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Heating and Cooling Systems
EEN-E4002 (5 cr)

Calculation of heating and cooling requirements

Learning objectives

Student will learn to

- know the elements of a building's heat balance and the basic terminology related to calculation of heating and cooling requirements
- comprehend the transient nature of calculating the cooling load
- calculate
  - heat loss of a building
  - heating energy requirements using the degree day method
  - heat load from occupants
  - intensity of solar radiation on surface (irradiance)
  - solar heat load through window
  - cooling load and room temperature in transient conditions

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Lesson outline

1. Heat balance of building
2. Terminology
3. Calculating heat loss and energy demand
4. Calculating human and solar heat loads
5. Calculating cooling loads and indoor temperatures based on the energy balance of a room

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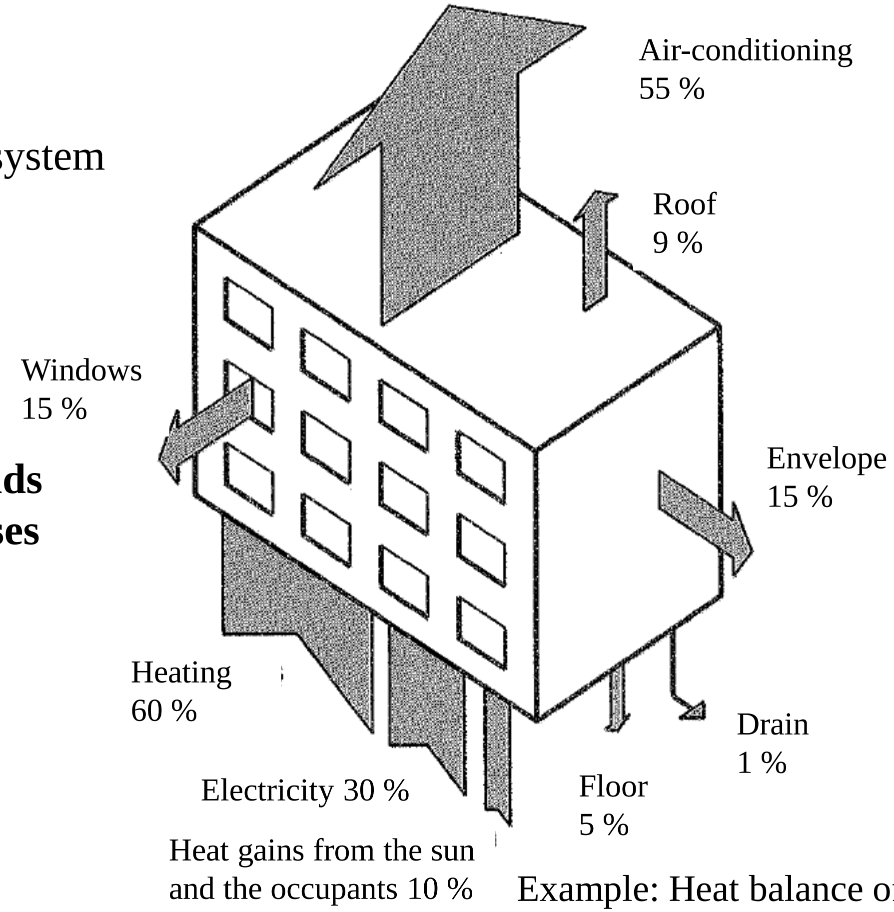
Heat balance of building

Heat to the building through heating system
– heat removed from the building through cooling system
+ heat from lighting/appliances
+ heat from solar and human sources
– heat (transmission) through windows
– heat (transmission) through floors
– heat (transmission) through roof
– heat (transmission) through envelope
– heat through ventilation/air leaks
– heat through drains

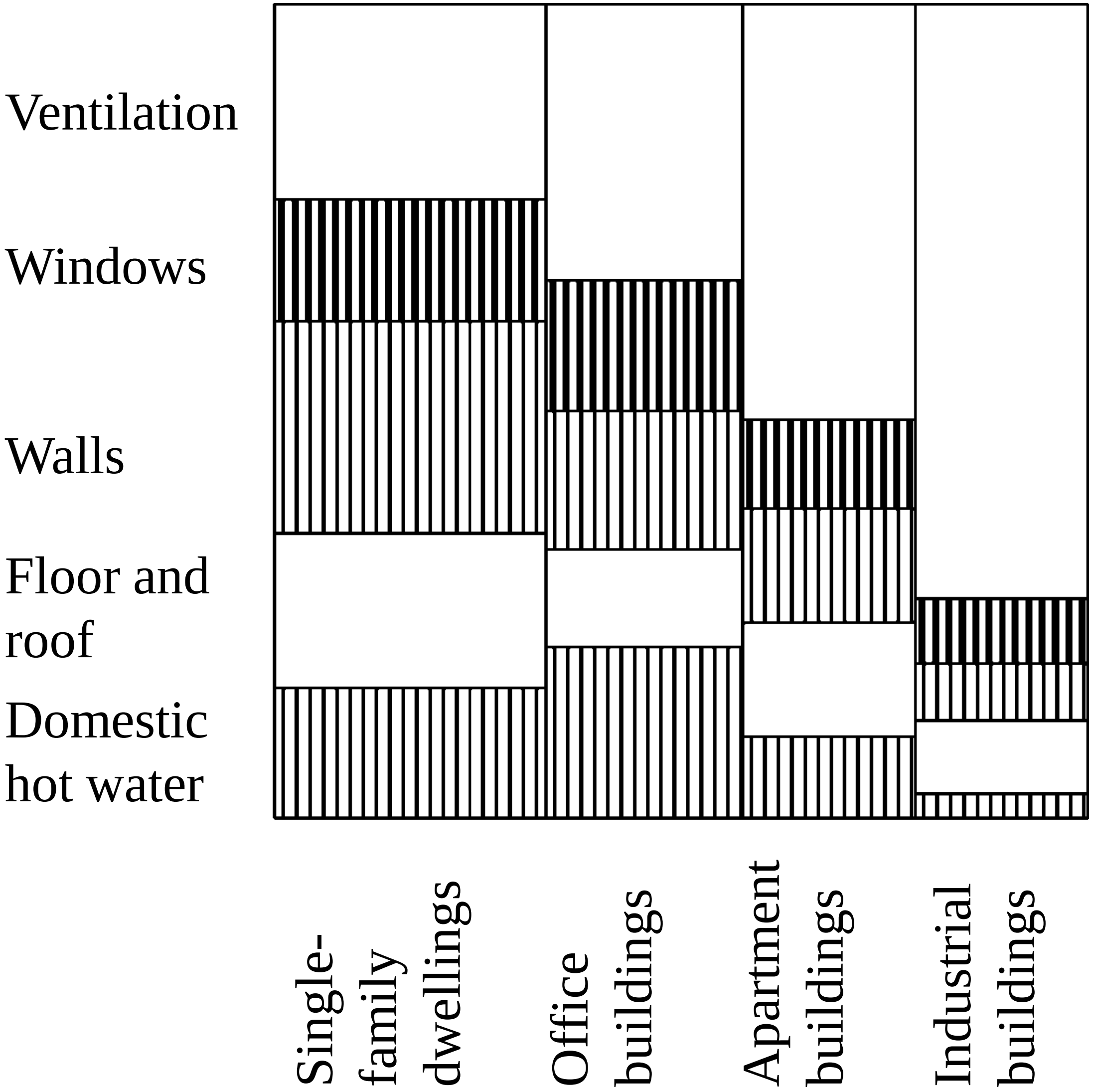
+ loads
- losses

= **heat stored in the building**
(= 0 for heating calculations)

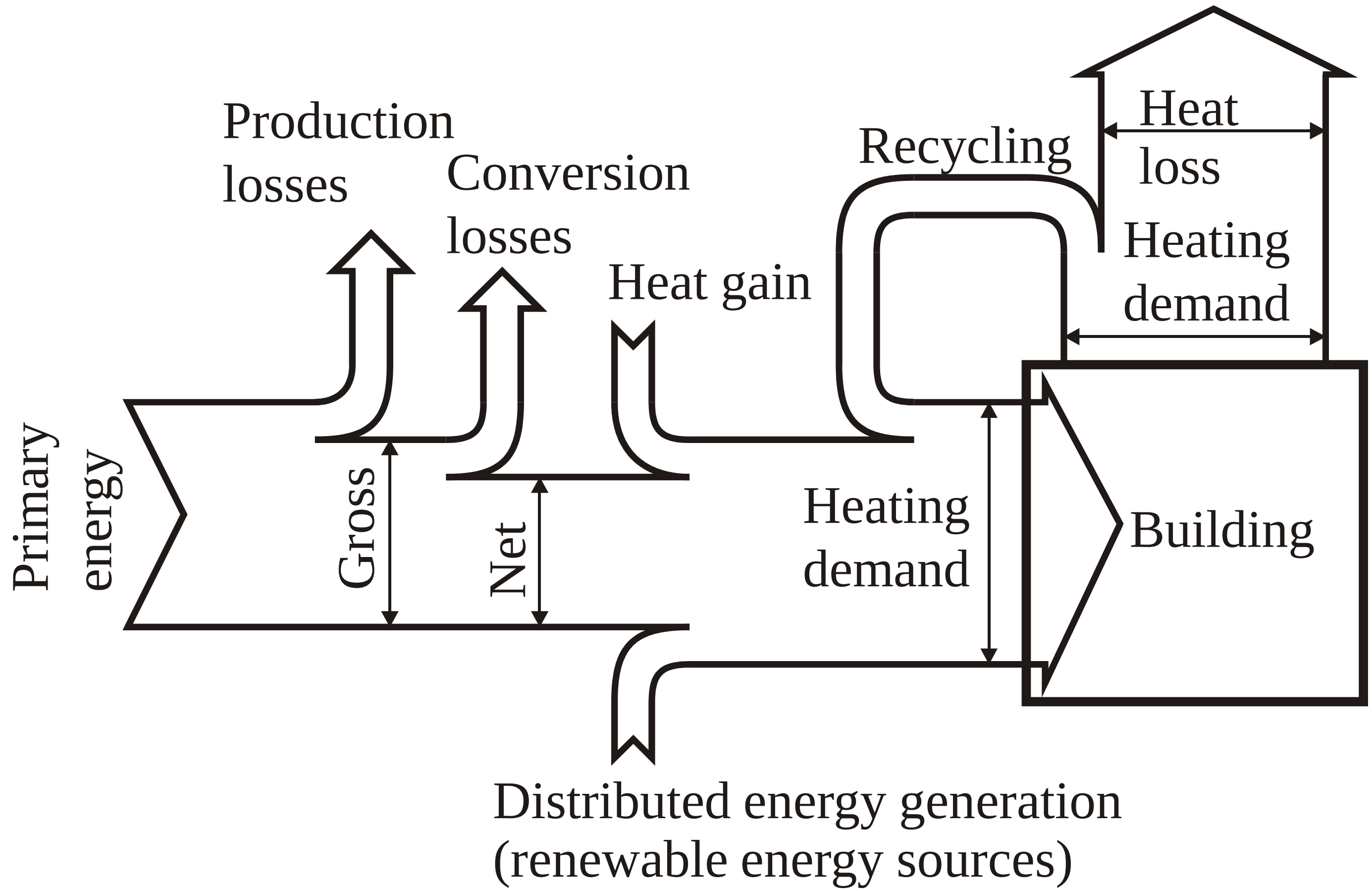


Example: Heat balance of an office building

Example: Share of energy need for heating by building types in Finland



Sankey diagram of building's heat supply



The National Building Code of Finland

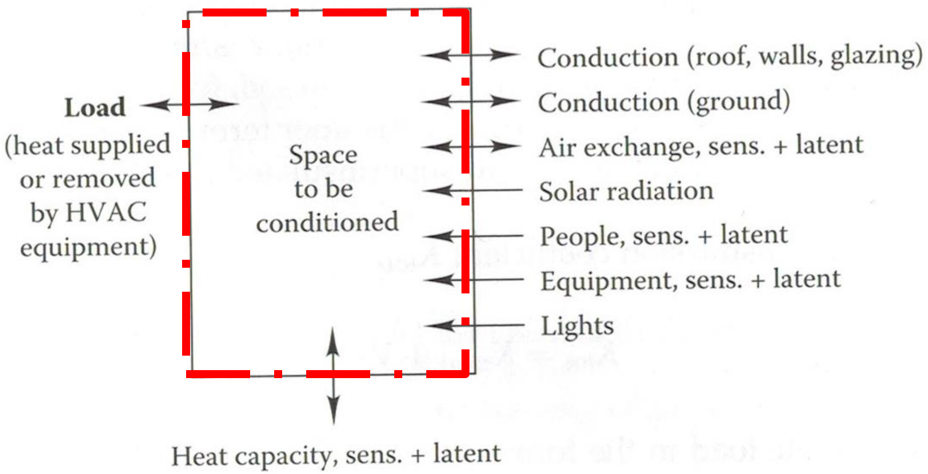
The key regulations for heating and cooling:

- C Insulation
  - C4 Thermal insulation (unofficial translation available)
- D Hepac and energy management (only in Finnish/Swedish)
  - D1 Water supply and drainage installations for buildings
  - D2 Indoor climate and ventilation of buildings
  - D3 Energy management in buildings
  - D4 HEPAC drawings
  - D5 Calculation of power and energy needs for heating of buildings
  - D7 Efficiency requirements for boilers

Terminology in nutshell

- *Heat load*: heat transfer into the building
- *Heat loss*: heat transfer outwards from the building
- *Heating/cooling load* is the heating/cooling power/energy, which is required to maintain the desired room temperature.
- *Heat gain* is *useful* heat to the room from occupants, lighting, equipment and the sun. Heat gains reduce the energy demand.
- Heat loads/gains may be *internal* (from inside of the system boundary, e.g. appliances) or *external* (from outside of the system boundary, e.g. solar).

Heating load = heat loss
Cooling load ≠ heat load



NOTE: Heating and cooling loads (gains) are expressed either as power [(k)W] or energy [(k)Wh] and the included energy needs are different.

1 kWh = 3,6 MJ

Heat gain vs. heat load

- In the literature, heat gains and heat loads are often treated without making a distinction.
- The word ”gain” has a positive meaning (”something wanted or valued that is gotten”), whereas ”load” has a negative meaning (”something that is lifted and carried”).
- The heat load can be utilized with a presumption that
  - heating demand exists
  - control equipment can take the advantage of the heat load by simultaneously reducing the heat supply
- If the above conditions are met, it is more appropriate to use the expression “heat gains” than “heat loads”.

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Latent and sensible heat load

1. *Latent heat load*: latent heat into the room with water vapour
2. *Sensible heat load*: heat load into the room through radiation, convection and conduction
3. *Total heat load* is the sum of latent and sensible heat loads.

Sensible heat load

$$\Phi_{sensible} = \Phi_{cond} + \Phi_{air} + \Phi_{floor} - \Phi_{gain} \pm \Phi_{stor}$$

Latent heat load

$$\Phi_{\text{latent}} = \sum \underbrace{q_v h_v (T_v)}_{\text{heat of evaporation from surroundings}} + \sum \underbrace{q_w c_{pv} T_w}_{\text{heat of evaporation from room}}$$

Subscripts:
v = vapour
w = water

Total heat load

$$\Phi_{\text{tot}} = \Phi_{\text{sensible}} + \Phi_{\text{latent}}$$

The theory of latent heat load will be treated in detail on the course EEN-E4003 Ventilation and Air-Conditioning systems.

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Sensible heat load (or loss)

$$\Phi = \Phi_{cond} + \Phi_{air} + \Phi_{floor} - \Phi_{gain} \pm \Phi_{stor}$$

General expression

where

$$\Phi_{cond} = G_{cond}\left(T_i - T_o\right) = \sum_{j=1}^{n} U_j A_j \cdot \left(T_i - T_o\right)$$

Conduction through envelope (other than floor), components 1...*j*...*n*

$$\Phi_{floor}$$

Conduction through floor to ground

$$\Phi_{air} = G_{air}\left(T_i - T_o\right) = q_V \rho c_p \left(T_i - T_o\right)$$

Heat due to ventilation/infiltration

$$\Phi_{gain} = \Phi_{sol} + \Phi_{lit} + \Phi_{equ} + \Phi_{occ}$$

Heat gains (solar, lighting equipment, occupants)

$$\Phi_{stor}$$

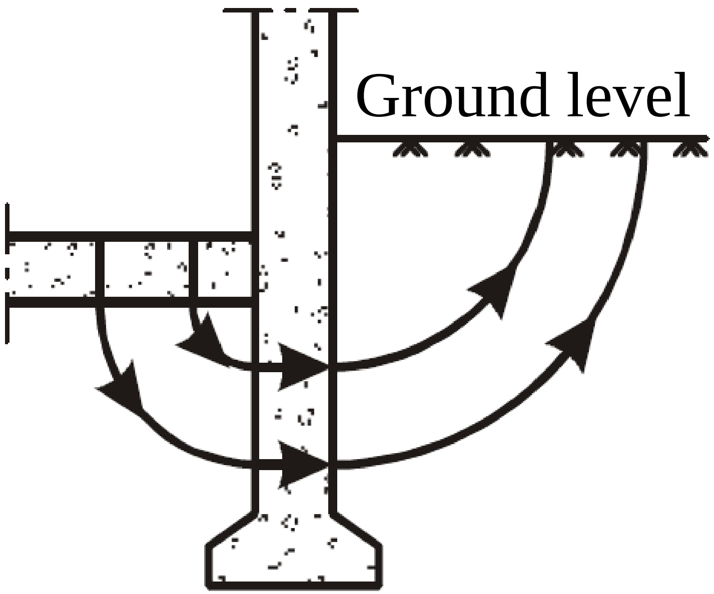
Heat stored into building (Φ_{stor} = 0 for heating calculations)

$$G_{tot} = G_{cond} + G_{air}$$

Total heat transmission coefficient aka *conductance*

Heating design conditions

- *Heating load in design conditions* [(k)W] depends on heat transmission through envelope, ventilation and domestic hot water and is defined as the sensible heating load *without* accounting for the impact of heat gains (gain) and energy storage (stor).
- Heat transmission (heat loss) through envelope is calculated on the basis of *outdoor (external) design temperature* (e.g. $T_{o,des}$ =–26°C in Helsinki area)
- Correspondingly, the *indoor design temperature* is commonly $T_{i,des}$ = +21°C.
- The heat transmission through floor (to ground) must be calculated separately, since it is not directly proportional to the temperature difference between indoor (internal) and outdoor (external) temperatures.
- Heating need of domestic hot water (DHW) [(k)W] is always estimated separately.



