Re} = \frac{vD_h}{\nu}$$

For laminar flow ($\mathrm{Re} < 3000$):

$$f = \frac{64}{\mathrm{Re}}$$

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# Moody Diagram

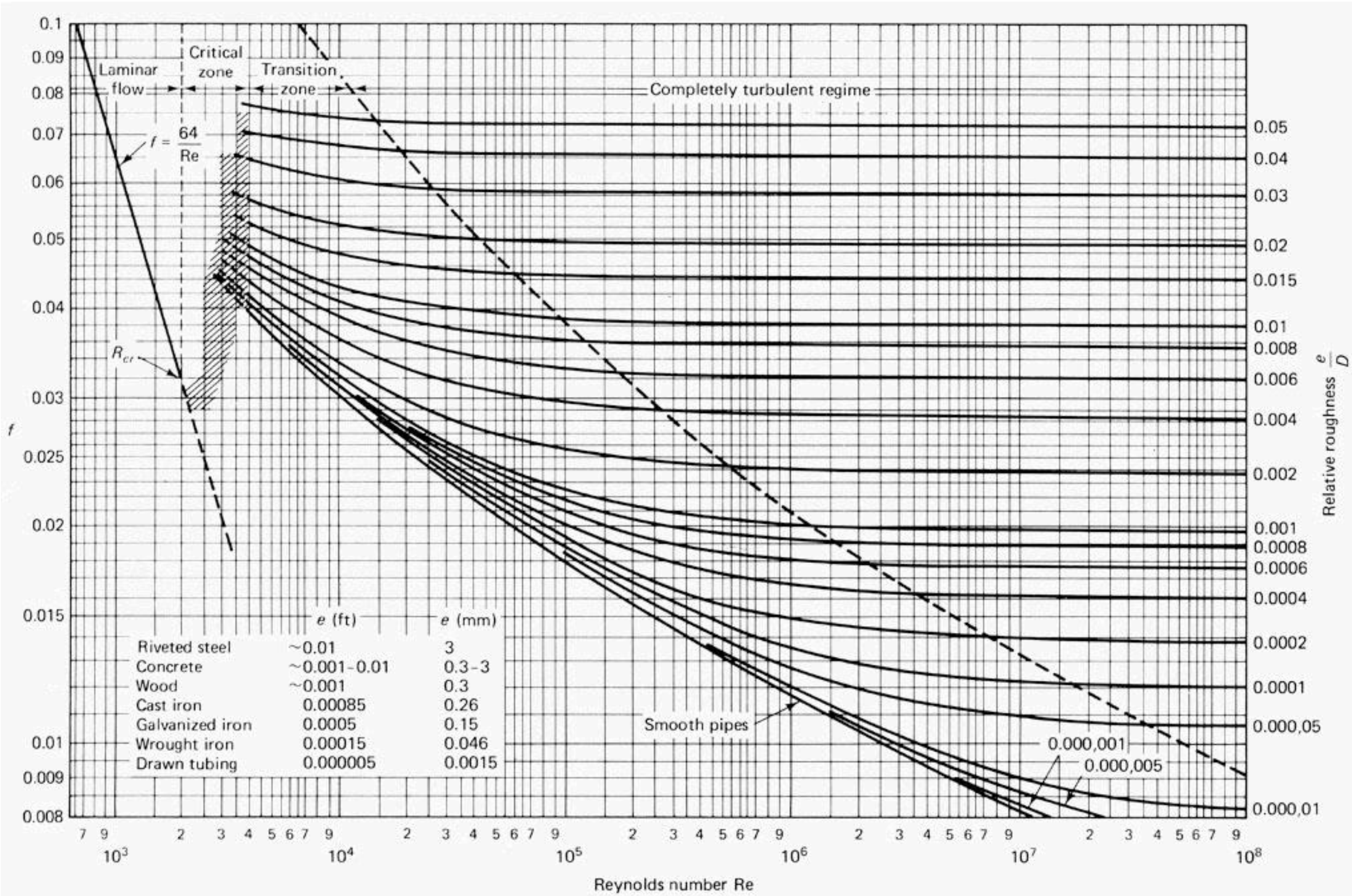
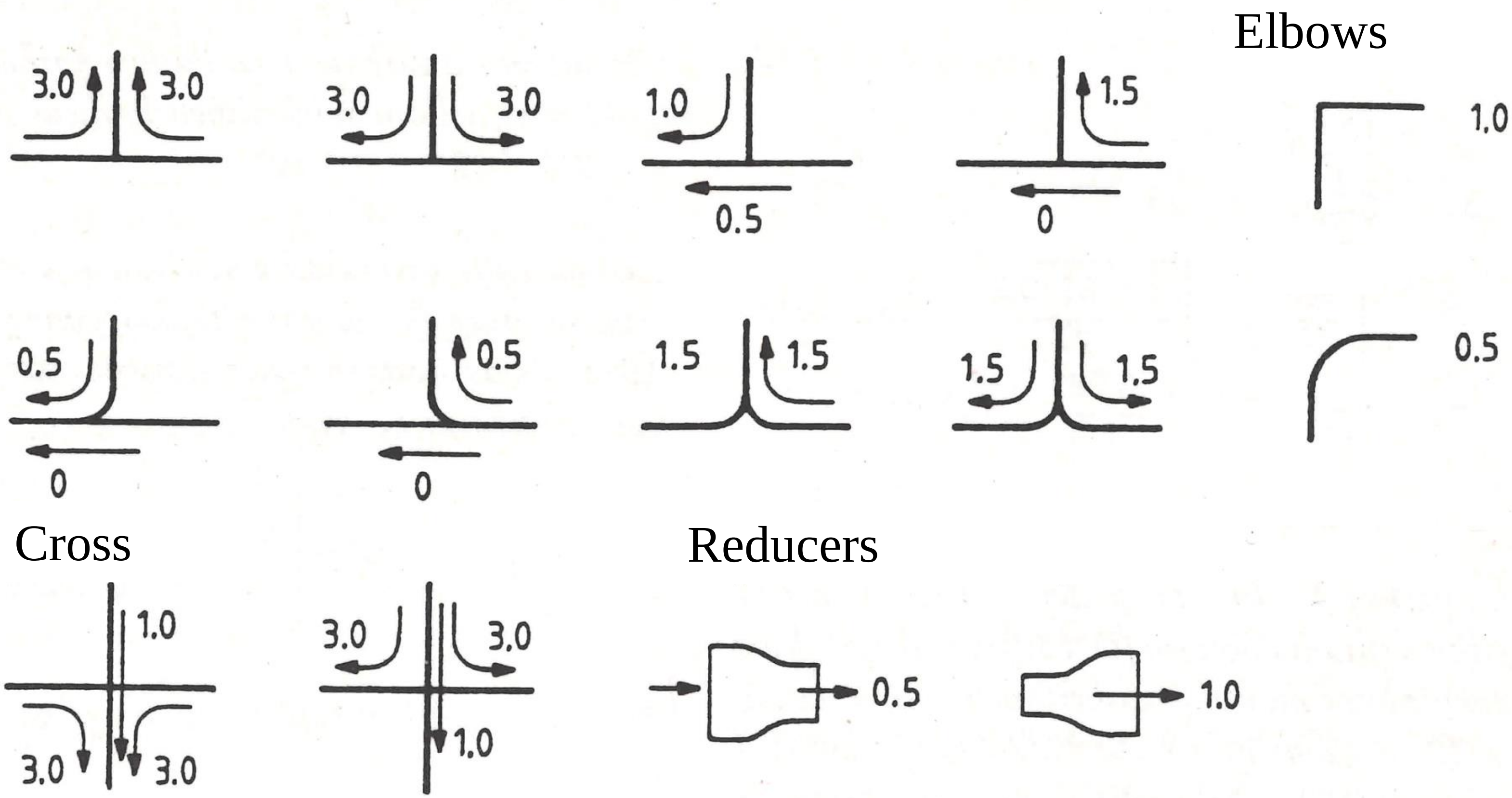



**Figure 7.13** Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

# Minor loss coefficients ($K$) for various pipe components



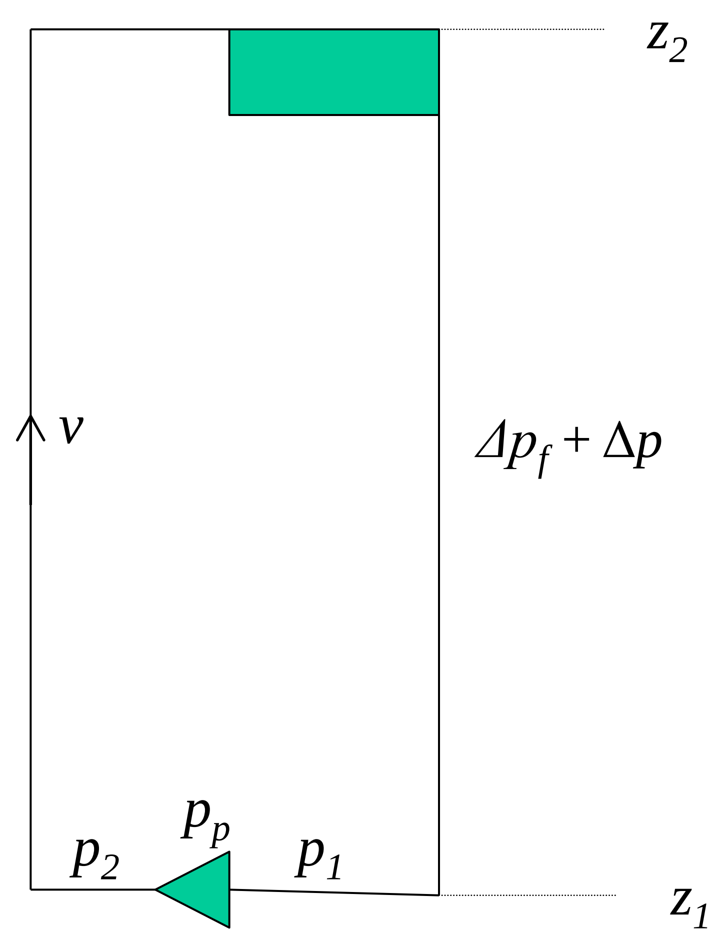
# Closed loop system (e.g. hydronic heating)

- *Pascal's principle*: Pressure is transmitted undiminished in an enclosed static fluid.