Rule of thumb: Heat transmission to ground is constant 5 W/m^2 throughout a year with a good accuracy.

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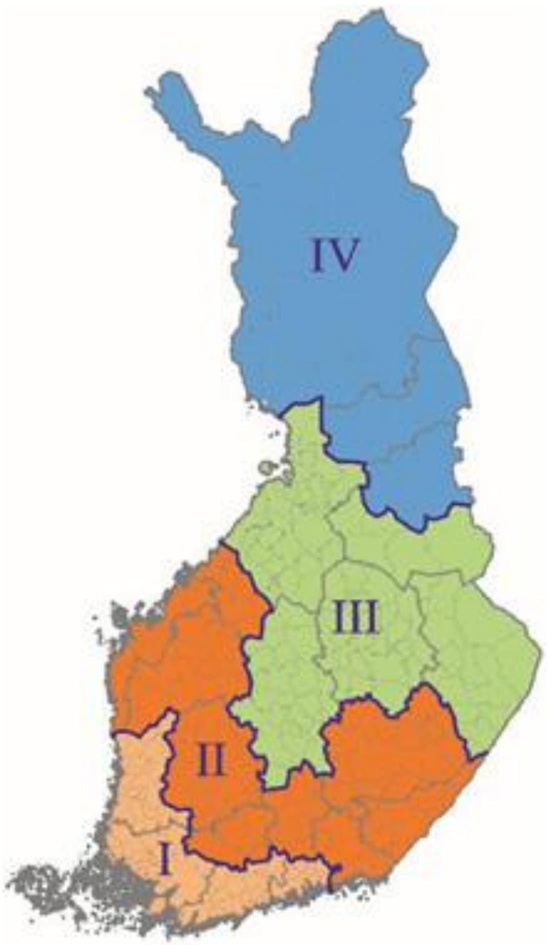
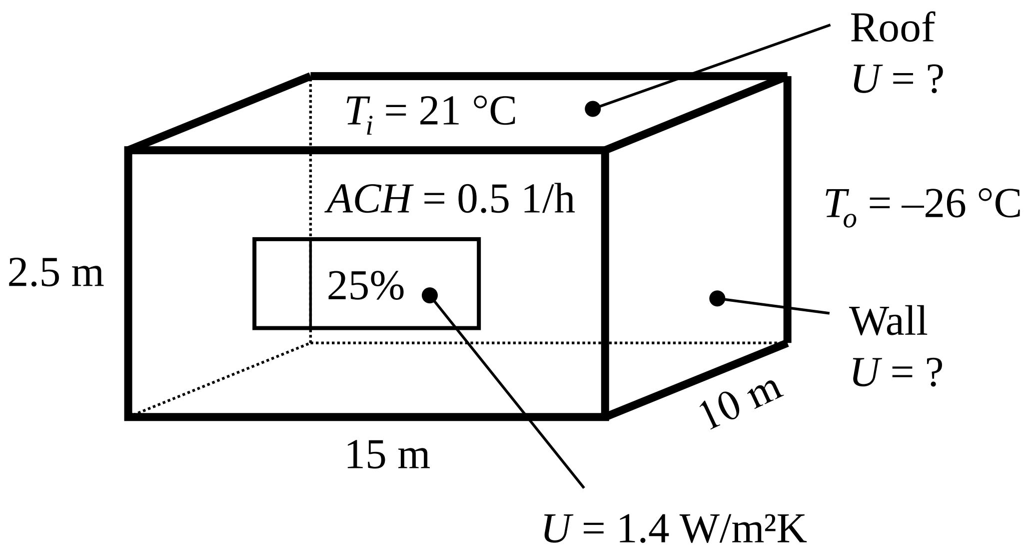
Outdoor design temperature

- Outdoor (external) design temperature is always somewhat less than the extreme temperature at the target location. Thus, oversizing the heating system can be avoided.
- Design temperatures in Finland:
  - Zone I (Helsinki): –26°C
  - Zone II (Jokioinen): –29°C
  - Zone III (Jyväskylä): –32°C
  - Zone IV (Sodankylä): –38°C



Example

A building (located in Helsinki) is assumed a rectangular box 15 m × 10 m × 2.5 m. The insulation is fiberglass (conductivity 0.06 W/mK, thickness 0.40 m in the roof and 0.25 m in the walls. The windows are triple-glazed with U = 1.4W/m^{2}K and they cover 25 % of the total wall area. The convective heat transfer coefficient of all the surfaces is 34 W/m^{2}K and the ***air change rate*** (***ACH***) is 0.5 1/h.



Air-(ex)change rate (ACH) is the number of air changes per one (1) hour due to ventilation. ACH is obtained by dividing the air flow rate [m^3/h] by the volume of the building/space [m^3].

Calculate the overall heat transmission coefficient (conductance) G_{tot} and the heating load (= heat loss in design conditions, indoor temperature 21°C). For the air ρ = 1.2 kg/m^3 and c_a = 1000 J/kgK. The heat transmission to ground is assumed as q_{floor} = 5 W/heated-m^2.

Solution – I

1. Calculations:
   - U-value (for each component)

$$R=\frac{1}{U}=\frac{1}{\alpha_i}+\frac{s}{\lambda}+\frac{1}{\alpha_o} \quad \rightarrow \quad U=\frac{1}{\frac{1}{\alpha_i}+\frac{s}{\lambda}+\frac{1}{\alpha_o}}$$

   - Heating load and overall heat transmission coefficient (conductance)

$$\Phi_{floor}=A_{net}\cdot q_{floor}$$

$$\Phi_{cond}=G_{cond}\left(T_i-T_o\right)=\sum U_j A_j\cdot\left(T_i-T_o\right)$$

$$\Phi_{air}=q_V\rho c_p\left(T_i-T_o\right)$$

$$G_{tot}=G_{cond}+q_V\rho c_p$$

Solution – II

2. Results (example substitutions)

– Heated area (assumption): A_{net} = 15 m × 10 m = 150 m^2

– Air volume: (15 × 10 × 2.5) m^3 = 375 m^3

$$\rightarrow G_{air} = q_V \rho c_p = \frac{ACH \cdot V}{3600\,\frac{\mathrm{s}}{\mathrm{h}}} \cdot \rho c_p = \frac{0.5\,\frac{1}{\mathrm{h}} \cdot 375\,\mathrm{m}^3}{3600\,\frac{\mathrm{s}}{\mathrm{h}}} \cdot 1.2\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 1000\,\frac{\mathrm{J}}{\mathrm{kgK}} = 62.5\,\frac{\mathrm{W}}{\mathrm{K}}$$

– Roof:

$$U_{roof} = \frac{1}{\frac{1}{\alpha_i} + \frac{s}{\lambda} + \frac{1}{\alpha_o}} = \frac{1}{\frac{1}{34\,\frac{\mathrm{W}}{\mathrm{m^2K}}} + \frac{0.4\,\mathrm{m}}{0.06\,\frac{\mathrm{W}}{\mathrm{mK}}} + \frac{1}{34\,\frac{\mathrm{W}}{\mathrm{m^2K}}}} = 0.15\,\frac{\mathrm{W}}{\mathrm{m^2K}}$$

$$\rightarrow G_{roof} = U_{roof} A_{roof} = 0.15\,\frac{\mathrm{W}}{\mathrm{m^2K}} \cdot (15 \cdot 10)\,\mathrm{m}^2 = 22.4\,\frac{\mathrm{W}}{\mathrm{K}}$$

Solution – III

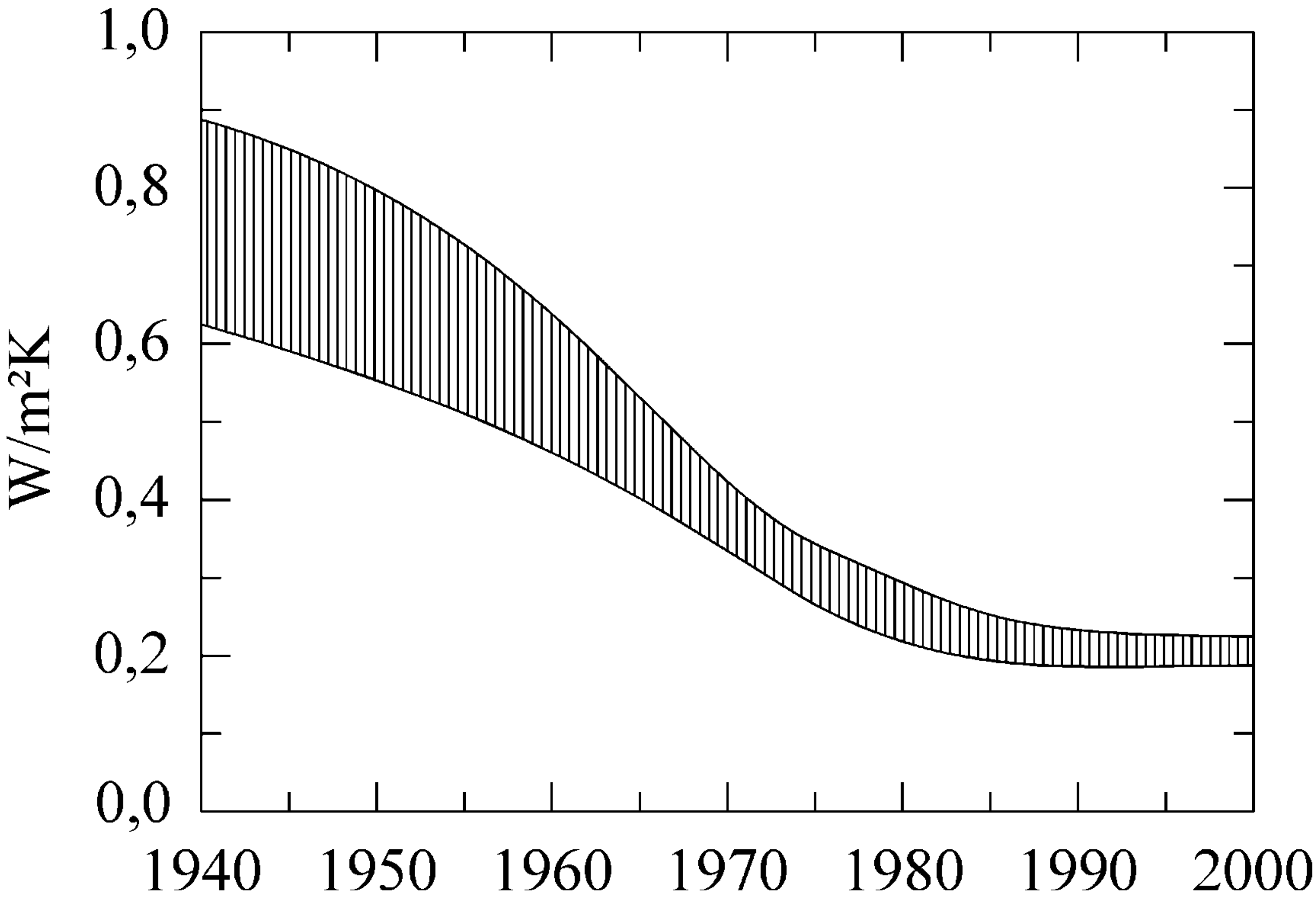
3. Summary of results

Component	A [m²]	s[m]	λ [W/mK]	α [W/m²K]	U [W/m²K]	G [W/K]
Roof	150	0.4	0.06	34	0.15	22.4
Walls (opaque)	93.75	0.25	0.06	34	0.24	22.3
Windows	31.25				1.4	43.8
Air exchange						62.5
G_{tot}						151.0

$$\rightarrow \text{Heating load}: \Phi = G_{tot}\left(T_i - T_o\right) + A_{net} \cdot q_{floor}$$

$$= 151\,\frac{\text{W}}{\text{K}} \cdot \left(21 - \left(-26\right)\right)\text{K} + 150\ \text{m}^2 \cdot 5\,\frac{\text{W}}{\text{m}^2} = \underline{\underline{7.8\ \text{kW}}}$$

Example: Evolution of U-values for outer walls of single-family dwellings



Annual energy need for heating

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Energy need for space heating [(k)Wh] includes the effect of heat transmission through envelope, ventilation and domestic hot water *plus* the impact of heat gains. The heating load [(k)W] is now defined as:

$$\Phi_{sh}(t) = \Phi_{cond} + \Phi_{air} + \Phi_{floor} - \Phi_{gain}$$

Note: Φ is a function of time, since the outdoor temperature varies over time.

Domestic hot water (DHW)

- Energy consumption is determined on the basis of how water demand:
  $$Q_{DHW} = m_{DHW} c_p \Delta T = V_{DHW} \rho c_p \Delta T$$
  where m_{DHW} = mass of DHW, kg
  V_{DHW} = volume of DHW, m^3
- Hot water temperature is at least +55°C (to prevent the growth of bacteria Legionella and burn risk) and cold water temperature is +5°C by default → $\Delta T = 50°C$

Space heating (sh)

- Energy = Power × Time

$$\Leftrightarrow Q_{sh} = \int_{t_1 = 0\,h}^{t_2 = 8760\,h} \Phi_{sh}(t)\,dt$$

where $\Phi_{sh}(t)$ = heating load, (k)W
t = time, h

Annual energy need for heating (tot)

$$Q_{tot} = Q_{sh} + Q_{DHW}$$

Methods for calculating need for heating energy

1. Degree day method

Need for heating energy over a period of time is proportional to the difference between the *effective indoor temperature* (+17 °C) and the external temperature.

2. Standard method

Based on the calculation of energy balance over a rough (e.g. monthly) time step. In Finland: Building Code D5

3. Simulation

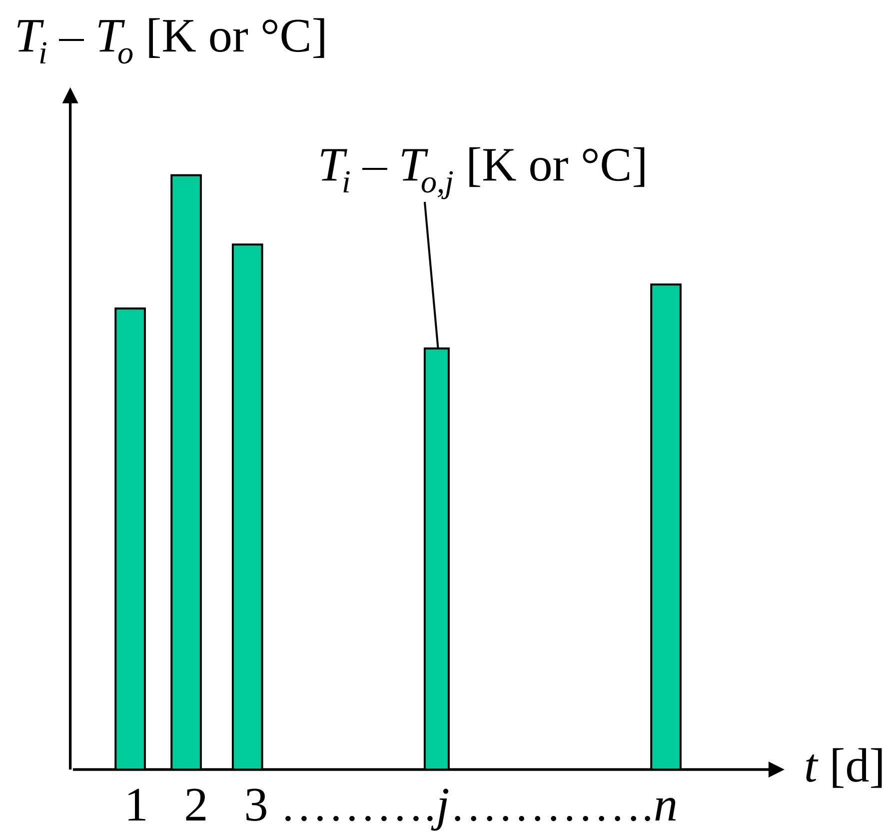
- IDA-ICE
- DOE
- ESP
- Energy Plus

Heating degree days – I

- *Heating degree days* (*HDD*) is a measure to depict the impact of the difference between indoor and outdoor temperatures on the heating energy demand caused by conduction through envelope (Φ_{cond}) and heat loss due to ventilation and air infiltration (Φ_{air}).
- Annual HDD is obtained as the sum of temperature differences ($T_i - T_{o,j}$) over the number of days:

$$HDD = \sum_{j=1}^{n=365} \left(T_i - T_{o,j} \right)$$

where $T_{o,j}$ = average outdoor temperature of the j-th day of the year [°C]



- The energy demand can be defined as:

$$Q_{cond} + Q_{air} = G_{tot} \int_{t_1}^{t_2} \left(T_i - T_o \right) dt$$

$$= G_{tot} \cdot 24 \cdot HDD$$

where G_{tot} = conductance [W/K],
HDD = heating degree days [Kd]
24 = conversion factor [h/d]

Heating degree days – II

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- When setting up the HDD, the *effective indoor temperature* can be chosen. It is usually chosen less than +21°C to include the impact of heat gains. Typically: T_i = +17ºC.
- The HDD is recorded for each year, which enables the comparison of energy demands within a building in different years and locations.
- The HDD for current climate (*normal period*) is defined as the average of HDDs over a period of 30 years.
- The HDDs can be often found from the web pages of Meteorological Institutes.
- A corresponding hourly measure can be defined as *degree hour* (*HDH*) [Kh].

Heating degree days during the normal period 1981-2010

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Year
Maarianhamina	592	567	551	406	216	34	3	17	135	308	432	542	3803
Vantaa	682	640	586	376	146	16	2	21	158	348	497	625	4097
Helsinki	647	612	566	383	153	11	1	12	125	316	464	588	3878
Pori	677	633	585	389	181	26	3	25	171	352	497	622	4161
Turku	663	625	575	377	161	19	2	18	149	338	486	608	4021
Tampere	724	675	612	400	176	28	5	34	192	382	529	667	4424
Lahti	726	677	610	395	159	20	4	31	191	383	528	668	4392
Lappeenranta	759	699	621	403	165	22	5	28	184	386	546	692	4510
Jyväskylä	785	721	646	440	206	40	10	56	227	414	569	718	4832
Vaasa	719	666	619	424	214	29	5	35	192	377	526	663	4469
Kuopio	812	741	653	445	198	31	7	38	194	400	571	735	4825
Joensuu	826	753	665	456	216	39	10	47	215	416	589	752	4984
Kajaani	864	777	695	479	251	57	17	75	245	441	618	785	5304
Oulu	824	742	677	465	249	47	9	55	224	423	593	749	5057
Sodankylä	946	838	760	548	345	106	49	136	316	523	722	891	6180
Ivalo	923	819	755	557	377	146	69	147	318	523	722	875	6231

Reference: Finnish Meteorological Institute (FMI)

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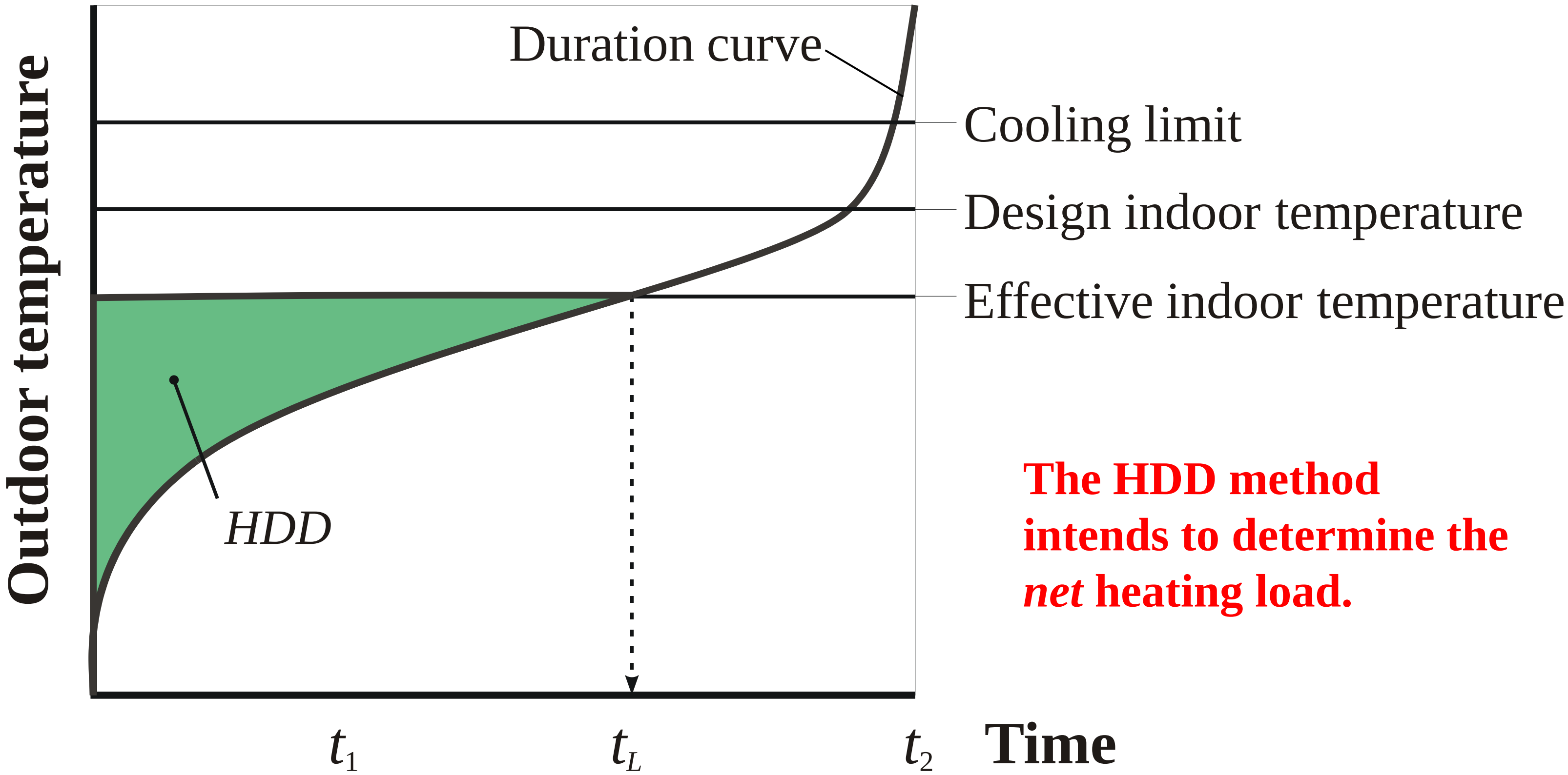
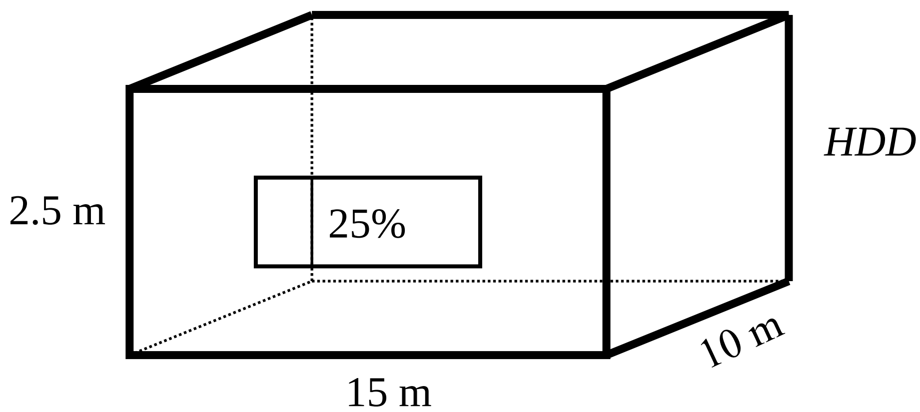
Heating degree days – III



The HDD method intends to determine the *net* heating load.

Length of heating period: t_L

Example



Consider the building of the previous example.

Calculate the annual heating energy demand of the building using the HDD method, given that the following input data/information are known:

- The building is located in Helsinki or Sodankylä.
- The consumption of DHW (55°C) is 100 L/a,gross-m^2.
- The constant heat loss to ground q_{floor} = 5 W/heated-m^2 can be assumed.
- The impact of heat gains is included in the *HDD* (effective T_i < 21°C).
- For water: ρ = 1000 kg/m^3 and c_p = 4.19 kJ/kgK.

Solution – I

1. Energy demand of DHW heating:
   - Gross area: A_{gross} = 15 m × 10 m = 150 m^2
   - Mass of water to be heated:

$$m_{DHW} = A_{gross} q_V \rho = 150\,\mathrm{m}^2 \cdot 100\,\frac{\mathrm{L}}{\mathrm{m}^2,\mathrm{a}} \cdot 0.001\,\frac{\mathrm{m}^3}{\mathrm{L}} \cdot 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} = 15000\,\frac{\mathrm{kg}}{\mathrm{a}}$$

   - Energy demand (assumption: the cold water temperature is 5°C):

$$Q_{DHW} = m_{DHW} c_p \Delta T_{DHW}$$

$$= 15000\,\frac{\mathrm{kg}}{\mathrm{a}} \cdot 4.19\,\frac{\mathrm{kJ}}{\mathrm{kg°C}} \cdot (55-5)\mathrm{°C} \cdot \frac{1}{3600\,\frac{\mathrm{kJ}}{\mathrm{kWh}}} = 873\,\frac{\mathrm{kWh}}{\mathrm{a}}$$

Solution – II

2. Annual heating energy demand:

- HDD by location is acquired from the normal period 1981-2010 data of FMI.
- Energy demand (cond + air):

Note: G_{tot} = constant (from previous example)

$$\text{Helsinki } Q_{cond} + Q_{air} = G_{tot} \cdot 24 \cdot HDD_{Hki} = 151\,\frac{\text{W}}{\text{K}} \cdot 24\,\frac{\text{h}}{\text{d}} \cdot 3878\,\frac{\text{Kd}}{\text{a}} = 14053\,\frac{\text{kWh}}{\text{a}}$$

$$\text{Sodankylä } Q_{cond} + Q_{air} = G_{tot} \cdot 24 \cdot HDD_{Skl} = 151\,\frac{\text{W}}{\text{K}} \cdot 24\,\frac{\text{h}}{\text{d}} \cdot 6180\,\frac{\text{Kd}}{\text{a}} = 22395\,\frac{\text{kWh}}{\text{a}}$$

- Energy demand (floor to ground) for both Helsinki and Sodankylä:

$$Q_{floor} = \Phi_{floor} \Delta t = A_{net} \cdot q_{floor} \cdot \Delta t = 137.8\,\text{m}^2 \cdot 5\,\frac{\text{W}}{\text{m}^2} \cdot 8760\,\frac{\text{h}}{\text{a}} = 6033\,\frac{\text{kWh}}{\text{a}}$$

- Energy demand of space heating:

Helsinki: $Q_{sh} = Q_{cond} + Q_{air} + Q_{floor}$ = (14053 + 6033) kWh/a = 20087 kWh/a

Sodankylä: Q_{sh} = (22395 + 6033) kWh/a = 28429 kWh/a

- Annual heating energy demand:

Helsinki: $Q_{tot} = Q_{sh} + Q_{DHW}$ = (20087 + 873) kWh/a = <u>20960 kWh/a</u>

Sodankylä: Q_{tot} = (28429+ 873) kWh/a = <u>29302 kWh/a</u>

Impact of ventilation heat recovery

- Heat loss due to ventilation and infiltration *without* heat recovery is calculated from:

$$\Phi_{air} = \Phi_{vent} + \Phi_{\inf} = G_{air}\left(T_i - T_o\right)$$
$$= \left(q_{V,vent} + q_{V,\inf}\right)\rho c_p\left(T_i - T_o\right)$$

- <u>*Temperature efficiency*</u> of heat recovery is defined as:

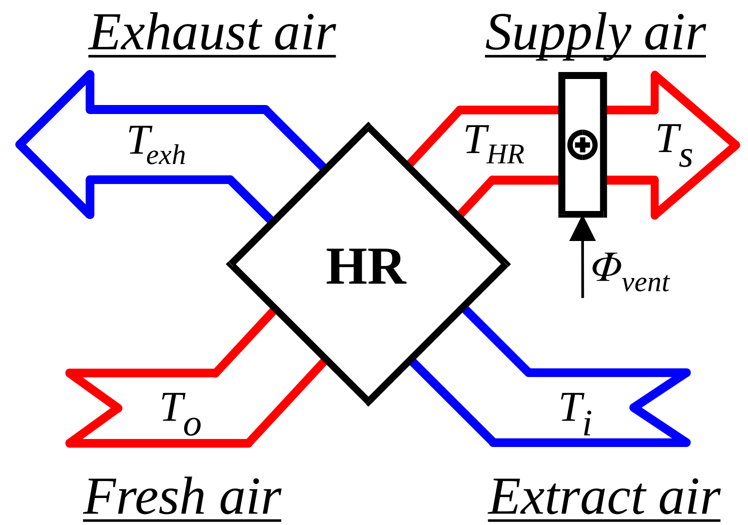
$$\eta_{HR} = \frac{T_{HR} - T_o}{T_i - T_o}$$

- Heat loss due to ventilation *with* heat recovery is calculated from:

$$\Phi_{vent} = (1-\eta_{LTO})\,\Phi_{vent} = (1-\eta_{LTO})\,q_{V,vent}\rho c_p\left(T_i - T_o\right)$$

- Heat loss due to infiltration is calculated from:

$$\Phi_{\inf} = q_{V,\inf}\rho c_p\left(T_i - T_o\right) = V \cdot ACH_{\inf} \cdot \rho c_p\left(T_i - T_o\right)$$



<u>*Infiltration*</u> is uncontrolled ventilation, which depends of the air tightness, heigth and location of the target building plus wind and outdoor temperature.

ACH_{inf} = <u>*infiltration air exchange rate*</u> [1/h]
V = heated volume [m^3]

Suggested infiltration air change rates [1/h]

Number of floors	**Mechanical exhaust only**		**Mechanical ventilation**	
	sheltered	windy	sheltered	windy
< 4	0.1	0.2	0.2	0.3
≥ 4	0.2	0.4	0.3	0.4

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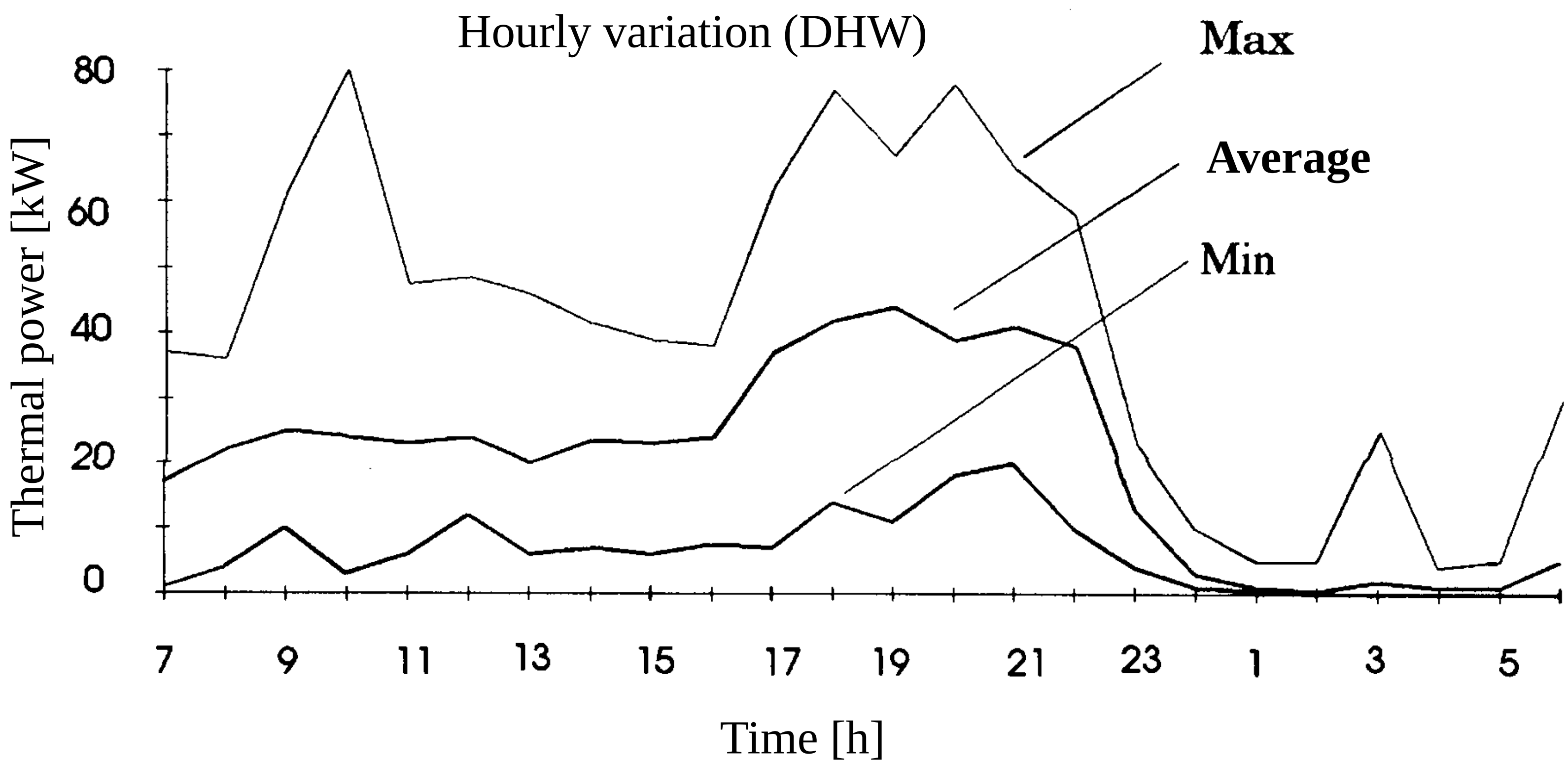
DHW heating requirements

- Share of hot water of the total water consumption:
  - residential buildings: 40%
  - other buildings: 30%
- Selected default values for DHW consumption by the Finnish Energy Agency Motiva [L/gross-m^2,a]:
  - residential buildings: 600
  - office buildings: 100
  - hospitals: 520
  - schools: 120
- DHW represents 20…30 % of the residential buildings' energy demand.
- Distribution by application:
  - shower: 40…60 %
  - kitchen: 20…30 %
  - toilet: 20…35 %