  $\rightarrow p_2 = p_1$

- The potential energy of the fluid reached at the elevation $z_2$ returns back to the system at the elevation $z_1$ due to gravity (weight of the fluid).

$\rightarrow$ To circulate the fluid, the pump only needs to overcome the friction of a piping system ($p_p = \Delta p_f + \Delta p$).




**Examine the above statements as an implication of the principle of energy conservation.**

# Example

City water is delivered at 300 kPa (3 bar) and the pressure loss of the water distribution system is 100 kPa.

Is the pressure (300 kPa) sufficient to distribute water at the volumetric flow rate of 0.2 L/s to a tap (inside diameter 10 mm) elevated at 20 m from the city water interface (inside diameter 16 mm)? (Inversely: Do we need a pump to deliver the requested water flow rate?)

The density of water $\rho = 1000\ \mathrm{kg/m^3}$ and the standard gravity $g = 9.81\ \mathrm{m/s^2}$

# Illustration

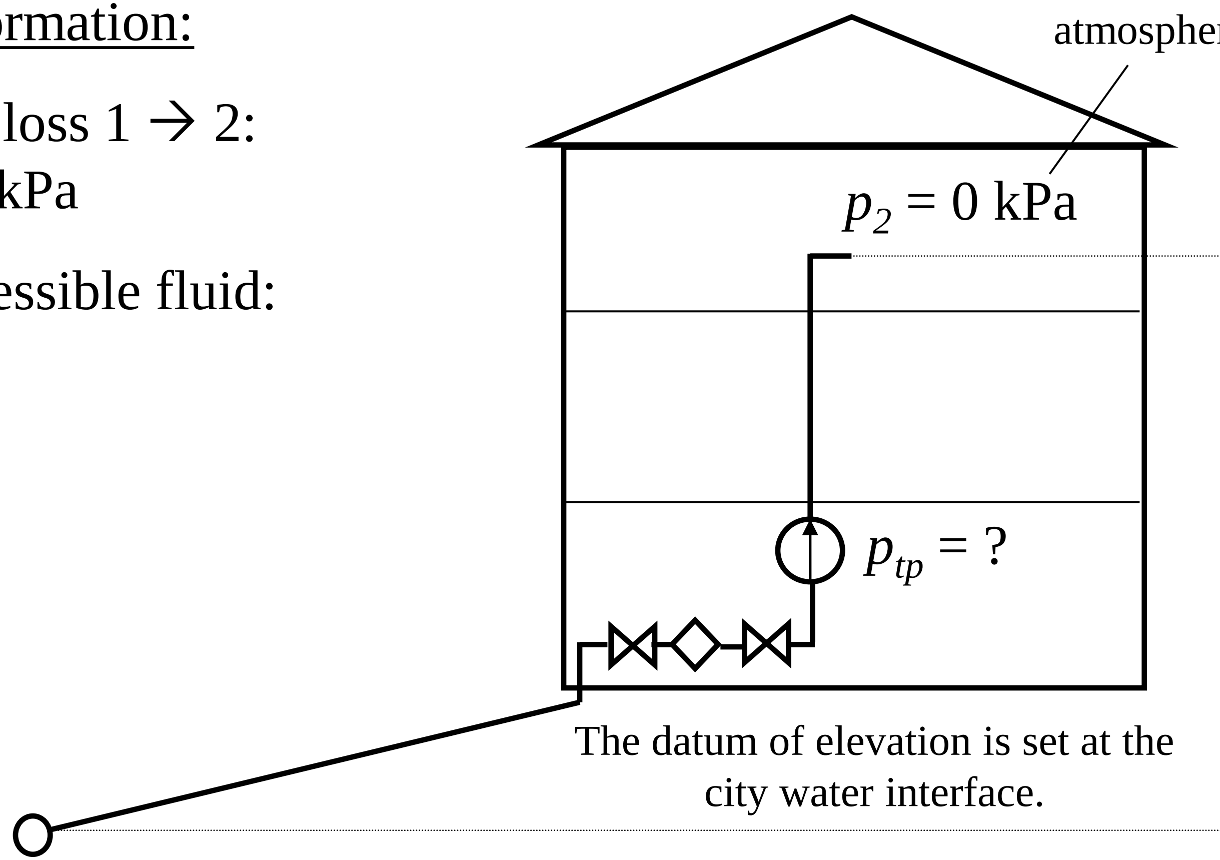
<u>Initial information:</u>

- Pressure loss 1 → 2:

$\Delta p_f = 100$ kPa

- Incompressible fluid:

$\rho_1 = \rho_2 = \rho$



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# Solution – I

1. Energy conservation:

$$p_1 + \frac{1}{2}\rho v_1^{\;2} + \rho g z_1 + p_{tp} = p_2 + \frac{1}{2}\rho v_2^{\;2} + \rho g z_2 + \Delta p_f$$

($\rho g z_1 = 0$, $p_2 = 0$)

$$\Rightarrow p_{tp} = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g z_2 + \Delta p_f - p_1$$

**Note: how to substitute the pump pressure in the energy equation?**

# Solution – II

2. Fluid velocity at 1 and 2
   - $v_1 = v_1\ (q_V = 0.2$ L/s, $D = 16$ mm$) = 0.99$ m/s
   - $v_2 = v_2\ (q_V = 0.2$ L/s, $D = 10$ mm$) = 2.55$ m/s
3. Pump pressure

$$p_{tp} = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g z_2 + \Delta p_f - p_1$$

$$= \frac{1}{2}\cdot 1000\,\frac{\text{kg}}{\text{m}^3}\cdot\left(\left(2.55\,\frac{\text{m}}{\text{s}}\right)^2 - \left(0.99\,\frac{\text{m}}{\text{s}}\right)^2\right) + 1000\,\frac{\text{kg}}{\text{m}^3}\cdot 9.81\,\frac{\text{m}}{\text{s}^2}\cdot 20\,\text{m} + \left(100000 - 300000\right)\text{Pa}$$

$$= -1052\,\text{Pa}$$

– The requested pump pressure is < 0 Pa, i.e. <u>300 kPa is enough to provide at least 0.2 L/s</u>.

# Example

Calculate the friction loss per 1 m of a straight air duct (diameter 500 mm, roughness 0.15 mm) at volumetric flow rates 1, 2 and 3 $m^3/s$. The air temperature is 300 K.

# Solution – I

1. Fluid velocity (substitutions for flow rate $q_V$ = 1 m$^3$/s)

$$q_V = Av = \frac{\pi D_h^{\,2}}{4} v \Rightarrow v = \frac{4\,\dot{q}_V}{\pi D_h^{\,2}} = \frac{4 \cdot 1\,\frac{\mathrm{m}^3}{\mathrm{s}}}{\pi \cdot (0.5\,\mathrm{m})^2} = 5.1\,\frac{\mathrm{m}}{\mathrm{s}}$$

2. Reynolds number
   – Kinematic viscosity at 300 K (Engineering Toolbox): $\nu$ = 0.00001568 m$^2$/s

$$\mathrm{Re} = \frac{vD_h}{\nu} = \frac{5.1\,\frac{\mathrm{m}}{\mathrm{s}} \cdot 0.5\,\mathrm{m}}{0.00001568\,\frac{\mathrm{m}^2}{\mathrm{s}}} = 162403 \rightarrow \text{Turbulent}$$

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# Solution – II

3. Friction factor

$$f = \frac{1.325}{\left\{\ln\left[\frac{(\varepsilon / D_h)}{3.7} + \frac{5.7}{\mathrm{Re}^{0.9}}\right]\right\}^2} = \frac{1.325}{\left\{\ln\left[\frac{(0.15\,\mathrm{mm} / 500\,\mathrm{mm})}{3.7} + \frac{5.7}{162403^{0.9}}\right]\right\}^2} = 0.018213$$

4. Pressure loss

– Density of air at 300 K (Engineering Toolbox): $\rho = 1.1726$ kg/m$^3$

$$\Delta p_f = f\left(\frac{L}{D_h}\right)\cdot\frac{1}{2}\rho v^2 = 0.018213\cdot\left(\frac{1\,\mathrm{m}}{0.5\,\mathrm{m}}\right)\cdot\frac{1}{2}\cdot 1.1726\,\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot(5.1\,\mathrm{m})^2 = \underline{\underline{0.55\,\frac{\mathrm{Pa}}{\mathrm{m}}}}$$

# Summary of results

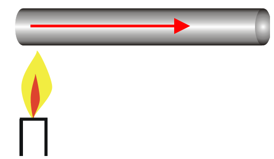
| $q_V$[m³/s] | $v$ [m/s] | Re | Quality | $f$ | $\Delta p_f$ [Pa/m] |
|---|---|---|---|---|---|
| 1 | 5.1 | 162403 | Turbulent | 0.018 | 0.55 |
| 2 | 10.2 | 324806 | Turbulent | 0.017 | 2.06 |
| 3 | 15.3 | 487209 | Turbulent | 0.016 | 4.49 |

# Introduction to heat transfer

- The origin of heat transfer: temperature differences tend to balance.
- The direction of ***heat flow*** (and ***heat flux***, i.e. heat flow per m$^2$) is from higher to lower temperature.
- Three (3) forms of heat transfer:

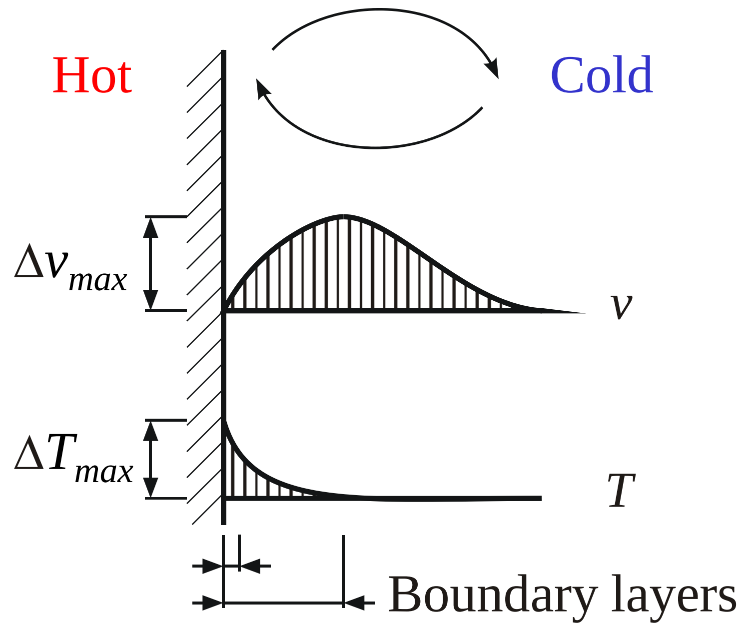
**1 CONDUCTION**

- Transfer of kinetic energy of particles
- Solid materials, fluids and gases



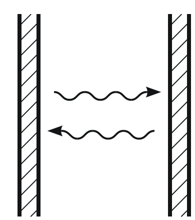
**2 CONVECTION**

- Heat transfer with the movement of fluid
- Forced or free (natural)
- Takes place between a solid surface and a fluid




**3 RADIATION**

- Electromagnetic wave motion
- All the solid bodies emit radiation



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# Fundamentals of conduction

- Governing equation: The *Fourier equation*

$$q = -\lambda \frac{\partial T}{\partial s} \left( = \frac{\Phi}{A} \right)$$

where $q$ = *heat flux*, W/m$^2$

$\lambda$ = thermal conductivity, W/mK

$T$ = temperature, K

$s$ = insulation thickness, m

$\Phi$ = *heat flow*, W

$A$ = surface area, m$^2$

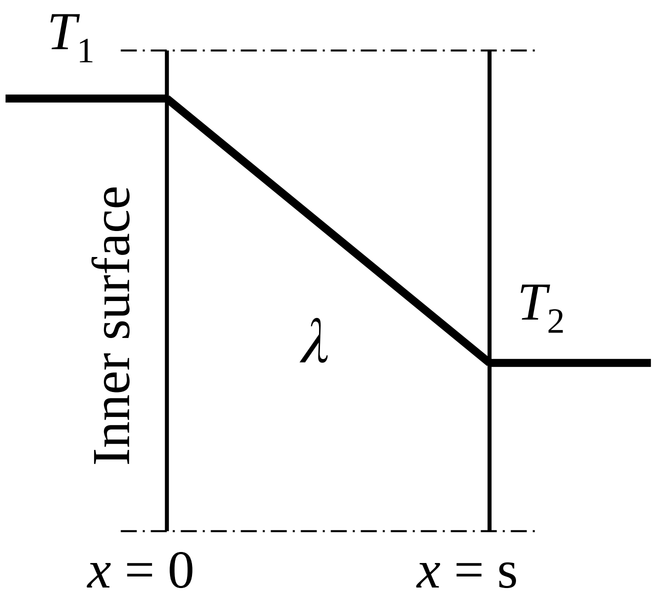
- Implementation (for the case on the right):

$$q = -\lambda \frac{\partial T}{\partial x} = -\lambda \frac{dT}{dx} \Rightarrow \int_0^s q\,dx = \int_{T_1}^{T_2} -\lambda\,dT$$

$$q = \frac{\lambda}{s}(T_1 - T_2) = \frac{T_1 - T_2}{\frac{s}{\lambda}} = \frac{\Delta T}{R}$$

where $R = \frac{s}{\lambda}$ = *thermal resistance*, m$^2$K/W

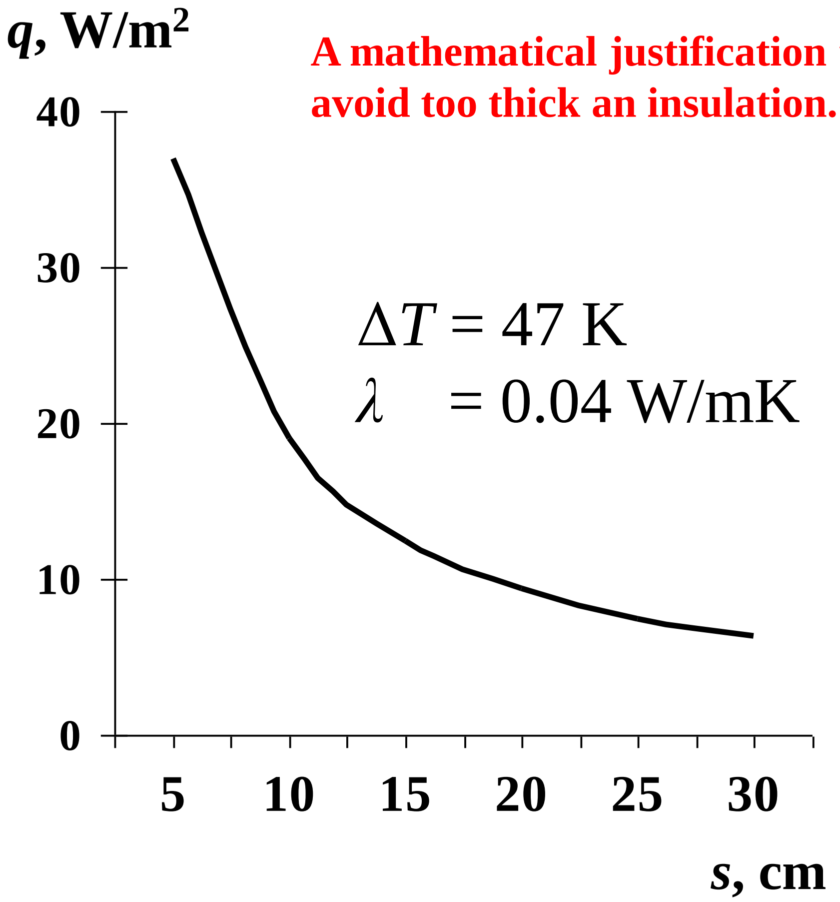
Assumption: one-dimensional conduction through surface (wall etc.)




**Self-studying: Formulate the heat conduction equation for a three-dimensional control volume.**

# Example: Impact of insulation thickness

| Thickness, cm | Heat flux, W/m$^2$ |
|---|---|
| 5 | 37 |
| 10 | 19 |
| 15 | 12.5 |
| 20 | 9.4 |
| 25 | 7.5 |
| 30 | 6.3 |




**A mathematical justification to avoid too thick an insulation.**

$\Delta T = 47$ K

$\lambda = 0.04$ W/mK

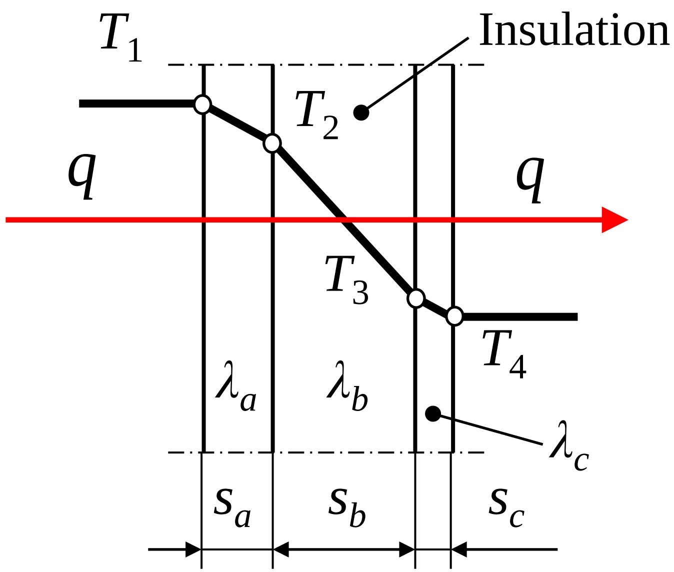
# Implementation: Conduction through wall element (”sandwich”)

Heat flux through wall remains the same through each layer (stationary conditions):

$$q = \frac{\lambda_a}{s_a}(T_1 - T_2) = \frac{\lambda_b}{s_b}(T_2 - T_3) = \frac{\lambda_c}{s_c}(T_3 - T_4)$$

$$\Leftrightarrow q = \frac{T_1 - T_4}{\frac{s_a}{\lambda_a} + \frac{s_b}{\lambda_b} + \frac{s_c}{\lambda_c}} \text{ (wall on the right)}$$

$$\Leftrightarrow q = \frac{T_1 - T_{n+1}}{\sum_{i=1}^{n} \frac{s_i}{\lambda_i}} \text{ (general expression)}$$




The temperature difference over the wall ($T_1 - T_4$) is apportioned according to the proportion of thermal resistances (i.e. the largest thermal resistance → the largest temperature difference etc.):

$\Delta T \sim R$

$$\Rightarrow \text{Surface temperatures } T_2 \text{ and } T_3: T_2 = T_1 - \frac{\frac{s_a}{\lambda_a}}{\sum_{i=1}^{n} \frac{s_i}{\lambda_i}}(T_1 - T_4) \qquad T_3 = T_1 - \frac{\frac{s_a}{\lambda_a} + \frac{s_b}{\lambda_b}}{\sum_{i=1}^{n} \frac{s_i}{\lambda_i}}(T_1 - T_4)$$

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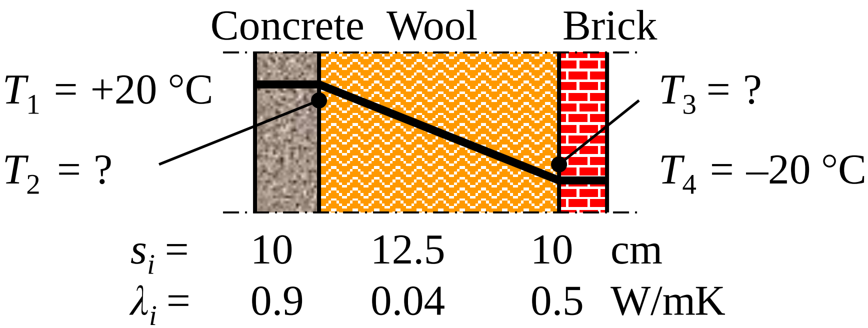
# Example

Calculate the heat flux and the surface temperatures $T_2$ and $T_3$ for the wall on the right.