Example: daily variation of DHW heating demand



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DHW heating design conditions

Challenges:

- DHW demand fluctuates vehemently
- Top demands are of short duration
- DHW storage tank is used for peak shaving

General principle (temperatures commonly applied in Finland):

$$\Phi_{DHW} = \rho_w c_{pw} q_{V,DHW,des} \left(T_{DHW} - T_{CW}\right)$$

$$T_{DHW} = 55\ °\mathrm{C}$$

$$T_{CW} = 5\ldots10\ °\mathrm{C}$$

$$q_{V,DHW,des} = \text{design flow rate}$$

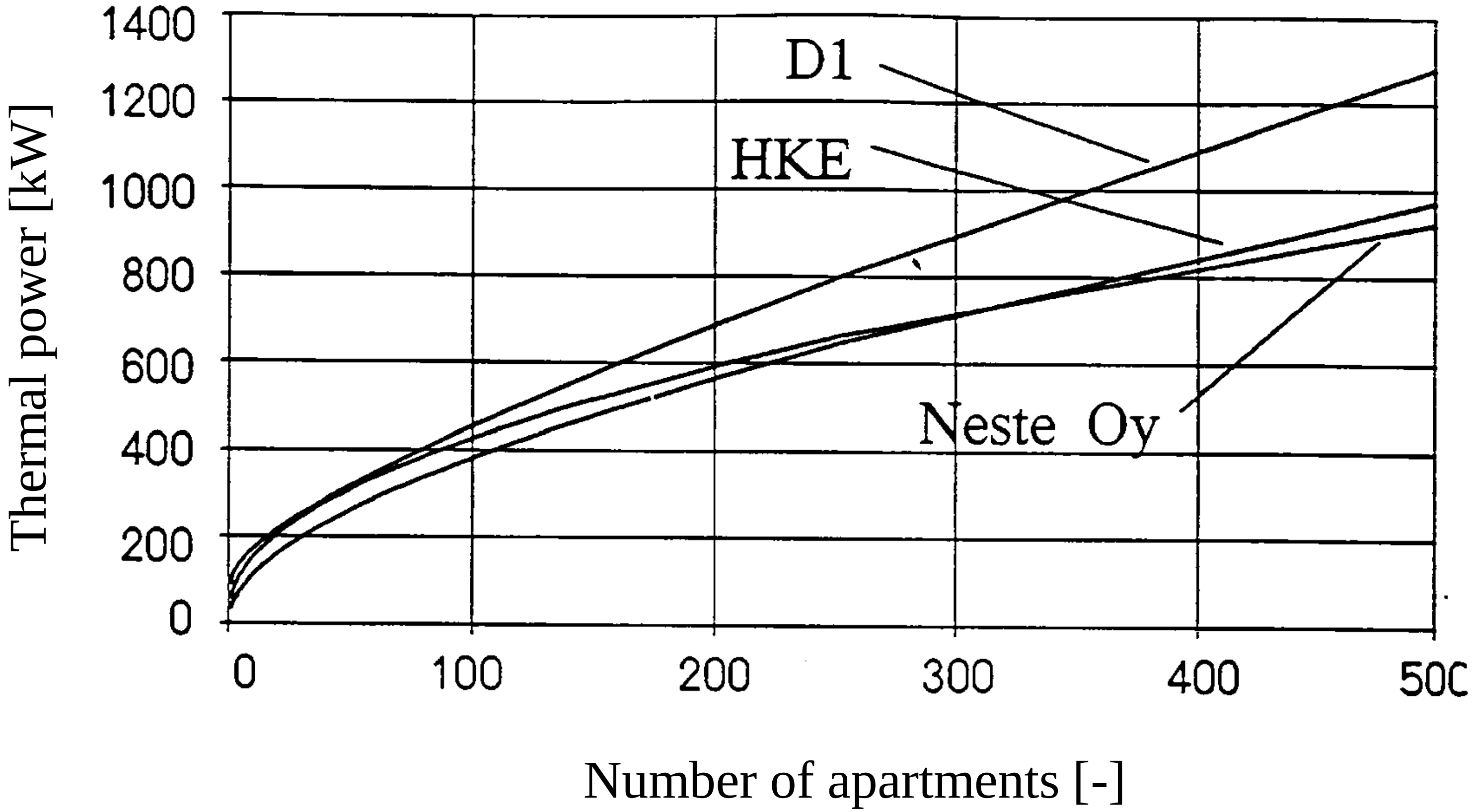
In typical detached houses the district heat exchanger for DHW is designed to provide 57 kW, corresponding to the design flow rate $q_{V,DHW,des}$ = 0.27 L/s. The size of the DHW storage tank is commonly 150...300 L (50 L/occupant).

Example: Design guide (HKE):

$$\Phi_{DHW} = 29 + 20 \cdot \sqrt{4 \cdot N - 2}\,; \qquad \left[\Phi_{DHW}\right] = \mathrm{kW}$$

where N = number of apartments

Example: DHW design thermal power according to selected Finnish design guides



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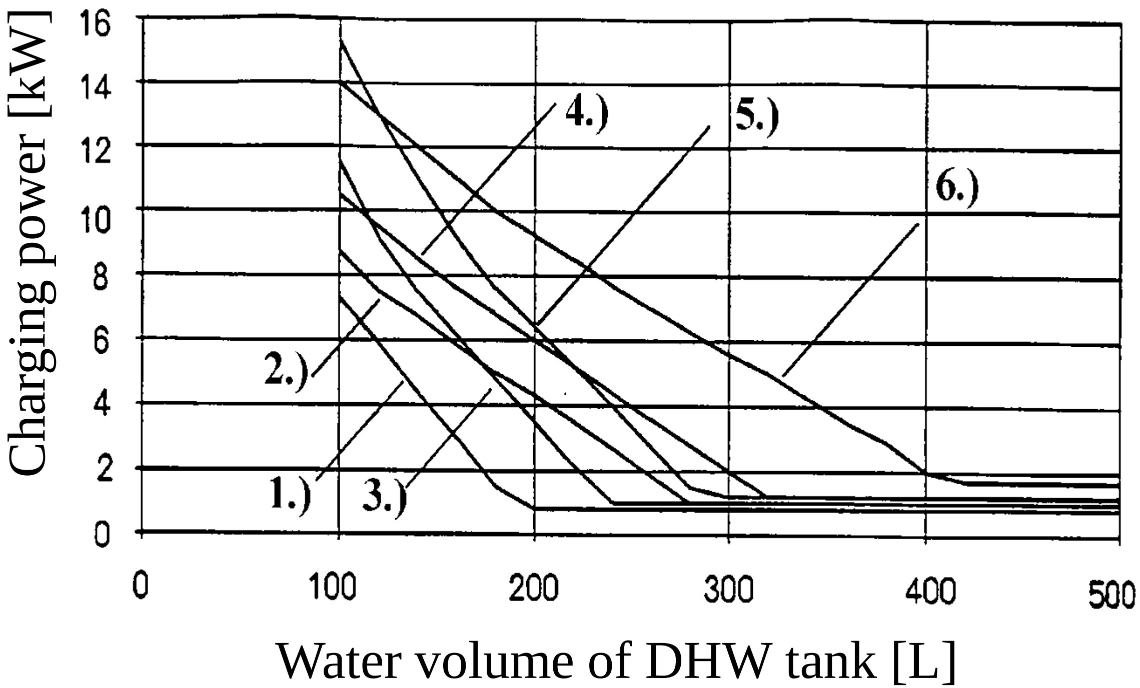
Example: Design guide for DHW storage tank of electrically heated detached house.

<u>Scenarios:</u>

1.) 3 occupants
2.) 3 occupants + sauna
3.) 4 occupants
4.) 4 occupants + sauna
5.) 5 occupants
6.) 5 occupants + sauna



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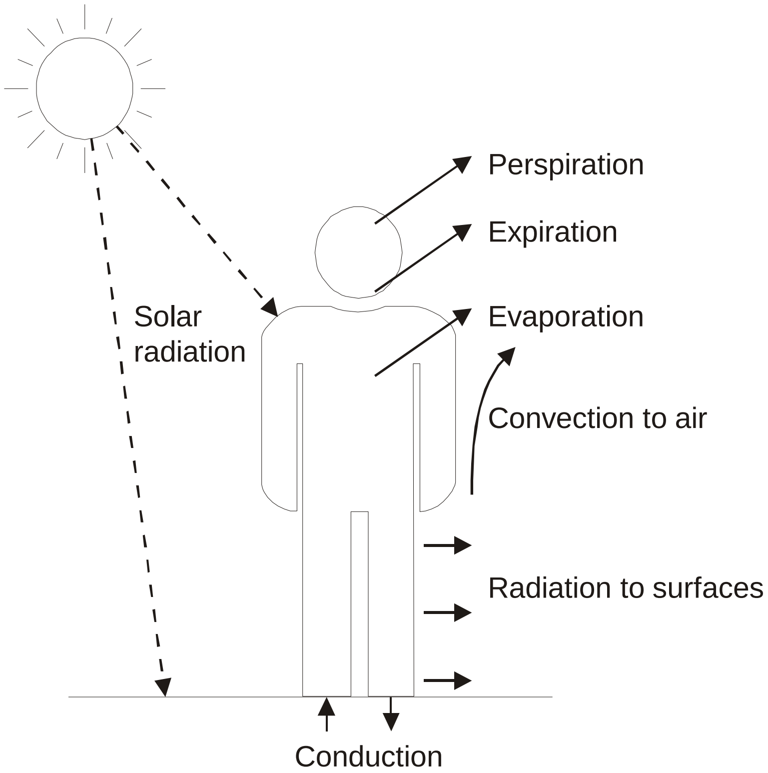
Heat load from occupants

- Based on the heat balance of a human body
  - sensible heat load: convection and radiation from the surface of clothing (skin)
  - latent heat load: evaporation from skin, expiration (exhalation)
- Depends on the level of clothing and physical activity
- Indicated by *metabolic equivalent* (*MET*)
- When MET = 1, one square meter of body surface releases 58.15 watts of heat (total heat load).
- Body surface area is commonly between 1.5…2.0 m^2.



MET-values for various activities

- Light intensity activities (MET < 3):
  - sleeping: MET = 0.9
  - sitting: MET = 1
- Moderate intensity activities (3 < MET < 6):
  - walking (4-5 km/h): MET ~ 3
  - bicycling: MET ~ 5
- Vigorous intensity activities (MET > 6):
  - jogging: MET ~ 7
  - heavy workout > 8

Example

Estimate the heat load released by a fitness cyclist.

<u>Solution:</u>

- Assumption: skin area 1.8 m^2
- Bicycling: $MET = 5$
- Released heat load:

$$\Phi_{tot,occ} = MET \cdot A_{skin} \cdot 58.15\,\frac{\text{W}}{\text{m}^2}$$

$$= 5 \cdot 1.8\,\text{m}^2 \cdot 58.15\,\frac{\text{W}}{\text{m}^2} = \underline{\underline{523\,\text{W}}}$$

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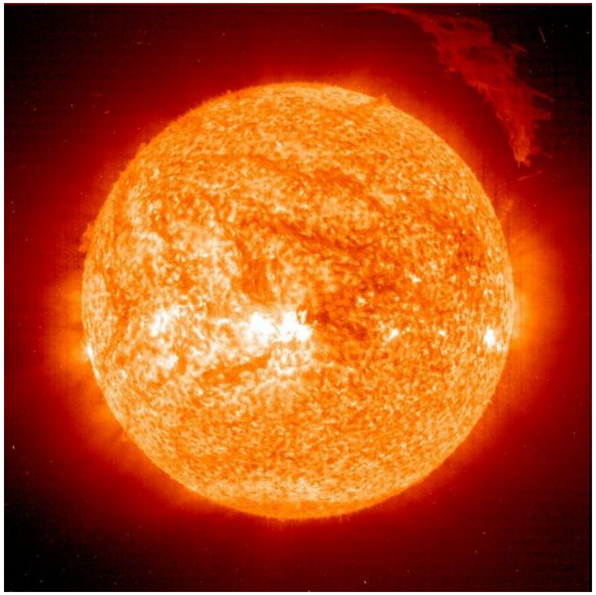
Solar radiation – definitions

- *Irradiance I* – amount of solar radiation received by given area and time, intensity of solar radiation [W/m²]
- *Total irradiance* I_{TOT} – sum of direct (I_D), reflected (I_R) and diffuse (I_d) irradiances:

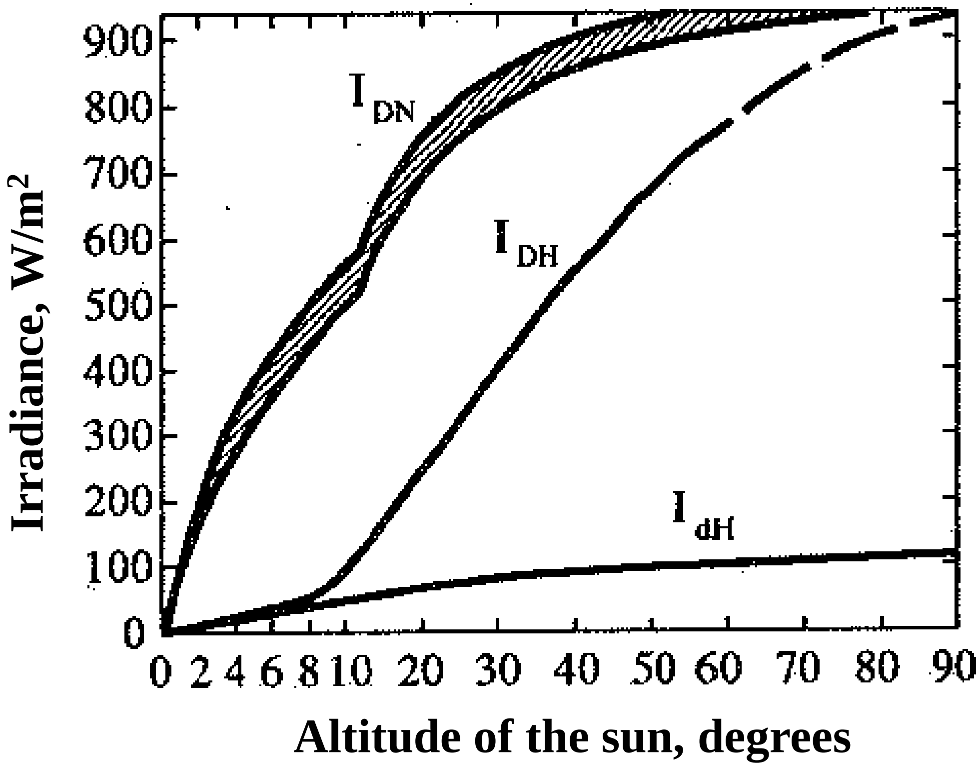
$$I_{TOT} = I_D + I_d + I_R$$

- Irradiance depends on:
  - position of surface against solar radiation
  - external shadings



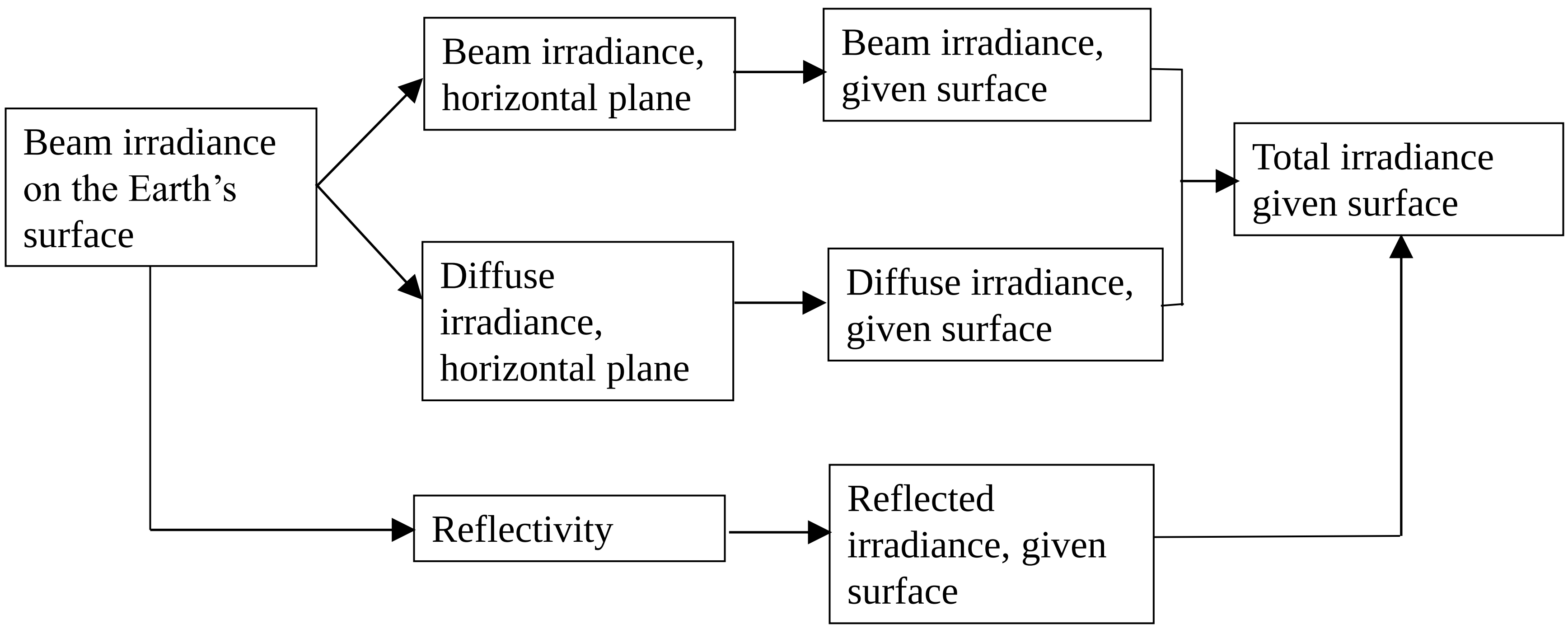
Solar radiation to horizontal plane is recorded in weather data.