# Solution – I

1. Thermal resistances

   – Concrete: $R_1 = \frac{s_1}{\lambda_1} = \frac{0.1\,\text{m}}{0.9\,\frac{\text{W}}{\text{mK}}} = 0.1\,\frac{\text{m}^2\text{K}}{\text{W}}$

   – Wool: $R_2 = \frac{s_2}{\lambda_2} = \frac{0.125\,\text{m}}{0.04\,\frac{\text{W}}{\text{mK}}} = 3.1\,\frac{\text{m}^2\text{K}}{\text{W}}$

   – Brick: $R_3 = \frac{s_3}{\lambda_3} = \frac{0.1\,\text{m}}{0.5\,\frac{\text{W}}{\text{mK}}} = 0.2\,\frac{\text{m}^2\text{K}}{\text{W}}$

2. Heat flux $q = \frac{\Phi}{A} = \frac{T_1 - T_4}{\sum_{i=1}^{3} R_i} = \frac{20-(-20)\,\text{K}}{(0.11+3.1+0.2)\,\frac{\text{m}^2\text{K}}{\text{W}}} = \underline{\underline{11.6\ \text{W/m}^2}}$

# Solution – II

3. Layer surface temperatures

$$T_2 = T_1 - \frac{R_1}{\sum R_i}(T_1 - T_4) = 20\,°\text{C} - \frac{0.11\,\frac{\text{m}^2\,°\text{C}}{\text{W}}}{(0.11+3.1+0.2)\,\frac{\text{m}^2\,°\text{C}}{\text{W}}} \cdot (20-(-20))\,°\text{C} = \underline{\underline{18.7\,°\text{C}}}$$

$$T_3 = T_1 - \frac{R_1 + R_2}{\sum R_i}(T_1 - T_4) = 20\,°\text{C} - \frac{0.11+3.1\,\frac{\text{m}^2\,°\text{C}}{\text{W}}}{(0.11+3.1+0.2)\,\frac{\text{m}^2\,°\text{C}}{\text{W}}} \cdot (20-(-20))\,°\text{C} = \underline{\underline{-17.7\,°\text{C}}}$$

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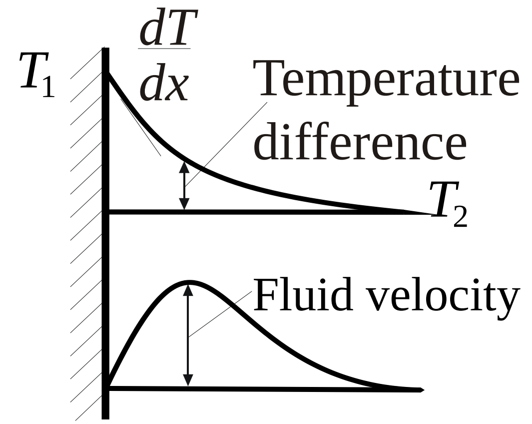
# Fundamentals of convection

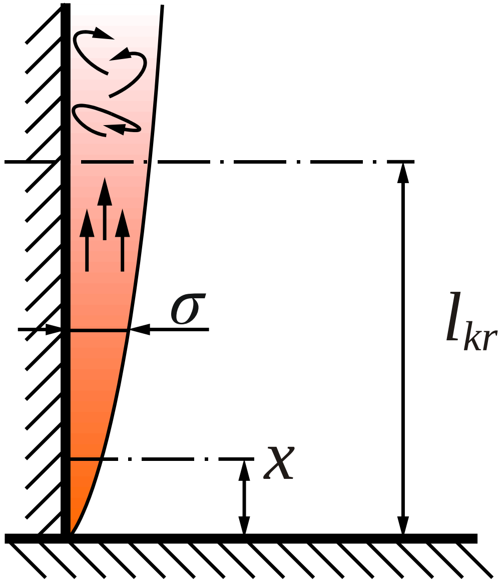
- Where there is a temperature difference between a fluid and solid surface, convective heat transfer to/from surface takes place.
- Types of convection:
  - free (natural): fluid flow is created by density differences caused by temperature differences
  - forced: fluid flow is created by external force (e.g. fan, ventilator etc.)
- Governing equation:

$$q = \alpha \left(T_1 - T_2\right) = \alpha\, \Delta T$$

where $\Phi$ = heat flow, W
$\alpha$ = heat transfer coefficient, W/m²K
$T_1$ = surface temperature, K or °C
$T_2$ = temperature of free fluid, K or °C




Free convection from a warm surface ($T_1 > T_2$)



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# Convective heat transfer coefficient

- For the sake of avoiding complicated analytical solutions, convective heat transfer coefficient $\alpha$ is determined experimentally for different cases of fluid flow (e.g. natural, forced) using dimensionless numbers. The following numbers (as derivatives from physical properties of fluid) are in use:

1. *Nusselt number*: $Nu_L = \frac{\alpha L}{\lambda}$

**Nusselt number is the dimensionless definition of convective heat transfer coefficient.**

$L$ = characteristic length for each geometry of flow (e.g. height of a wall), m
$\lambda$ = thermal conductivity of the fluid, W/mK

2. *Prandtl number*: $Pr = \frac{\nu}{a}$

**In addition to these three numbers, Reynolds number is used to define whether the fluid flow is laminar or turbulent.**

$\nu$ = kinematic viscosity of the fluid, $m^2/s$
$a$ = thermal diffusivity of the fluid, $m^2/s$

3. *Grashoff number*: $Gr = \frac{gL^2 x\rho}{\nu^2 \Delta\rho}$

**Self-studying: Review the meanings (interpretations) of the dimensionless numbers.**

$x$ = height of surface (in free convection), $m^2/s$ (local heat transfer coefficient: $x = L$)
$\rho$ = density of the fluid (free flow, far from the surface) $kg/m^3$
$\Delta\rho$ = difference between the density of the free flow and that of the flow close to the surface

# Implementation 1:
# Natural convection between vertical surface and air

*Nusselt correlation* at height $x$:

$$Nu_x = 0.356 \cdot (Gr)^{0.25}$$

Thickness of the boundary layer:

$$\frac{\sigma}{x} = 5.34\, Gr^{-0.25}$$

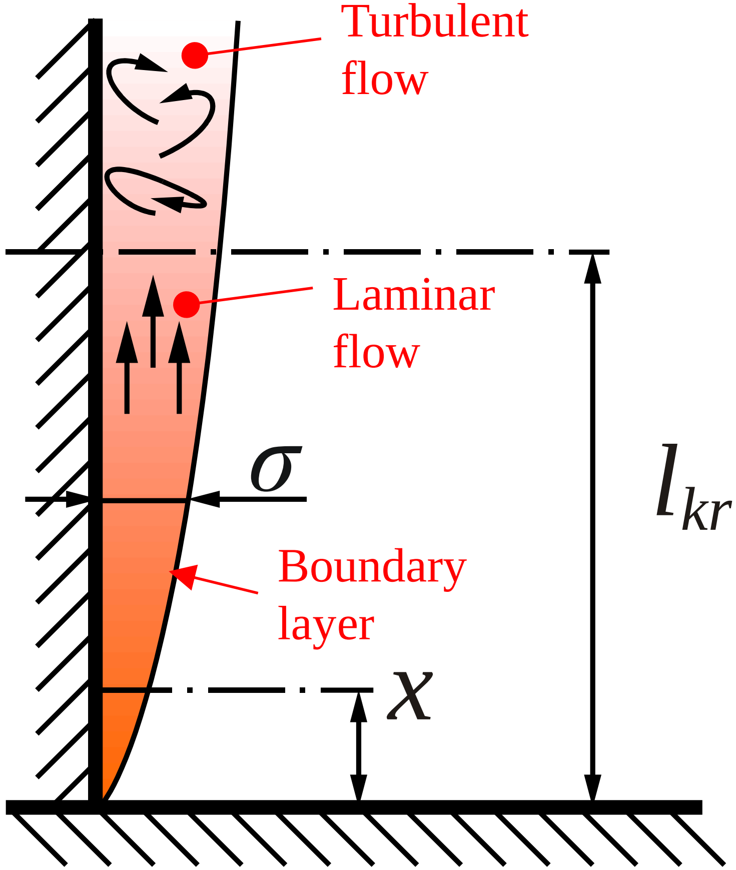
For room air ($T$ = +20 °C, $L = x$):

→ Heat transfer coefficient at height $x$:

$$\alpha_x = 0.96 \cdot \left( \frac{\Delta T}{x} \right)^{0.25}$$

→ Transition from laminar to turbulent at height:

$$l_{kr} = 1.89 \cdot \Delta T^{-1/3}$$




→ *Average* heat transfer coefficients:

$$\alpha = 1.17 \cdot \Delta T^{1/3} \quad \text{(laminar zone)}$$

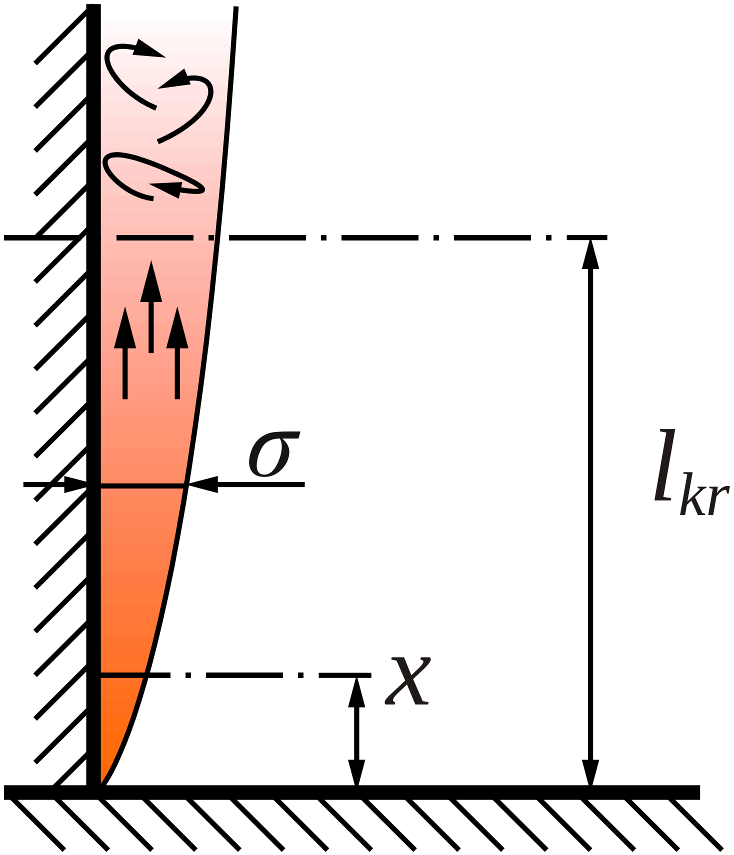
$$\alpha = 1.66 \cdot \Delta T^{1/3} \quad \text{(turbulent zone)}$$

# Example

a) Derive the convective heat transfer coefficient at height *x* ($\alpha_x$) from the Nusselt number and Nusselt correlation for air at $T = 20°C$.

b) Calculate the thickness of the boundary layer ($\sigma$) and the position of the transition zone ($l_{kr}$) at $x = 1$ m and $\Delta T = 2°C$.

c) Sketch a graph for the average heat transfer coefficient of the *laminar zone* for $\Delta T = 0...20°C$.



# Solution – I

1. Grashoff number:
   - Kinematic viscosity at 293 K (Engineering Toolbox): $\nu$ = 0.0000157 $m^2/s$
   - Ideal gas: $\Rightarrow \rho \sim \frac{1}{T} \Leftrightarrow \frac{\rho}{\Delta\rho} = \frac{\Delta T}{T}$
   - Characteristic lengths: $L = x$

$$\Rightarrow Gr = \frac{gL^2 x\Delta T}{\nu^2 T} = \frac{gx^3\Delta T}{\nu^2 T} = \frac{9.81 \cdot x^3 \Delta T}{(0.0000157)^2 \cdot 293} = 135831996 \cdot x^3 \Delta T$$

**Units are omitted for the sake of readability.**

2. Heat transfer coefficient at $x$:
   - Thermal conductivity at 293 K (Engineering Toolbox): $\lambda$ = 0.025 W/mK

$$Nu_x = \frac{\alpha_x x}{\lambda} = 0.356 \cdot (Gr)^{0.25} = 0.356 \cdot (135831996 \cdot x^3 \Delta T)^{0.25}$$

$$\Rightarrow \underline{\underline{\alpha_x}} = \lambda \cdot 0.356 \cdot (135831996)^{0.25} \cdot \left(\frac{\Delta T}{x}\right)^{0.25} = \underline{\underline{0.96 \cdot \left(\frac{\Delta T}{x}\right)^{0.25}}}$$

# Solution – II

3. Grashoff number:
   - Air at 293 K: $\nu$ = 0.0000157 m$^2$/s
   - We choose the characteristic length: $L = x = 1$ m
   - At $\Delta T$ = 2°C (K):

$$Gr = \frac{gx^3\Delta T}{\nu^2 T} = \frac{9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\cdot(1\,\mathrm{m})^3\cdot 2\,\mathrm{K}}{(0.0000157\,\frac{\mathrm{m}^2}{\mathrm{s}})^2\cdot 293\,\mathrm{K}} = 2.6\cdot 10^8$$

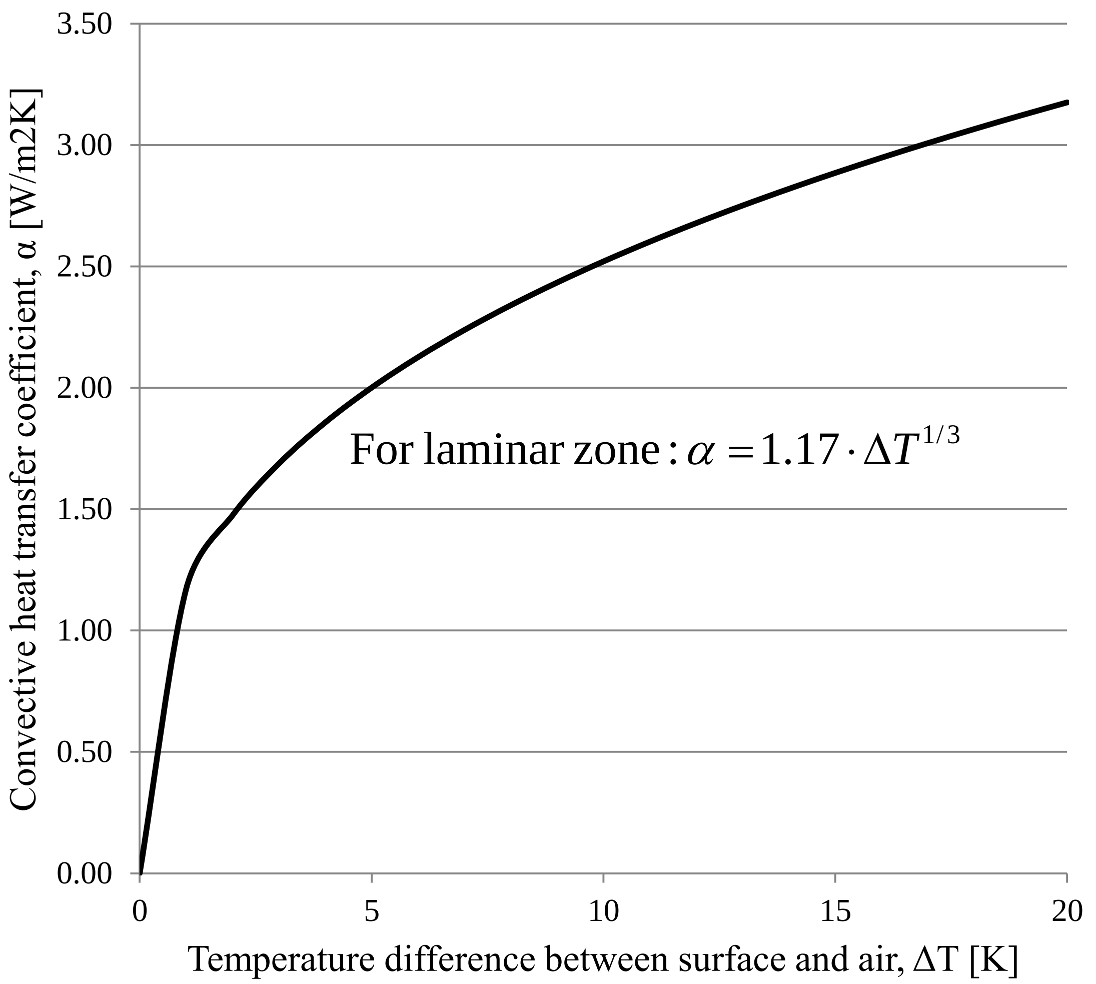
4. Thickness of the boundary layer:

$$\underline{\underline{\delta}} = 5.34\,Gr^{-0.25} = 5.34\cdot(2.6\cdot 10^8)^{-0,25} = 0.04\,\mathrm{m} = \underline{\underline{4\,\mathrm{cm}}}$$

5. Position of the transition zone:

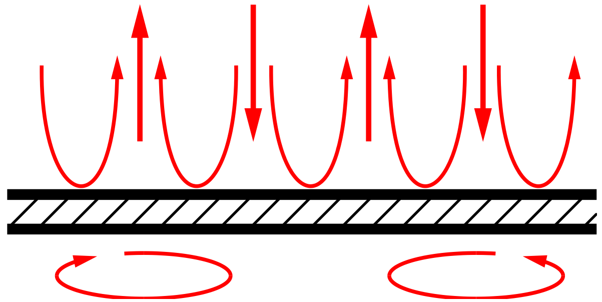
$$\underline{\underline{l_{kr}}} = 1.89\cdot\Delta T^{-1/3} = 1.89\cdot(2\,\mathrm{K})^{-1/3} = \underline{\underline{1.5\,\mathrm{m}}}$$

## Solution – III



# Implementation 2: Natural convection between horizontal surface and air

The convective heat transfer from horizontal surface such as warm floor or cold ceiling is more efficient than that from a vertical surface.