Irradiance as a function of altitude of the sun



Source: Seppänen: Heating of buildings (in Finnish)

Calculating irradiances



Direct (beam) radiation

- An estimate of beam irradiance on the earth's surface (on **surface oriented *perpendicular* to the sun's rays**) I_{DN} can be calculated from

$$I_{DN} = I_0 \tau^{\frac{1}{\sin(h)}}$$

where I_0 = solar constant (~1353 W/m^2)

h = altitude of the sun

τ = transmittance of the atmosphere;
= 0.62 (cloudy sky)…0.74 (clear cky)

- Beam irradiance on **horizontal plane** I_{DH}

$$I_{DH} = I_{DN} \sin(h)$$

- Beam irradiance on **given surface** I_D

$$I_D = I_{DN} \cos(i)$$

where i = angle of incidence

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Diffuse radiation – I

- Diffuse irradiance on **horizontal plane** I_{dH}

$$I_{dH} = C \cdot I_{DN} = C \cdot I_0 \tau^{\frac{1}{\sin(h)}}$$

where C = monthly fraction of diffuse radiation [-]

- Diffuse irradiance on **given surface** I_d

$$I_d = I_{dH} \cdot F_{pt}$$

where F_{pt} = view factor of the sky observed from surface p [-]

Diffuse radiation – II

Monthly fractions of diffuse radiation:

January	0.058	August	0.122
February	0.060	September	0.092
March	0.071	October	0.073
April	0.097	November	0.063
May	0.121	December	0.057
June	0.134		
July	0.136		

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Diffuse radiation – III

View factor of the sky F_{pt} is calculated as follows:

$$F_{pt} = \frac{1+\cos(\gamma)}{2}; \quad \text{diffuse radiation distributed evenly}$$

$$F_{pt} = \frac{1}{2}; \text{ special case } \gamma = 90° \text{ (vertical surface) } \leftrightarrow \cos(\gamma) = 0$$

View factor for vertical surface, radiation distributed unevenly:

$$F_{pt} = 0.55 + 0.437\cos(i) + 0.313\cos^2(i); \quad \cos(i) > -0.2$$

$$F_{pt} = 0.45; \quad \cos(i) < -0.2$$

Reflected radiation – I

Reflected irradiance on **given surface** I_R

$$I_R = (I_{DH} + I_{dH}) F_{pm} r$$

where r = reflectivity (reflection factor) of (reflecting) surface [-]

F_{pm} = view factor between given and reflecting surfaces [-]

View factor between given surface and reflecting surroundings F_{pm} is defined as

$$F_{pm} = \frac{1}{2}\left(1 - \cos(\gamma)\right)$$

Reflected radiation – II

Values of reflectivity for various surfaces:

• Asphalt	0.07
• Concrete	0.2 (soiled)…0.45 (clean)
• Water	0.1…0.5
• Dry ground	0.1…0.2
• Snow (average)	0.7
• Soil (black)	0.05
• Coloured surfaces	0.1 (black)…0.75 (white)

Angles to be calculated

1. Angle of declination
2. Altitude of the sun
3. Angle of azimuth
4. Angle of incidence

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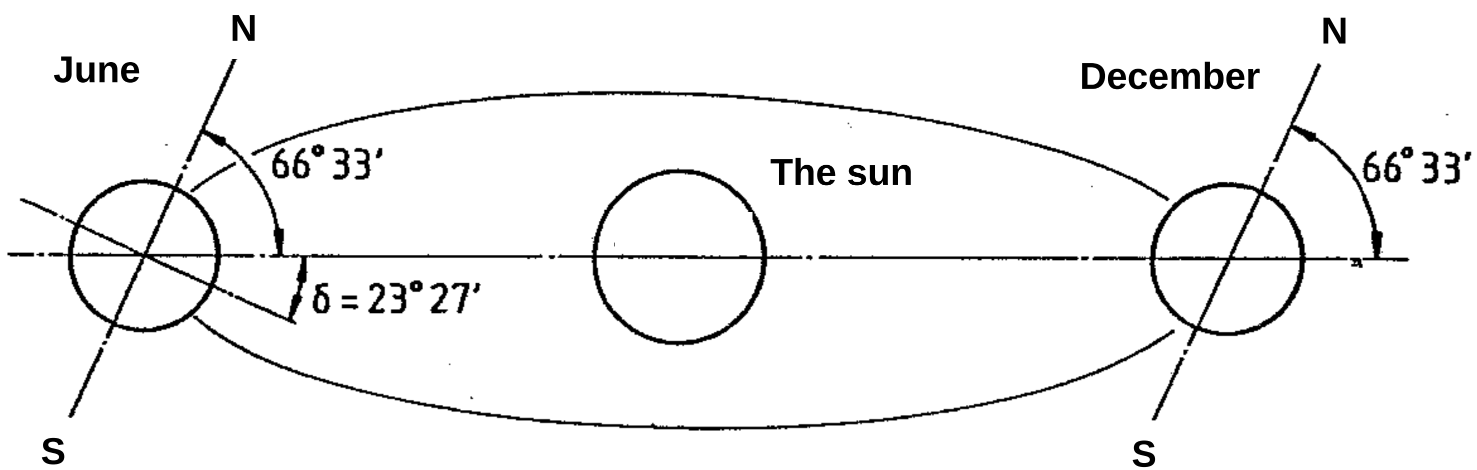
Angle of declination – I

Angle of declination δ is determined by how many degrees **the earth's axis is tilted to the axis of the plane** in which it orbits the sun.



Angle of declination – II

Angle of declination δ is calculated from

$$\delta = 23°27'\sin\left(360\cdot\frac{284+n}{365}\right)$$

Minutes: as a decimal number 27’= 27/60 = 0.45

where n = the number of days from January 1

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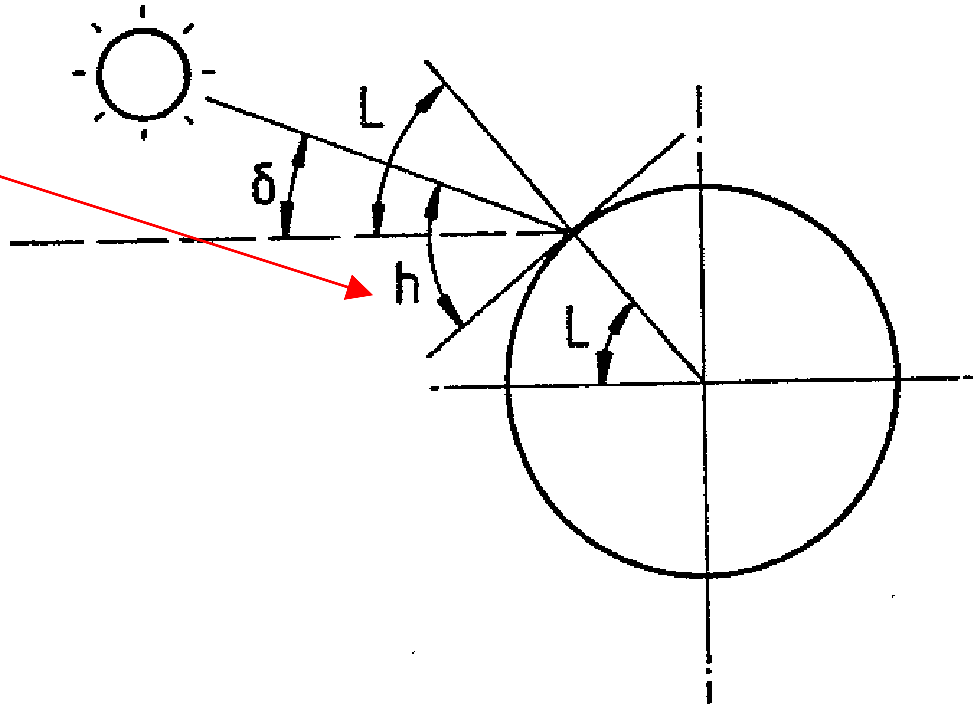
Altitude of the sun – I

<u>Altitude of the sun</u> h is **the angle of inlet of solar radiation** in relation to the earth's surface at the latitude *L*.



Altitude of the sun – II

Altitude of the sun h at the latitude L declination being δ is calculated from

$$\sin(h) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(\theta)$$

where *hour angle* θ is degree of apparent **deviation of the sun from the direction of south** (noon) at the given time so that an hour corresponds to 15 degrees.

Altitude of the sun – III

- *Apparent solar time* t_{au}
  - is required for calculating hour angle
  - deviates from *coordinated universal time* (UTC), which is the official time.
- Apparent solar time [h] is calculated from

$$t_{au} = t_{klo} + \frac{4\left(P_{klo} - P_{tod}\right)}{60} + \frac{E}{60}$$

$$\text{where} \begin{cases} E = 9.87\sin 2B - 7.53\cos B - 1.5\sin B \quad (= \text{time deviation [min]}) \\ B = \frac{360}{364}\left(n - 81\right) \\ P_{klo} = \text{longitude of time zone (Finland: 30°)} \\ P_{tod} = \text{longitude of location of interest} \\ n = \text{number of days from January 1} \end{cases}$$

Daylight saving time: $t_{\text{au,daylight saving time}} = t_{\text{au}} - 1\ \text{h}$

Example

Calculate the altitude of the sun.

- Time: January 1, at 13:00
- Location: 60° N, 29° E

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Solution – I

1. Angle of declination

$$\delta = 23.45° \cdot \sin\left(360 \cdot \frac{284+1}{365}\right) = 23.45° \cdot \sin(281°) = -23.01°$$

2. Apparent solar time

$$B = \frac{360}{364}(n-81) = \frac{360}{364}(1-81) = -79.12°$$

$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$$

$$= 9.87 \sin(2 \cdot (-79.12°)) - 7.53 \cos(-79.12°) - 1.5 \sin(-79.12°)$$

$$= -3.6 \text{ min}$$

$$t_{au} = t_{klo} + \frac{4(P_{klo} - P_{tod})}{60} + \frac{E}{60}$$

$$= 13 \text{ h} + \frac{4 \cdot (-30° - (-29°))}{60} + \frac{-3.6 \text{ min}}{60} = 12.87 \text{ h}$$

Solution – II

3. Hour angle
   - Deviation from the south (noon): (12.87 – 12) h = (+)0.87 h
   - As degrees: 0.87 · 15° = (+)13.1°

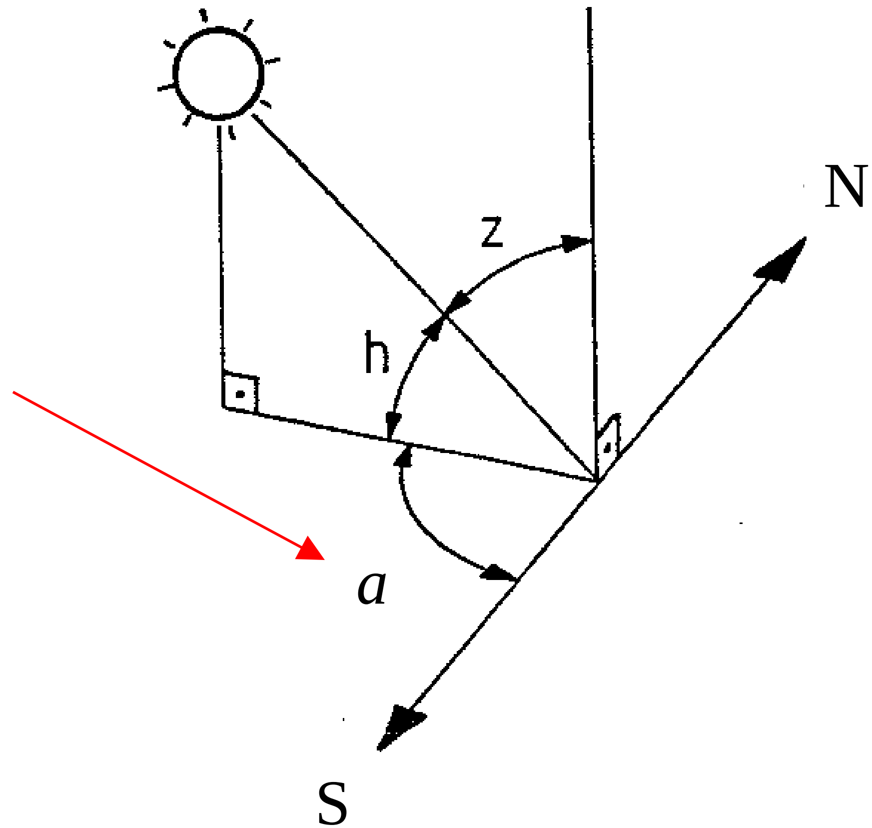
4. Altitude of the sun

$$\sin(h) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(\theta)$$
$$= \sin(60°)\sin(-23.01°) + \cos(60°)\cos(-23.01°)\cos(13.1°)$$
$$= 0.108$$
$$\rightarrow h = \arcsin(0.108) = \underline{\underline{6.21°}}$$

Angle of azimuth

- *Angle of azimuth* *a* alias *solar azimuth (azimuth of the sun)* is the angle within the horizontal plane measured clockwise from the south (in some cases: north).
- Angle of azimuth is defined as

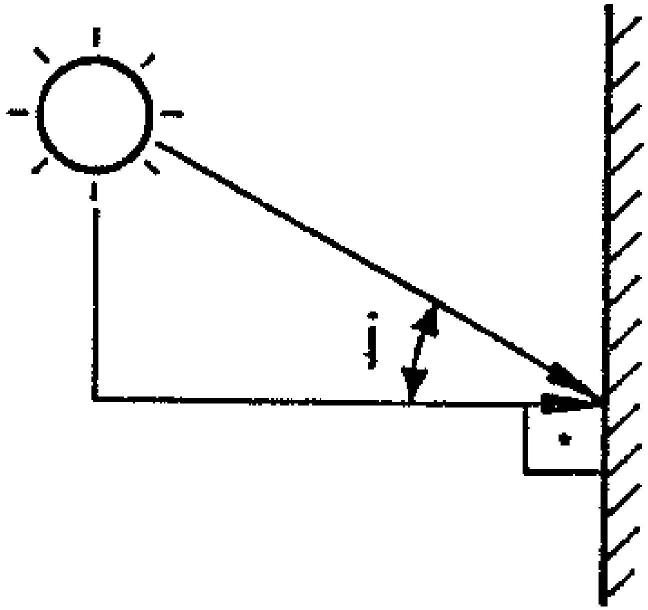
$$\cos(a) = \frac{\sin(h)\sin(L) - \sin(\delta)}{\cos(h)\cos(L)}$$



Angle of incidence – I

<u>Angle of incidence</u> i

– represents the angle the radiation hits a given surface (perpendicular component of irradiance on the surface)
– is required to calculate the heat transfer through surface



Angle of incidence – II

Angle of incidence is calculated from

$$\cos(i) = \cos(h)\cos(a \pm \alpha)\sin(\gamma) + \sin(h)\cos(\gamma)$$

before noon (+) ja afternoon (−)

where γ = pitch angle of surface to horizontal plane

α = orientation angle of surface from due south clockwise (+) or counter-clockwise (−)

Example

For the previous time and location data, calculate the angle of incidence for a vertical wall facing south-east.

Solution – I

1. Applying definition for specific case (tilt angle γ = 90º)
   – γ = 90º → cos(γ) = 0, sin(γ) = 1

$$\rightarrow \cos(i) = \cos(h)\cos(a \pm \alpha)$$

2. Angle of azimuth

$$\cos(a) = \frac{\sin(h)\sin(L) - \sin(\delta)}{\cos(h)\cos(L)}$$

$$= \frac{\sin(6.21^\circ)\sin(60^\circ) - \sin(-23.01^\circ)}{\cos(6.21^\circ)\cos(60^\circ)} = 0.97$$

$$\rightarrow a = \arccos(0.97) = 12.87^\circ$$

Solution – II

3. Orientation of surface
   - South-east → $\alpha = -(360^\circ/8) = -45^\circ$

4. Angle of incidence
   - Afternoon (at 13:00)

$$\cos(i) = \cos(h)\cos(a - \alpha)$$

$$= \cos(6.21^\circ) \cdot \cos\left(12.87^\circ - \left(-45^\circ\right)\right) = 0.53$$

$$\rightarrow i = \arccos\left(0.53\right) = \underline{\underline{58.1^\circ}}$$

External shadings – I

- Purpose – to protect windows from solar radiation
  - recesses (niches)
  - juts (lintels)
- Impact on irradiances
  - The shaded area only exposes to diffuse and reflected radiation.

    → The geometry of shadows must be known.

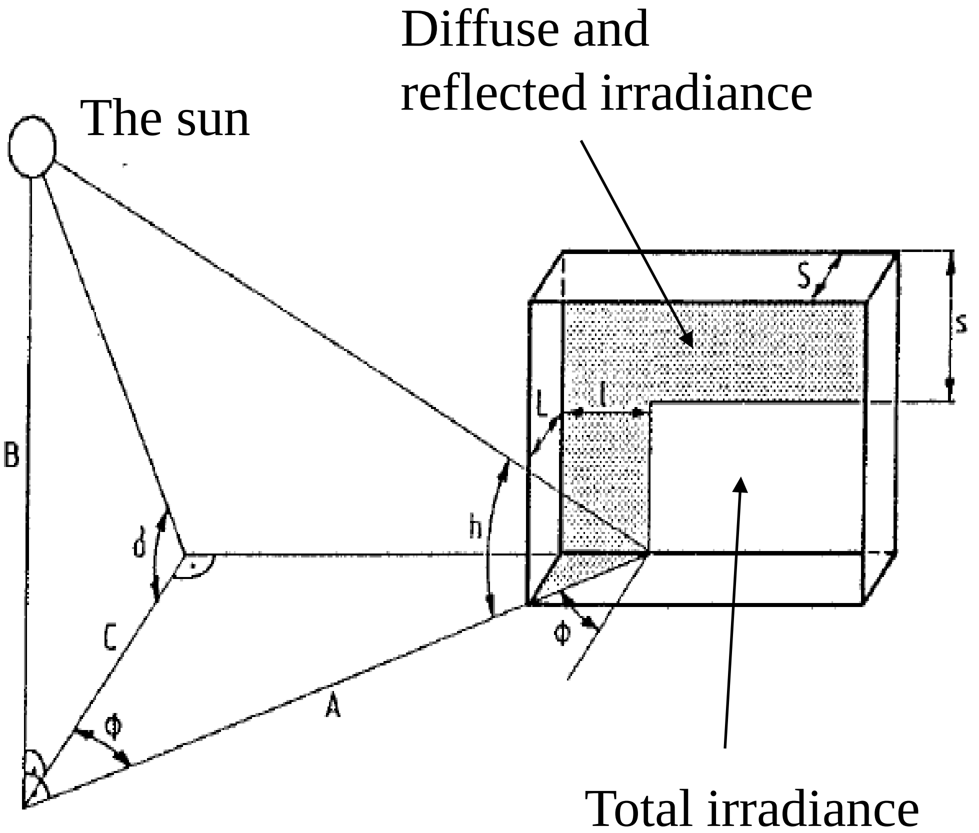
External shadings – II

Geometry of shadows is calculated from:

$$l = L\tan(\phi) = L\tan(a \pm \alpha)$$

$$s = S\tan(\sigma) = S\frac{\tan(h)}{\cos(a \pm \alpha)}$$

where α = angle of orientation



Example

Calculate the sunlit proportion of a 2 m × 2 m window which is installed in the vertical wall described in previous examples and recessed by 0.1 m.

Solution

1. Geometry of shadows

$$l = L\tan(a-\alpha) = 0.1\text{ m}\cdot\tan\left(12.87^\circ-(-45^\circ)\right) = 0.16\text{ m}$$

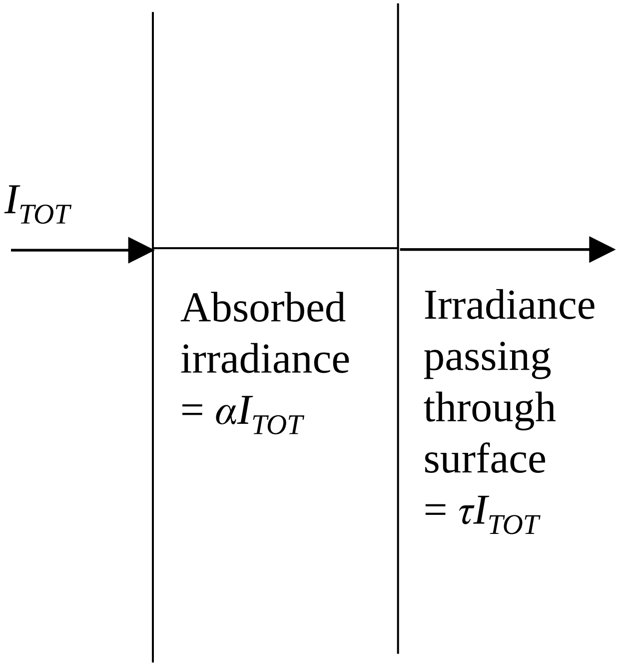
$$s = S\,\frac{\tan(h)}{\cos(a-\alpha)} = 0.1\text{ m}\cdot\frac{\tan(6.21^\circ)}{\cos\left(12.87^\circ-(-45^\circ)\right)} = 0.02\text{ m}$$

2. Sunlit proportion (= proportion exposed to direct radiation)

$$\frac{A_{sol}}{A_{tot}} = \frac{(2-0.16)\,\text{m}\times(2-0.02)\,\text{m}}{2\text{ m}\times 2\text{ m}} = 0.91 = \underline{\underline{91\%}}$$

Radiation through surface

- Some fraction of radiation is
  - reflected to ambient
  - absorbed by surface
  - passed through by surface
- *Absorption factor* α expresses the fraction of radiation absorbed into surface.
- *Transmittance* τ expresses the fraction of radiation passing through surface.



Calculating heat load through surface

Heat flow through a surface (wall, window etc.) is calculated from

$$\Phi = \tau_D I_D A_{sol} + \tau_d I_d A_{tot} + \tau_R I_R A_{tot}$$

where τ_D, τ_d and τ_R are transmittances for direct, diffuse and reflected radiation [-]
A_{sol} and A_{tot} are sunlit and total area of surface [m^2]

Transmittance depends on wavelength, angle of incidence and window type. Reflected radiation can be assumed diffuse as behaviour, when more accurate data are not available.

Example

Calculate the solar heat load through the window making use of the data from the previous examples, when the transmittance (both direct and diffuse radiation) is 0.6, the weather is clear and the ground is covered by an average blanket of snow.

Solution – I

1. Beam irradiance
   - From graph: in January I_0 = 1390 W/m^2
   - Clear sky: τ = 0.74

$$I_{DN} = I_0 \tau^{\frac{1}{\sin(h)}} = 1390\,\frac{\mathrm{W}}{\mathrm{m}^2} \cdot 0.74^{\frac{1}{\sin(6.21^\circ)}} = 85.9\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

$$I_{DH} = I_{DN}\sin(h) = 85.9\,\frac{\mathrm{W}}{\mathrm{m}^2} \cdot \sin(6.21^\circ) = 9.3\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

$$\rightarrow I_D = I_{DN}\cos(i) = 85.9\,\frac{\mathrm{W}}{\mathrm{m}^2} \cdot \cos(58.1^\circ) = 45.4\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

Solution – II

2. Diffuse irradiance
   - Diffuse radiation – II (table): in January $C = 0.058$
   - For a vertical wall: $F_{pt} = 0.5$

$$I_{dH} = C \cdot I_{DN} = 0.058 \cdot 85.9 \frac{\mathrm{W}}{\mathrm{m}^2} = 5.0 \frac{\mathrm{W}}{\mathrm{m}^2}$$

$$\rightarrow I_d = I_{dH} F_{pt} = 5.0 \frac{\mathrm{W}}{\mathrm{m}^2} \cdot 0.5 = 2.5 \frac{\mathrm{W}}{\mathrm{m}^2}$$

Solution – III

3. Reflected irradiance
   - Reflected radiation – II (table): snow (average) r = 0.7
   - For a vertical wall : $F_{pm} = 0.5$

$$I_R = \left(I_{DH} + I_{dH}\right) F_{pm} r$$

$$= \left(9.3 + 5.0\right) \frac{\mathrm{W}}{\mathrm{m}^2} \cdot 0.5 \cdot 0.7$$

$$= 5.0 \frac{\mathrm{W}}{\mathrm{m}^2}$$

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Solution – IV

4. Total irradiance

$$I_{TOT} = I_D + I_d + I_R = (45.4 + 2.5 + 5.0)\frac{\mathrm{W}}{\mathrm{m}^2} = 52.9\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

5. Heat load through window
   – Given data (and assumption): $\tau_D = \tau_d\ (= \tau_R) = 0.6$

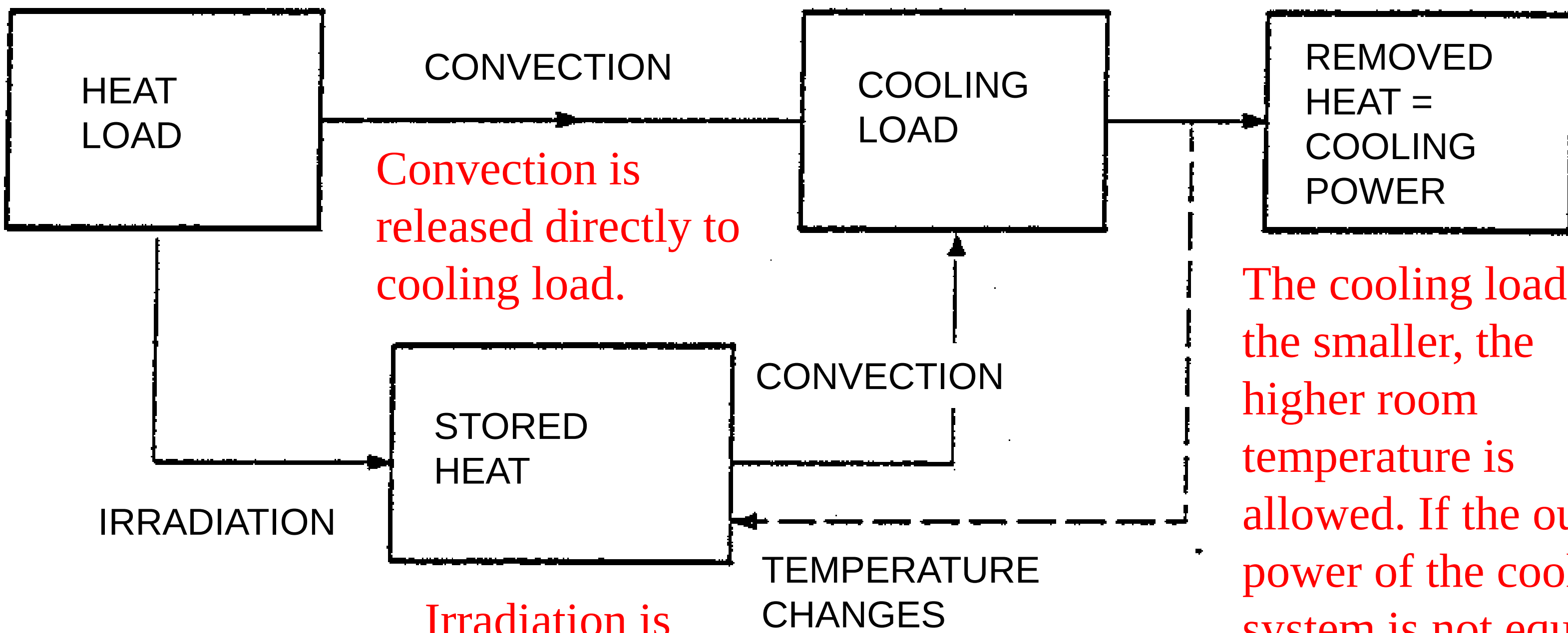
$$\Phi = \tau\left(A_{sol} I_{TOT} + \left(A_{tot} - A_{sol}\right)\left(I_d + I_R\right)\right)$$

$$= 0.6\cdot\left(\left(0.91\cdot 4\,\mathrm{m}^2\right)\cdot 52.9\,\frac{\mathrm{W}}{\mathrm{m}^2} + \left(1 - 0.91\right)\cdot 4\,\mathrm{m}^2\cdot\left(2.5 + 5.0\right)\frac{\mathrm{W}}{\mathrm{m}^2}\right)$$

$$= \underline{\underline{117\ \mathrm{W}}}$$

From heat load to cooling load

Heat load ≠ Cooling load



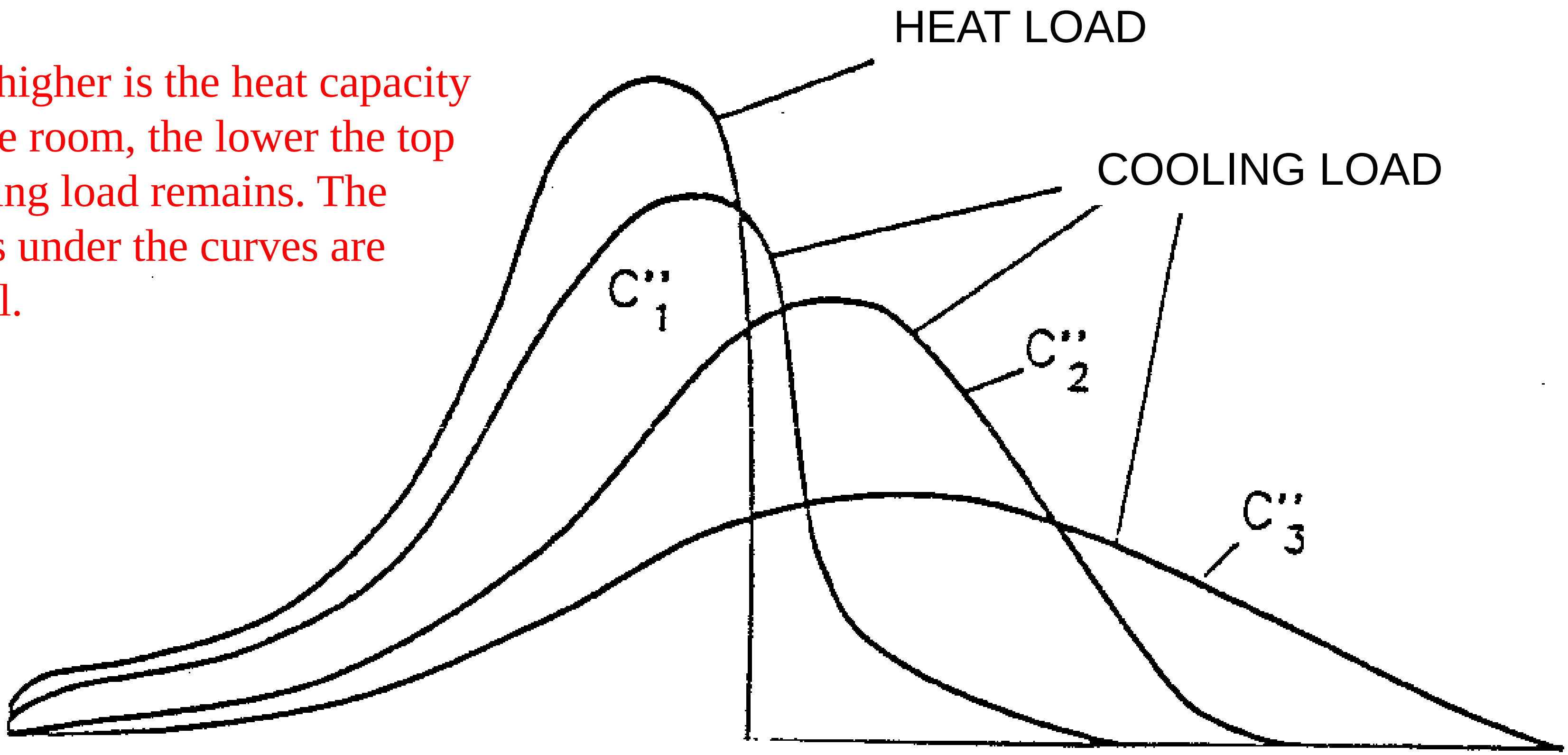
Convection is released directly to cooling load.

Irradiation is stored into structures.

The cooling load is the smaller, the higher room temperature is allowed. If the output power of the cooling system is not equal to cooling load, the room temperature changes.

Heat load vs. cooling load

The higher is the heat capacity of the room, the lower the top cooling load remains. The areas under the curves are equal.



Calculating cooling load

- Characteristics:
  - significant changes
  - periodicity: alternating heating and cooling loads
  - internal loads more significant than external loads (solar)
- Consequences:
  - Cooling is **transient in nature**. (Heating load: steady-state)
  - Cooling load calculation is more complicated than heating load calculation.