Average heat transfer coefficient:

$$\alpha = 2.26 \cdot \Delta T^{1/3}$$

Example: Warm floor

$\Delta T = 5$ K $\qquad \alpha = 3.84$ W/m$^2$K

$\Delta T = 20$ K $\qquad \alpha = 6.07$ W/m$^2$K

# Implementation 3: Natural convection between tube and air

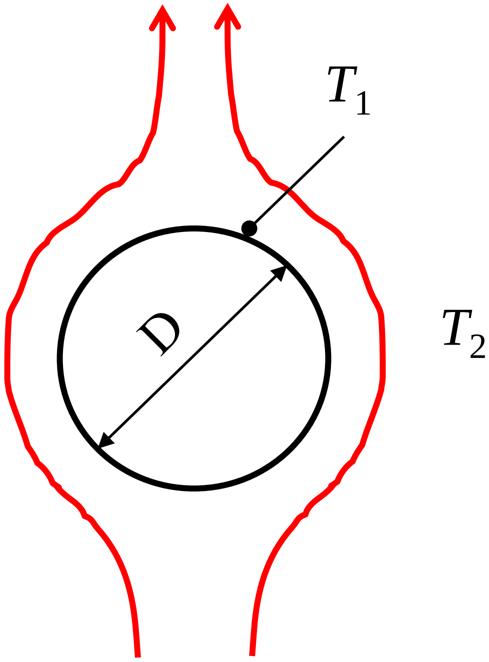
Average heat transfer coefficient:

$$\alpha = 5.0 \cdot \sqrt[4]{\frac{T_1 - T_2}{T_2\, D}}$$

Example:

$T_1$ = 40°C, $T_2$ = 20°C, $D$ = 15 mm

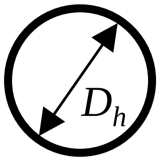
$$\alpha = 5.0 \cdot \sqrt[4]{\frac{20\,\mathrm{K}}{293\,\mathrm{K} \cdot 0.015\,\mathrm{m}}} = \underline{\underline{7.3\,\mathrm{W/m^2K}}}$$

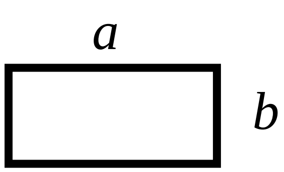


# Implementation 4: Forced convection

Nusselt correlation:

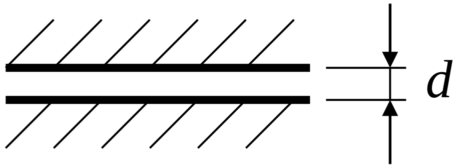
$$Nu_D = 0.023\,Re^{0.8}Pr^{0.33}, \text{ where } Re = \frac{vD_h}{\nu}$$




For rectangle: $D_h = \frac{4A}{P} = \frac{4ab}{2(a+b)}$

For gap: $D_h = 2d$




Average heat transfer coefficients ($v$ = fluid velocity, m/s):

Surfaces in room: $\alpha = 6.4 + 4.2\,v$ $(\text{default} = 10.6\ \text{W/m}^2\text{K})$

Outer wall against wind: $\alpha = 11.1\,v^{0.51}$ $(\text{default} = 11.1\ \text{W/m}^2\text{K})$

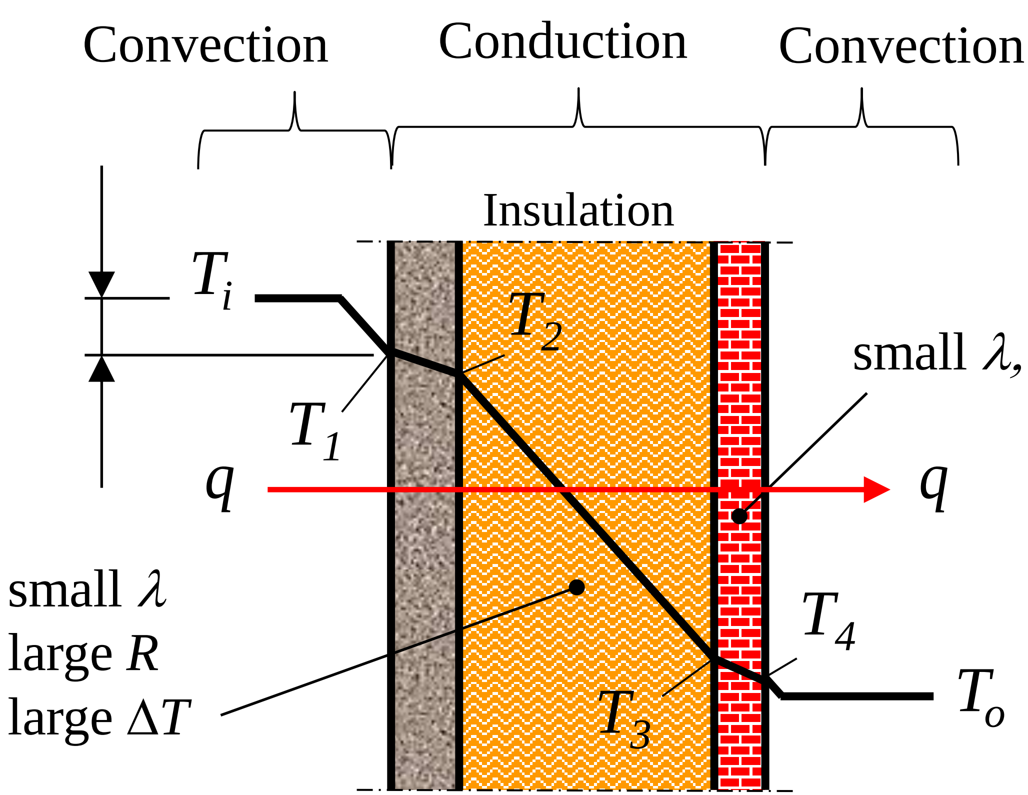
Outer wall windbreak: $\alpha = 7.0\,v^{0.36}$ $(\text{default} = 7.0\ \text{W/m}^2\text{K})$

# Temperature distribution through wall element

Internal temperature $T_i$ > External temperature $T_o$

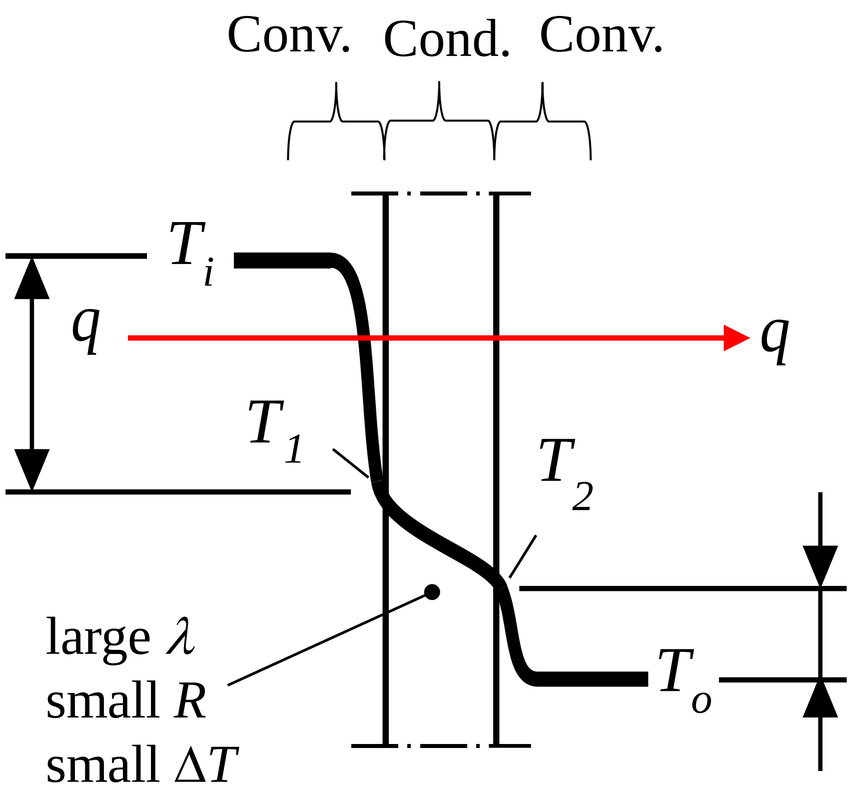
$$R = \frac{s}{\lambda}$$

$$R_s = \frac{1}{\alpha}$$




**Thermal resistance of the insulation (*R*) is conclusive.**

# Temperature distribution through window




$T_i > T_o$

$$R = \frac{s}{\lambda}$$

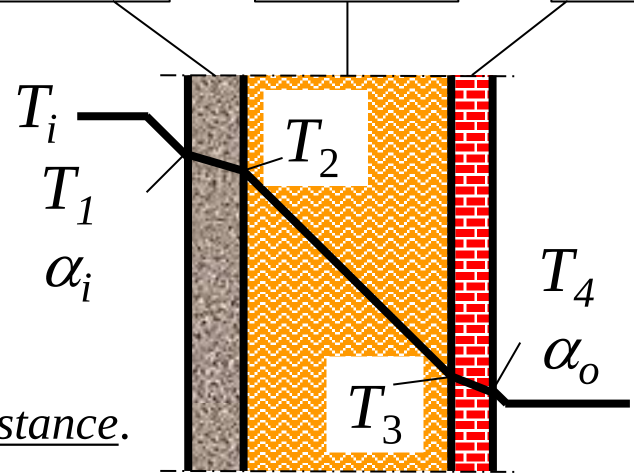
$$R_s = \frac{1}{\alpha}$$

**Thermal resistance of the boundary layer ($R_s$) is conclusive.**

# How to calculate the structure temperatures?

$$R_1 = \frac{s_1}{\lambda_1} \qquad R_2 = \frac{s_2}{\lambda_2} \qquad R_3 = \frac{s_3}{\lambda_3}$$

$$R_{si} = \frac{1}{\alpha_i}$$




⊕ $R_{si}$ is known as *internal surface resistance*.

⊖ $R_{so}$ is known as *external surface resistance*.

$$R_{so} = \frac{1}{\alpha_o}$$

Heat flux through wall remains the same through each layer (stationary conditions):

$$q = \frac{T_i - T_1}{R_{si}} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_o}{R_{so}}$$

$$\Rightarrow \text{E.g.} \quad \frac{T_i - T_1}{T_i - T_o} = \frac{R_{si}}{R_{si} + \sum_{i=1}^{3} R_i + R_{so}} = \frac{\frac{1}{\alpha_i}}{\frac{1}{\alpha_i} + \sum_{i=1}^{3} \frac{s_i}{\lambda_i} + \frac{1}{\alpha_o}}$$

# U-value and conductance

Total heat transfer coefficient aka *thermal transmittance* aka *U-value* [W/m²K]:

$$U = \frac{1}{R_{si} + \sum_{i=1}^{n} R_i + R_{so}}$$

Specific heat transfer coefficient aka *conductance* [W/K]:

$$G = UA = \frac{1}{R_{si} + \sum_{i=1}^{n} R_i + R_{so}} \cdot A$$

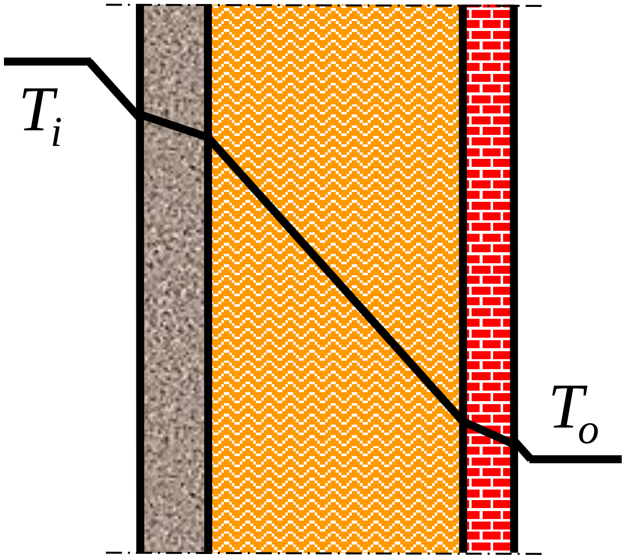
where $A$ = area of the wall [m²]

Heat flow through wall [W]:

$$\Phi = qA = UA(T_i - T_o)$$

Thermal resistances

$R_{si}$, $R_1 \ldots R_i \ldots R_n$, $R_{so}$



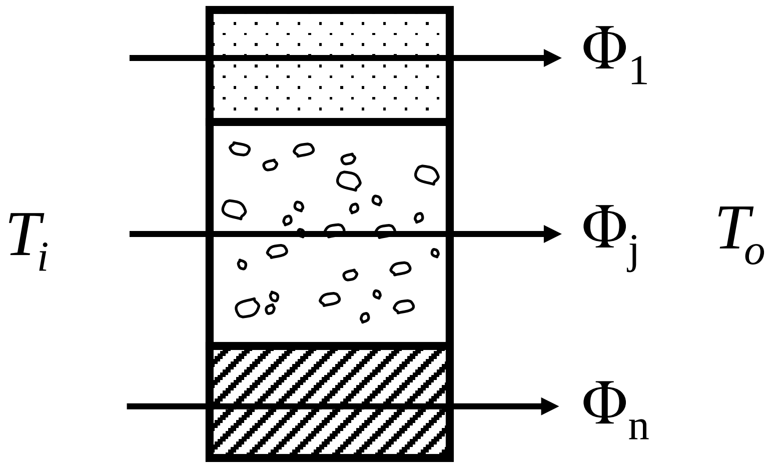
# Wall element with various materials – "U-value method"

- In practice, a wall element consists of several components that differ from each other in properties.
- In the ”U-value method”, the U-value of the whole wall is calculated as the weighted average of U-values based on the areas of each material:

$$\overline{U} = \frac{\sum_{j=1}^{n} U_j A_j}{\sum A_j}$$

- The sum heat flow through the wall is

$$\Phi = \sum_j \Phi_j = \sum_j \left(U_j A_j (T_i - T_o)\right) = \overline{U} A (T_i - T_o)$$




**The U-value method assumes one-dimensional heat transfer, which underestimates the sum heat flow through the wall.**

# Wall element with various materials – “Standard method”

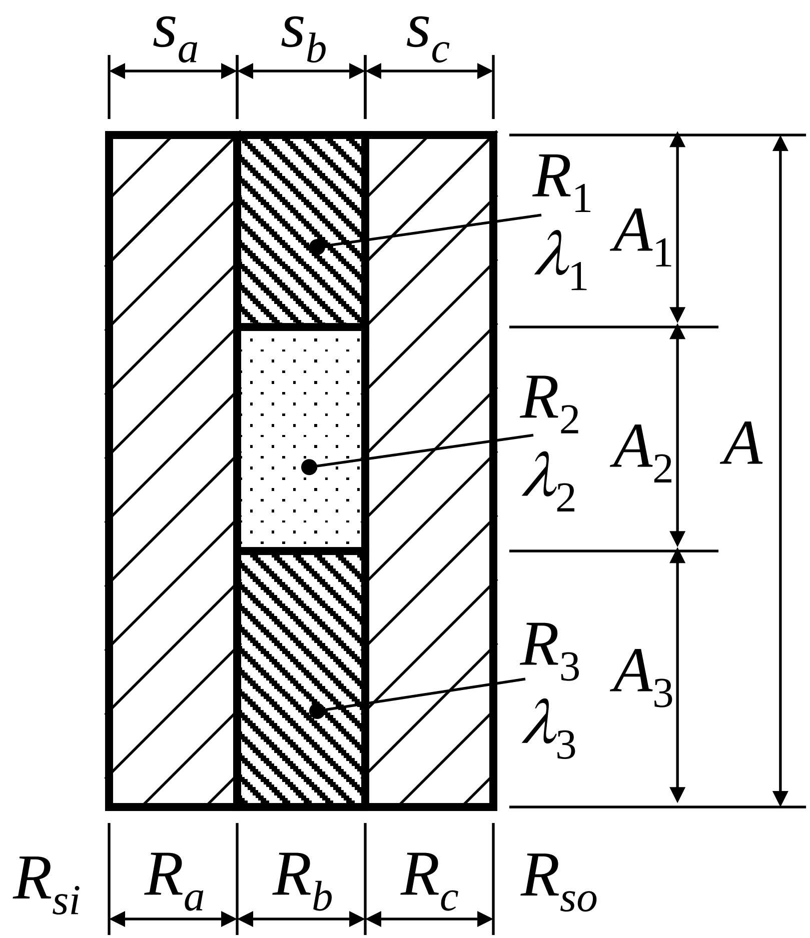
- In the ”Standard method” (based on the Finnish Building Code C4), the λ-value of a wall structure consisting of various materials is calculated as the weighted average of λ-values based on the areas of each material:

$$\lambda_b = \frac{A_1\lambda_1 + A_2\lambda_2 + A_3\lambda_3}{A_1 + A_2 + A_3}; R_b = \frac{s_b}{\lambda_b}$$

- The total thermal resistance of the wall is

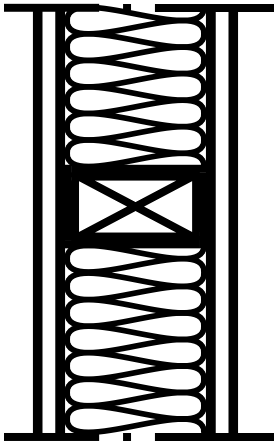
$$R = R_{si} + \sum_{j=a}^{c} R_j + R_{so}$$

- The method assumes the temperature between each layer (a, b, c) is constant throughout the area, which allows new layers to be easily added.



# Thermal bridges

- A *thermal bridge* (aka cold bridge or heat bridge) is an area in a building envelope with a significantly higher heat transfer (poorer thermal insulation) than the surrounding materials.
- The area is commonly interpreted as a point or a line in shape.
- The impact of thermal bridges on the U-value of the wall is calculated by way of *additional thermal transmittances* ($\Delta U_X$ and $\Delta U_\psi$).



- The U-value of a wall with thermal bridges:

$$U = U_0 + \Delta U_X + \Delta U_\psi$$
$$= U_0 + \frac{n}{A} X + \frac{l}{A} \Psi$$

where $U_0$ = U-value without thermal bridges, W/m$^2$K
$n$ = number of identical point thermal bridges
$X$ = *additional conductance* due to point thermal bridges, W/K
$l$ = total length of linear thermal bridges, m
$\psi$ = additional conductance due to linear thermal bridges, W/mK
$A$ = area of the wall, m$^2$

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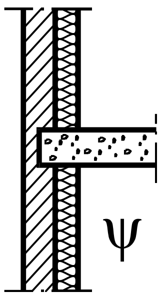
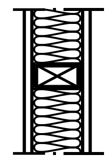
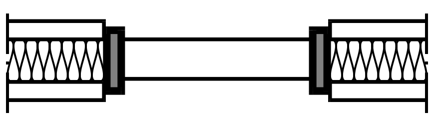
# Additional conductances for linear thermal bridges