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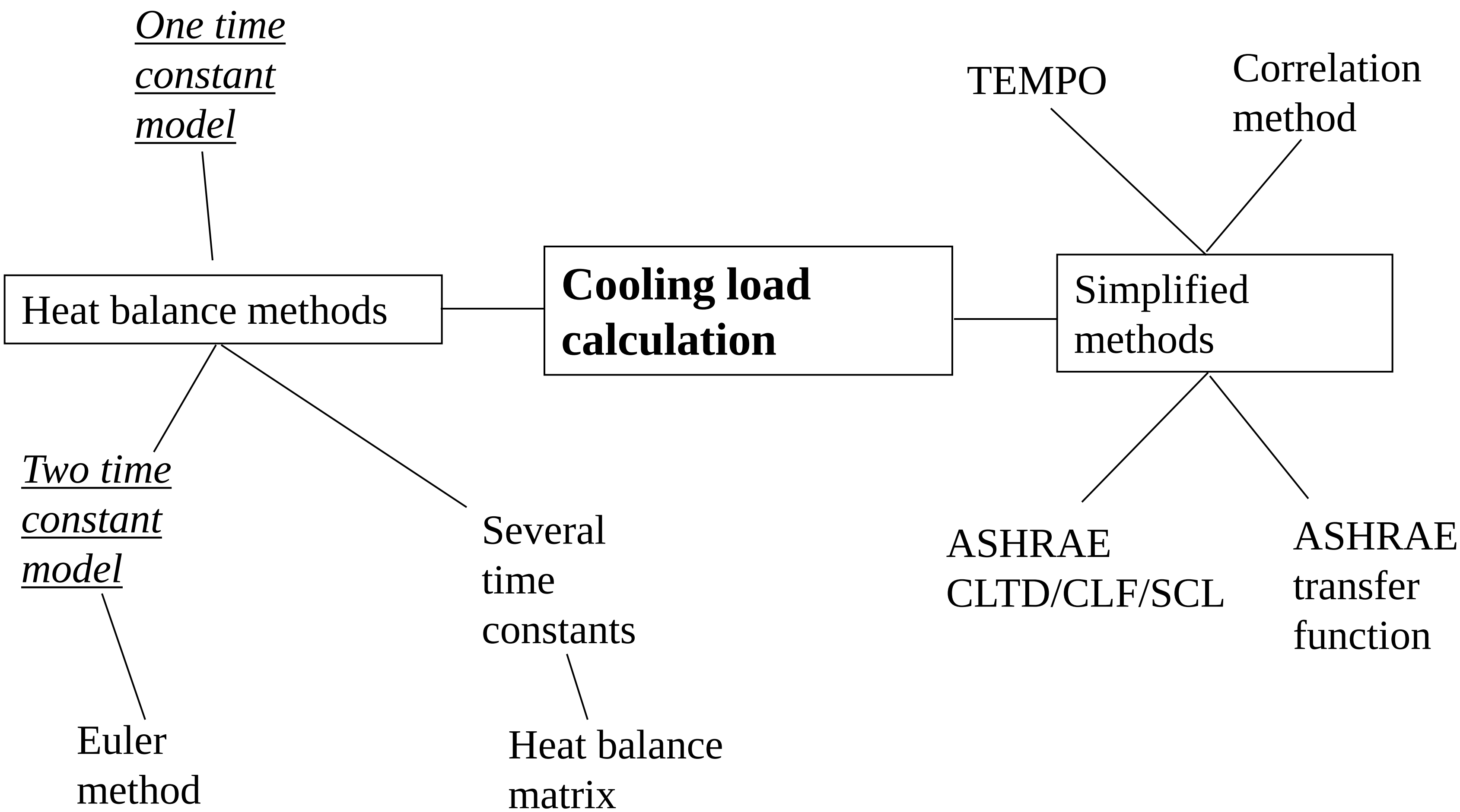
Calculation methods



Heat balance methods

- Purpose:
  - to find out the thermal performance of a room for the cooling load calculation
- Method:
  - Separate heat balance for each surface and the air control volume of the room is constructed.
  - Set of balance equations is solved.

- Assumptions:
  - evenly distributed irradiation
  - one-dimensional surfaces
  - all surfaces at uniform temperature
  - each wall represents a surface
  - surrounding rooms at uniform temperature
  - solid structures (walls, furniture) at uniform temperature

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Heat balance – air control volume

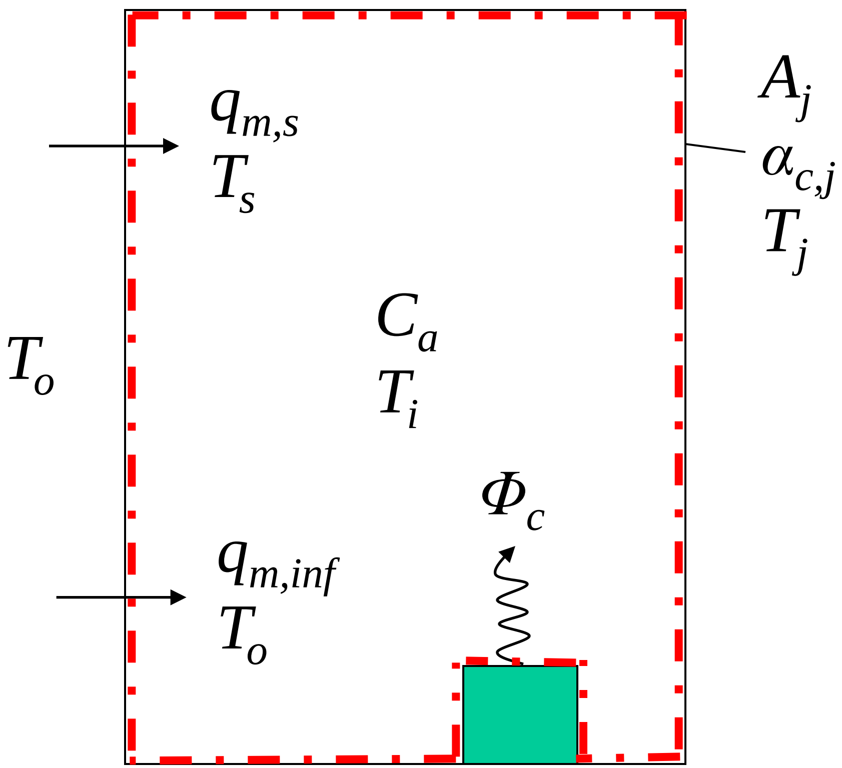
$$C_a \frac{dT_i}{dt} = \Phi_c + q_{m,\inf} c_{pa}(T_i - T_o) - q_{m,s} c_{pa}(T_i - T_s) - \sum_j \alpha_{c,j} A_j (T_i - T_j)$$

Storage; Cooling power of air leak; Cooling power of ventilation; Convective heat flow to surfaces

Note: the direction of heat flows.

where

A_j	area of j-th surface [m^2]
c_{pa}	specific heat capacity of air [J/kgK]
C_a	heat capacity of air control volume [J/K]
$q_{m,s}$	supply air flow [kg/s]
$q_{m,inf}$	infiltration air flow [kg/s]
T_j	temperature of j-th surface [ºC]
T_i	indoor temperature [ºC]
T_o	outdoor temperature [ºC]
$\alpha_{c,j}$	convective heat transfer coefficient of the j-th surface [W/m^{2}K]
Φ_c	convective heat flow from the internal heat gains/loads [W]



Note: the heat transfer is convective only.

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Models for solving the heat balance

1. *<u>One time-constant (one heat capacity) model</u>*
   - **one homogeneous control volume (surfaces + air)** at uniform temperature
   - one balance equation
2. *<u>Two time-constant (heat capacity) model</u>*
   - **separate control volumes** for surfaces and air
   - two balance equations

The above models are applicable, when the temperature distribution is not required. More accurate calculation requires *<u>several time-constant model</u>*, where the number of elements (heat capacities) is not constrained.

One time-constant model – I

Heat balance is expressed as:

$$C_{tot}\frac{dT_i}{dt} = \Phi - G_{tot}\left(T_i - T_o\right)$$

where Φ (net) heat flow to the room [W]

C_{tot} combined heat capacity (surfaces + air) [J/K]

G_{tot} specific thermal loss (conductance) for ventilation and heat loss through envelope [W/K]

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One time-constant model – II

- Air control volume:

$$C_a = m_a c_{pa} = V_a \rho_a c_{pa}$$

where V_a = air volume of the room [m^3]

- *Active heat capacity* of the surface (wall):

$$C_w = \sum_i (mc)_i = \sum_i (V\rho c)_i = \delta \sum_i (A\rho c)_i$$

δ = active thickness $\left[\mathrm{m}\right]$

$\rho =$ density of the surface material $\left[\mathrm{kgm^{-3}}\right]$

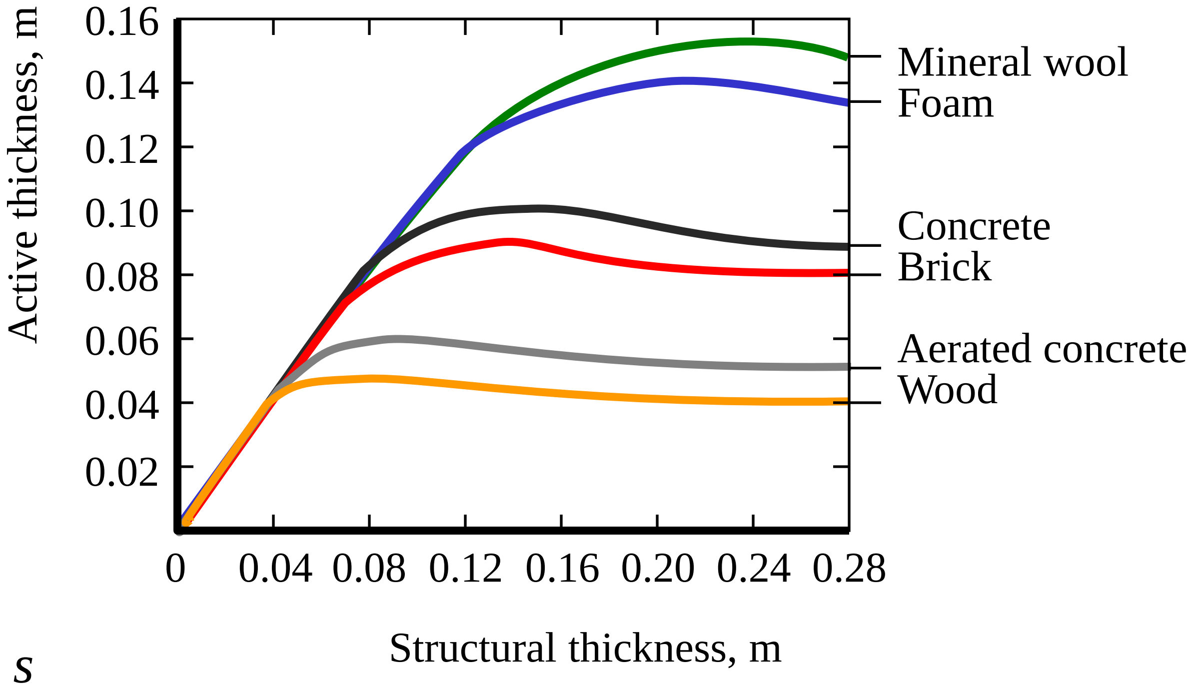
$c =$ specific heat capacity of the surface material $\left[\mathrm{J\left(kgK\right)^{-1}}\right]$

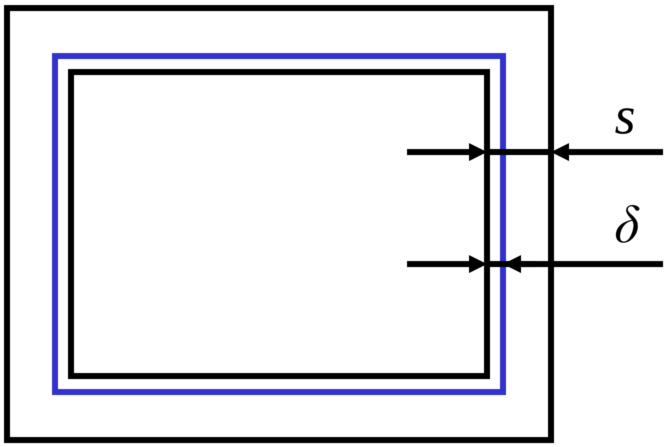
- Total heat capacity:

$$C_{tot} = C_a + C_w$$

One time-constant model – III

- Only certain part of wall that acts as heat storage. *Active thickness* is a quantitative measure for the thickness of that part.
- Active thickness is characteristic for each material.





Active thickness is calculated from

$$\delta = [a\,\tau]^{0.5}$$

a = thermal diffusivity, m²/s

τ = length of calculation period, s

Thermal diffusivity is a description of the rate of heat transfer into a body of material.

One time-constant model – IV

- Specific heat loss through conduction:

$$G_{cond} = \sum_j U_j A_j$$

where U_j = U-value of the j-th surface [W/m^{2}K]

- Specific heat loss through ventilation:

$$G_{vent} = q_{m,s} c_{pa}$$

- Aggregated (total) specific heat loss:

$$G_{tot} = G_{cond} + G_{vent}$$

One time-constant model – V

Analytical solution:

- step change in outdoor temperature (t = 0)
- using the definitions beside

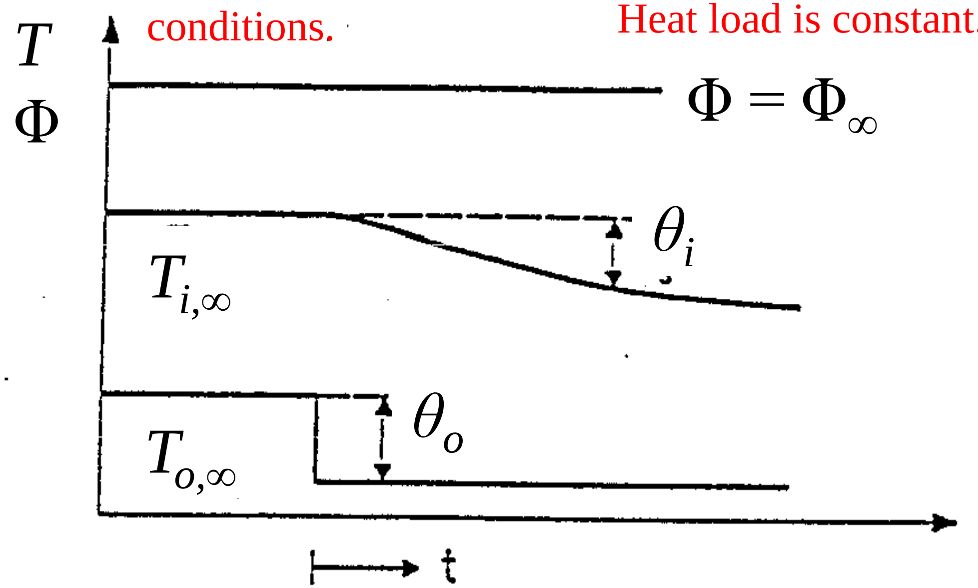
$$\theta_i = \theta_o \left(1 - e^{\frac{-t}{\tau}} \right)$$

- time constant

$$\tau = \frac{C_{tot}}{G_{tot}}$$

At t = 0 the room is in steady-state conditions.

Heat load is constant.



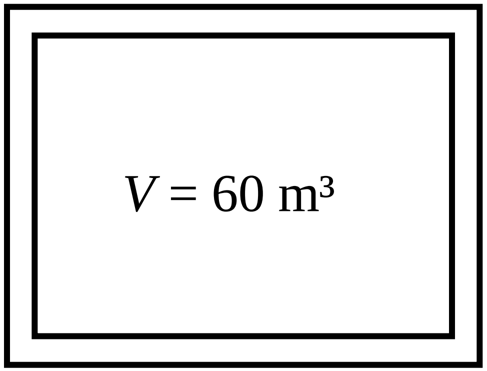
Discrete solution:

- The temperature of the future time step is calculated from that of the previous time step.

$$T_{i,n} = T_{i,n-1} + \frac{G_{tot}}{C_{tot}} \left(T_o - T_{i,n-1} \right) \Delta t$$

where Δt = length of the time step [s]