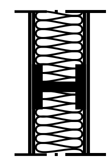
ψ = 0.35…0.57 W/mK



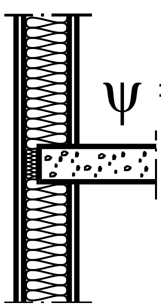
ψ = 0.02…0.03 W/mK



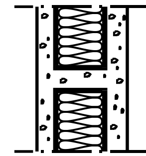
ψ = 0.002 W/mK



ψ = 0.6…0.8 W/mK



ψ = 0.12…0.17 W/mK



ψ = 0.1…0.5 W/mK

# Fundamentals of thermal radiation

- Thermal radiation is electromagnetic radiation emitted by a body or a surface on the basis of its temperature, not depending on the state of its surroundings.
- *Radiant heat flux* (aka *radiance*) [$W/m^2$] of a *black body* (i.e. idealized physical body that absorbs all incident electromagnetic radiation) is calculated from

$$M_m = \sigma T^4$$

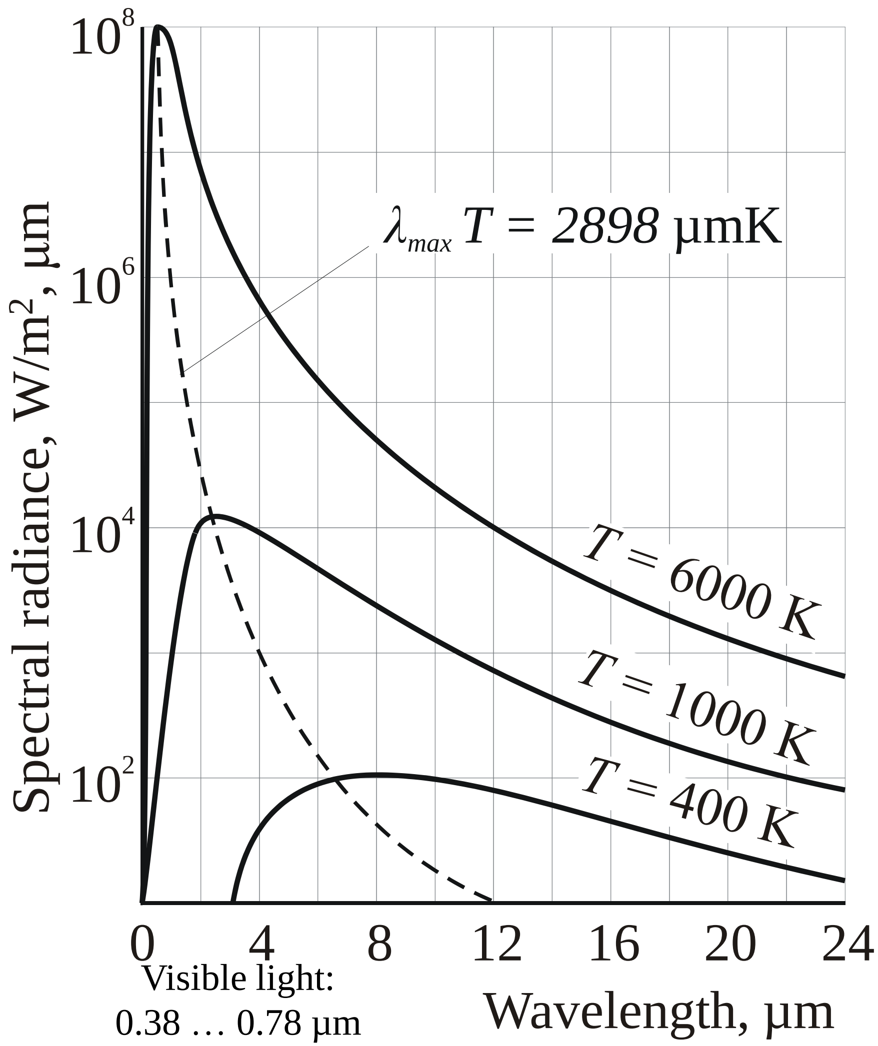
where $\sigma$ = Stefan-Boltzmann constant ($5.67 \cdot 10^{-8}$ $W/m^2K^4$)

- In practice (realistic surface *i*) the radiance is calculated from

$$M_i = F_i \varepsilon_i M_m$$

where $\varepsilon_i$ = emissivity of surface *i* [-]
$F_i$ = view factor of surface *i* (i.e. fraction of the surroundings the surface *i* represents) [-]

$$\sum_{i=1}^{n} F_i = 1$$



Visible light:
0.38 … 0.78 μm
Blue…Red

# Radiative heat transfer coefficient

- Surfaces visible to each other receive (absorb and reflect) thermal radiation they emit
- Net heat transfer between two surfaces depends on surface temperatures, emissivities and their position with respect to each other as follows:

$$q_{12} = \sigma \frac{T_1^{\,4} - T_2^{\,4}}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

- Net heat transfer can be also expressed as

$$q_{12} = q_1 - q_2 = \alpha_r (T_1 - T_2)$$

where $\alpha_r$ = *radiative heat transfer coefficient*, W/m$^2$K

- For parallel, infinite surfaces, $\alpha_r$ can be defined as

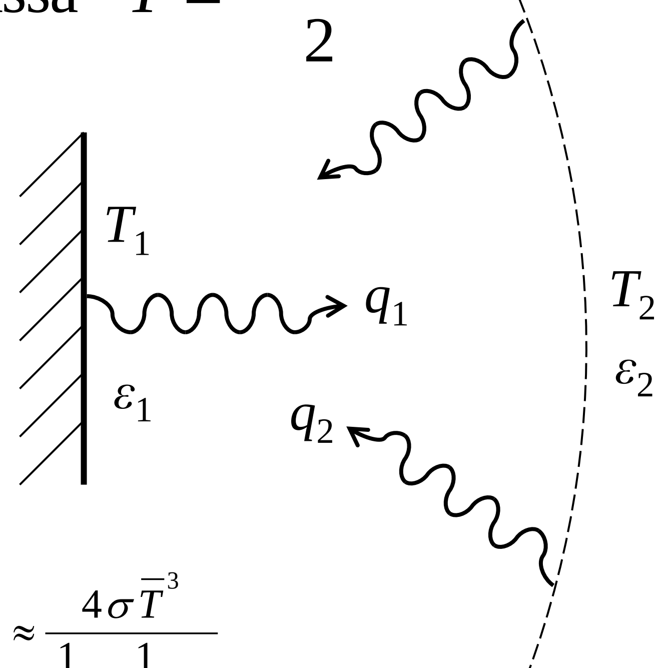
$$\alpha_r = 4\varepsilon\sigma\bar{T}^3;\ \bar{T} = \frac{T_1 + T_2}{2}$$

- When surface 2 is large in comparison with surface 1: $\alpha_r \approx \dfrac{4\sigma\bar{T}^3}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$

Net heat transfer

$$q = q_1 - q_2$$

missä $\bar{T} = \dfrac{T_1 + T_2}{2}$



# Total heat transfer coefficient

- Total heat transfer to/from a surface entails both convection (c) and radiation (r):

  $q = q_c + q_r$

- The share of convection in the total heat transfer is calculated from

  $q_c = \alpha_c (T_{s,i} - T_a)$

  where $\alpha_c$ = convective heat transfer coefficient
  $T_{s,i}$ = temperature of surface $i$
  $T_a$ = air temperature

- The share of convection in the total heat transfer is calculated from

  $q_r = \alpha_r (T_{s,i} - T_s)$

  where $\alpha_r$ = radiative heat transfer coefficient
  $T_{s,i}$ = temperature of surface $i$
  $T_s$ = temperature of other surfaces (excluding $i$) (assumption: they have equal temperature)

- The *total heat transfer coefficient* is

$$\alpha = \alpha_c + \frac{T_{s,i} - T_s}{T_{s,i} - T_a} \alpha_r$$

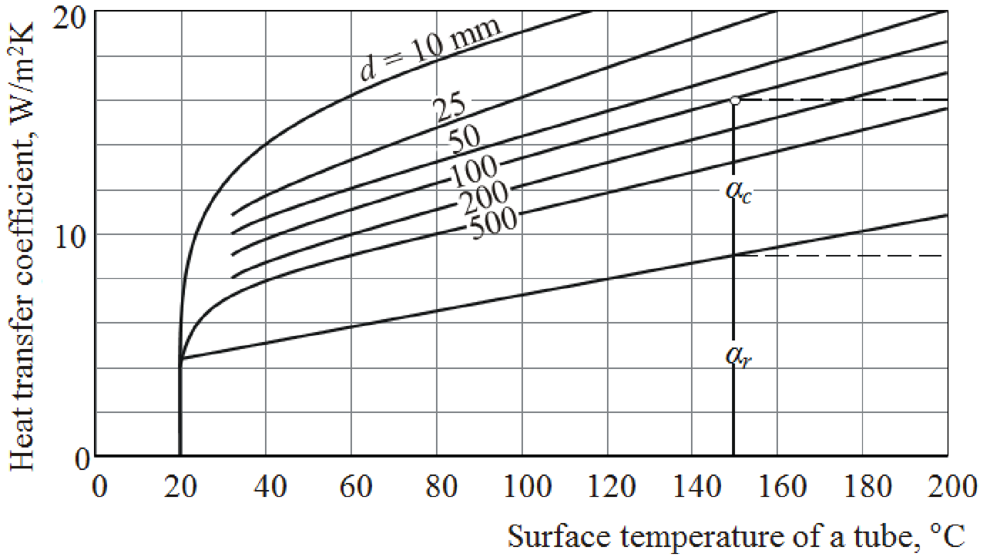
**$T_s$ commonly unknown**

- With an acceptable accuracy can be stated: $q = (\alpha_c + \alpha_r)(T_{s,i} - T_a) = \alpha (T_{s,i} - T_a)$

# Example

The graph summarizes the total heat transfer coefficient of a horizontal tube, indicating the share of convective and radiative heat transfer. The data are valid for $T_a$ = 20°C (293 K). The emissivities for both the tube (1) and its surroundings (2) can be assumed $\varepsilon_1 = \varepsilon_2 = 0.93$.

Show through calculation the radiative heat transfer coefficient ($\alpha_r$) at the surface temperature of 150°C (423 K).




Solution:

$$\overline{T} = \frac{(423 + 293)\,\text{K}}{2} = 358\,\text{K}$$

$$\alpha_r = \frac{4\sigma\overline{T}^3}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{4 \cdot 5.67 \cdot 10^{-8}\,\frac{\text{W}}{\text{m}^2\text{K}^4} \cdot (358\,\text{K})^3}{\frac{1}{0.93} + \frac{1}{0.93} - 1} = 9.0\,\frac{\text{W}}{\text{m}^2\text{K}}$$