Example: Interpretation of time constant for two materials



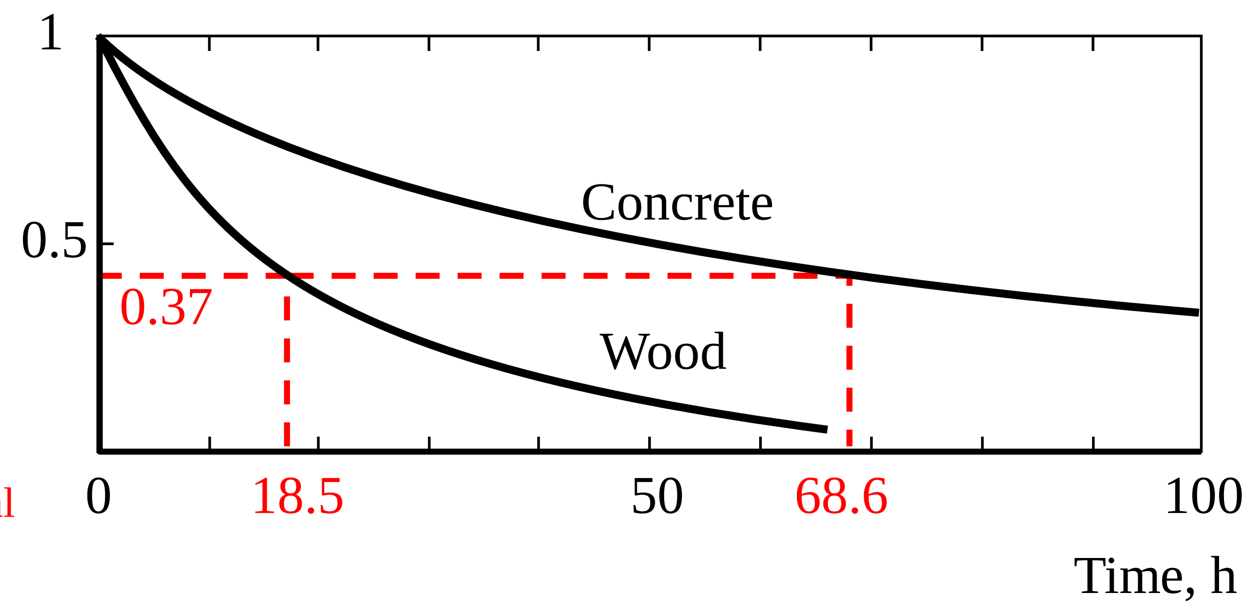
$C_a = 72$ kJ/K

$G_{tot} = 24.15$ W/K

Concrete: $C_w = 5888$ kJ/K $\tau = 68.6$ h

Wood: $C_w = 1536$ kJ/K $\tau = 18.5$ h

$$\frac{T - T_0}{T_1 - T_0}$$



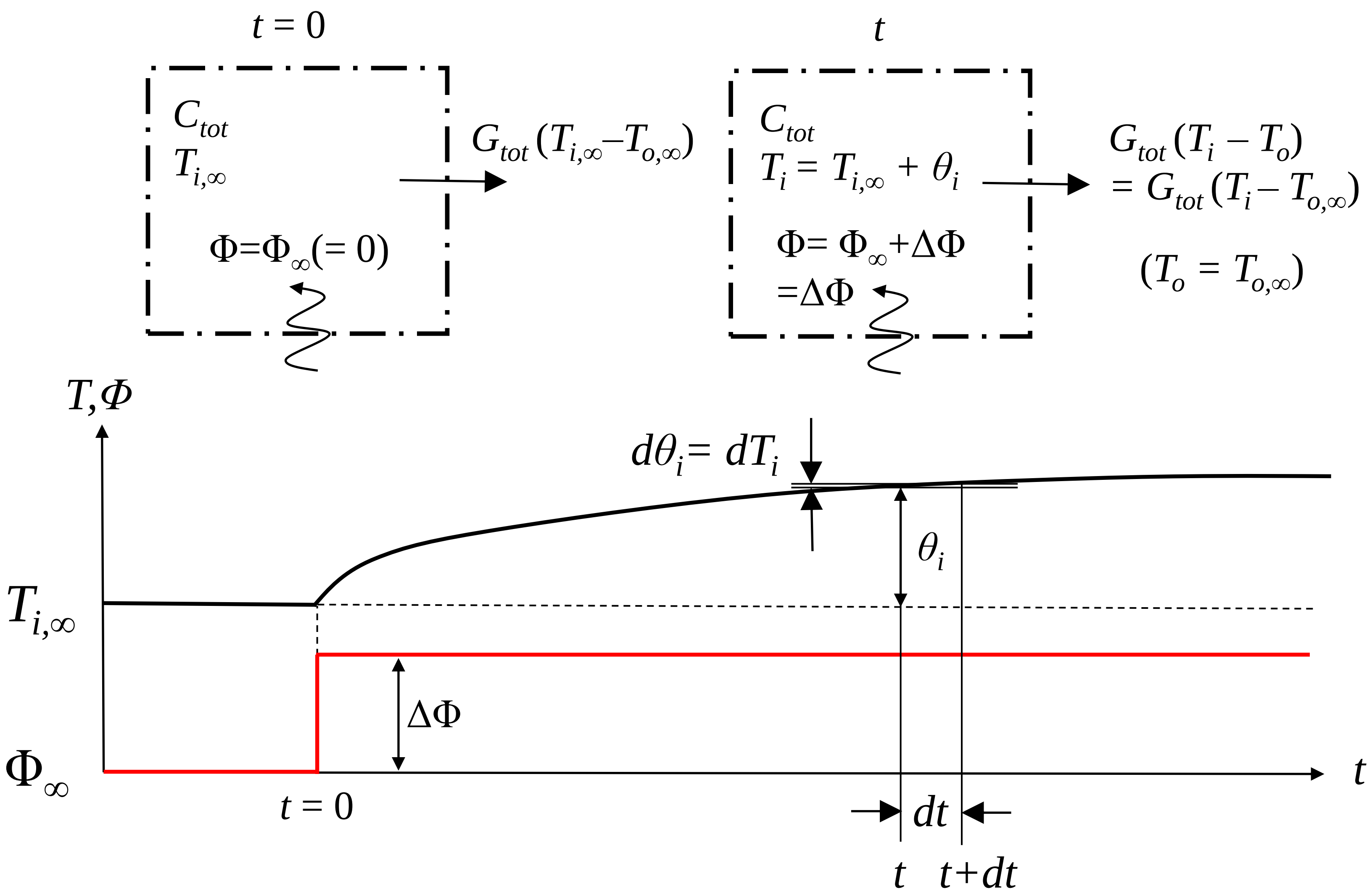
Cooling law: the time constant is the time for the system's step response to reach 63 % of its final (asymptotic) value (from a step increase).

Example

A room is in steady-state conditions. From the time $t = 0$ onwards there is a constant heat load $\Delta\Phi$ (stepwise change from $\Phi = \Phi_\infty = 0$ to $\Phi = \Phi_\infty + \Delta\Phi$) to the room. The outdoor temperature does not change.

Derive the analytical solution of the governing heat balance equation following the one time constant model.

Illustration



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Solution – I

1. Heat balance (transient conditions at time *t*):

$$\text{Define}: \theta_i = T_i - T_{i,\infty}, dT_i = d\theta_i$$

$$C_{tot}\frac{d\theta_i}{dt} = \Delta\Phi - \left[G_{tot}\left(T_i - T_o\right) - G_{tot}\left(T_{i,\infty} - T_{o,\infty}\right)\right]$$

$$= \Delta\Phi - \left[G_{tot}\left(T_i - T_{o,\infty} - T_{i,\infty} + T_{o,\infty}\right)\right] = \Delta\Phi - G_{tot}\theta_i$$

– dividing by G_{tot} and substituting the definition of the time constant τ gives:

$$\frac{C_{tot}}{G_{tot}}\frac{d\theta_i}{dt} = \tau\frac{d\theta_i}{dt} = \frac{\Delta\Phi}{G_{tot}} - \theta_i$$

– by reducing the above equation we obtain:

$$\frac{1}{\tau}dt = \frac{d\theta_i}{\frac{\Delta\Phi}{G_{tot}} - \theta_i}$$

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Solution – II

2. Final function

- differential equation → integration
- required function: $\theta_i = \theta_i(t)$

→ integration 0 → t and 0 → θ_i:

$$\frac{1}{\tau}\int_0^t dt = \int_0^{\theta_i} \frac{d\theta_i}{\frac{\Delta\Phi}{G_{tot}} - \theta_i}$$

$$\rightarrow \frac{(t-0)}{\tau} = \ln\left|\frac{\Delta\Phi}{G_{tot}} - \theta_i\right| - \ln\left|\frac{\Delta\Phi}{G_{tot}} - 0\right| = \ln\left|\frac{\frac{\Delta\Phi}{G_{tot}} - \theta_i}{\frac{\Delta\Phi}{G_{tot}}}\right| \rightarrow e^{\frac{t}{\tau}} = 1 - \frac{\Delta\Phi}{G_{tot}}\theta_i$$

$$\rightarrow \theta_i = \frac{\Delta\Phi}{G_{tot}}\left|1 - e^{\frac{t}{\tau}}\right| \rightarrow \theta_i = \frac{\Delta\Phi}{G_{tot}}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Intrinsic value is always > 0

→ $t/\tau < 0$

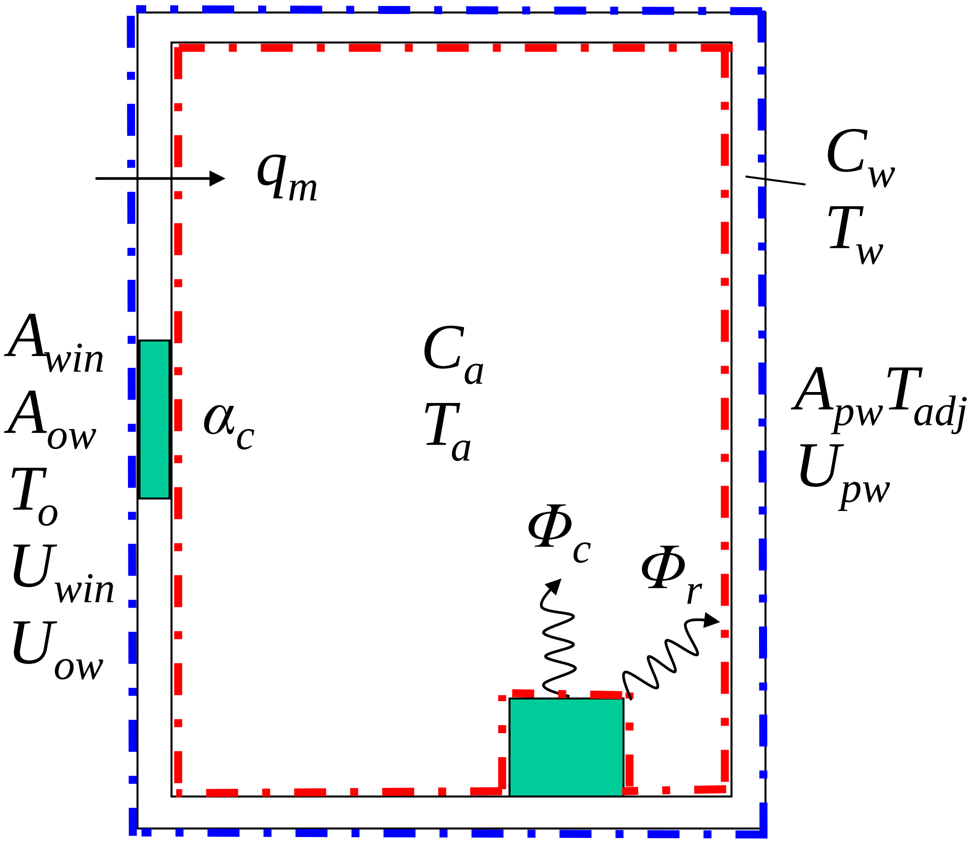
Mathematical operations:

$$\begin{cases} \int \frac{f'}{f}dx = \ln|f| + C \\ \log\left(\frac{x}{y}\right) = \log(x) - \log(y) \\ e^{\log_e(x)} = e^{\ln(x)} = x \end{cases}$$

Two time-constant model – I

A_{ow},A_{pw}	area of outer wall (*ow*) and partition walls (*pw*) [m^2]
C_a	heat capacity of room air [J/K]
C_w	heat capacity of the active layer of the walls [J/K]
T_i,T_o,T_w	indoor (*i*)-, outdoor (*o*)- and wall (*w*) temperature [ºC]
T_{adj}	temperature of adjacent rooms [ºC]
U_{win}	heat transfer coefficient of windows (U-value) [W/m^2K]
A_{win},A_w	area of windows (*win*) and walls (*w*) [m^2]
q_m	mass flow of fresh air [kg/s]
U_{ow},U_{pw}	U-value of outer wall (*ow*) and partition walls (*pw*) [W/m^2K]
α_c	convective heat transfer coefficient from the inner surface to the room air [W/m^2K]
Φ_r	radiative heat load[W]
Φ_c	convective heat load [W]

Two separate control volumes



$$A_w = A_{ow} + A_{pw}$$

Two time-constant model – II

1. Heat balance for air control volume:

$$C_a \frac{dT_i}{dt} = \alpha_c A_w (T_w - T_i) + U_{win} A_{win} (T_o - T_i) - q_m c_{pa} (T_o - T_i) + \Phi_c$$

Convective heat load is released directly to cooling load. Hence, it is placed in the heat balance of the air control volume.

2. Heat balance for surfaces:

$$C_w \frac{dT_w}{dt} = \alpha_c A_w (T_i - T_w) + \left(\frac{1}{U_{ow}} - \frac{1}{\alpha_c} \right)^{-1} A_{ow} (T_o - T_w)$$

$$+ \left(\frac{1}{U_{pw}} - \frac{1}{\alpha_c} \right)^{-1} A_{pw} (T_{adj} - T_w) + \Phi_r$$

Only heat storing surfaces (the ones with active thickness) are counted in the heat balance for surfaces. (Windows are counted in the heat balance of air control volume.)

Radiative heat load is stored in the surfaces. Hence, it is placed in the heat balance of the surface control volume.

Two time-constant model – III

Heat balance equations can be solved numerically, by discretizing them through Euler method. Discretized heat balance for the air control volume at *n*-th time step is:

$$C_a \frac{T_{i,n} - T_{i,n-1}}{\Delta t} = \alpha_c A_w \left(T_{w,n-1} - T_{i,n-1}\right) + U_{win} A_{win} \left(T_{o,n-1} - T_{i,n-1}\right) - q_{m,j-1} c_{pa} \left(T_{o,j-1} - T_{i,j-1}\right) + \Phi_{c,j-1}$$

Discretized heat balance for surfaces at *n*-th time step is:

$$C_w \frac{T_{w,n} - T_{w,n-1}}{\Delta t} = \alpha_c A_w \left(T_{i,n-1} - T_{w,n-1}\right) + \left(\frac{1}{U_{ow}} - \frac{1}{\alpha_c}\right)^{-1} A_{ow} \left(T_{o,n-1} - T_{w,n-1}\right)$$

$$+ \left(\frac{1}{U_{pw}} - \frac{1}{\alpha_c}\right)^{-1} A_{pw} \left(T_{adj,n-1} - T_{w,n-1}\right) + \Phi_{r,n-1}$$

T_i and T_w are solved using the data of the previous (*n* – 1) time step. The length of the time step is Δt [s].

Example

A windowless building is at $t = 0$ at the outdoor temperature 15ºC. Heat load 1000 W is divided evenly between convection and radiation. Calculate the air and surface temperatures after 15 minutes given that

- area and *U*-value of the envelope are 50 m^2 and 0.35 W/m^2K, respectively
- heat capacities of air and surface layer are 20 and 600 kJ/K, respectively
- convective heat transfer coefficient between air and structures is 3 W/m^2K
- fresh air mass flow is 0.012 kg/s

Solve the problem applying the two time-constant model and the Euler method, the time step being $\Delta t = 1$ min.

Solution – I

1. Air and surface temperatures at the *n*-th time step from heat balances:
   – assumption: windowless building, adjacent to outdoor air (all directions)

$$T_{i,n} = T_{i,n-1} + \frac{\Delta t}{C_a}\left[\alpha_c A_w\left(T_{w,n-1} - T_{i,n-1}\right) + q_{m,j-1} c_{pa}\left(T_{o,j-1} - T_{i,j-1}\right) + \Phi_{c,j-1}\right]$$

$$T_{w,n} = T_{w,n-1} + \frac{\Delta t}{C_w}\left[\alpha_c A_w\left(T_{i,n-1} - T_{w,n-1}\right) + \left(\frac{1}{U_{ow}} - \frac{1}{\alpha_c}\right)^{-1} A_{ow}\left(T_{o,n-1} - T_{w,n-1}\right) + \Phi_{r,n-1}\right]$$

Solution – II

2. Substitution (example): after the 1st time step (t = 1 min)

$$T_{i,1\,\text{min}} = 15\,°\text{C} + \frac{60\,\text{s}}{20000\,\frac{\text{J}}{°\text{C}}} \cdot \left[3\,\frac{\text{W}}{\text{m}^2\,°\text{C}} \cdot 50\,\text{m}^2 \cdot (15-15)\,°\text{C} + 0.012\,\frac{\text{kg}}{\text{s}} \cdot 1006\,\frac{\text{J}}{\text{kg}\,°\text{C}} \cdot (15-15)\,°\text{C} + 500\,\text{W} \right]$$

$$= 16.5\,°\text{C}$$

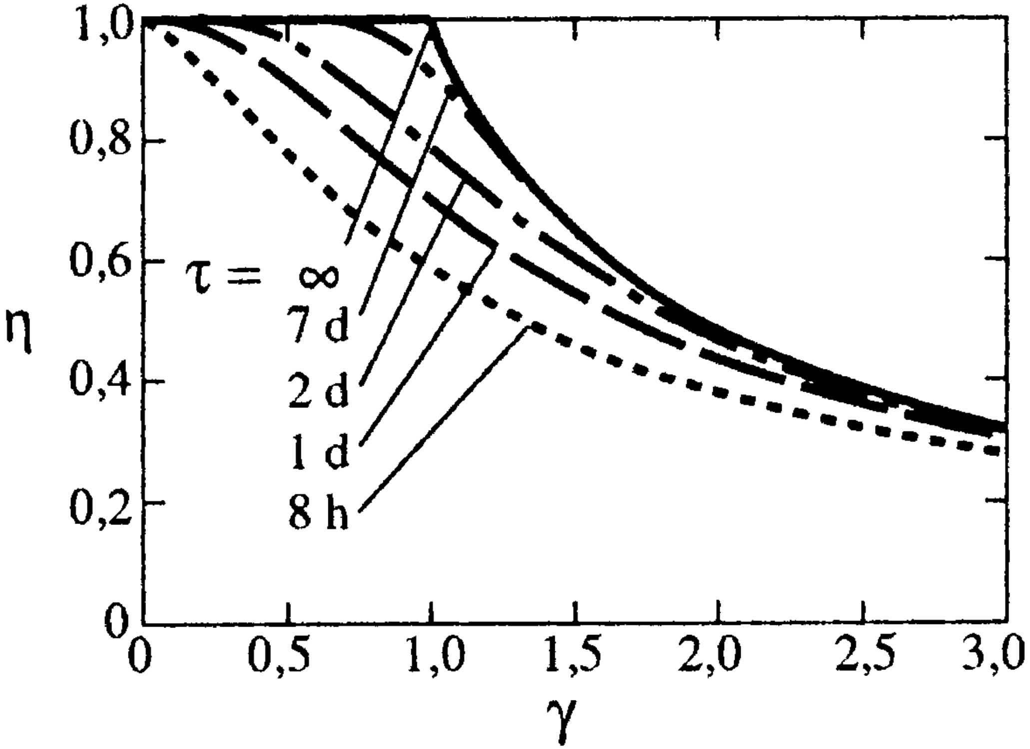
$$T_{w,1\,\text{min}} = 15\,°\text{C} + \frac{60\,\text{s}}{600000\,\frac{\text{J}}{°\text{C}}} \cdot \left[3\,\frac{\text{W}}{\text{m}^2\,°\text{C}} \cdot 50\,\text{m}^2 \cdot (15-15)\,°\text{C} + \left(\frac{1}{0.35\,\frac{\text{W}}{\text{m}^2\,°\text{C}}} - \frac{1}{3\,\frac{\text{W}}{\text{m}^2\,°\text{C}}} \right)^{-1} \cdot 50\,\text{m}^2 \cdot (15-15)\,°\text{C} + 500\,\text{W} \right]$$

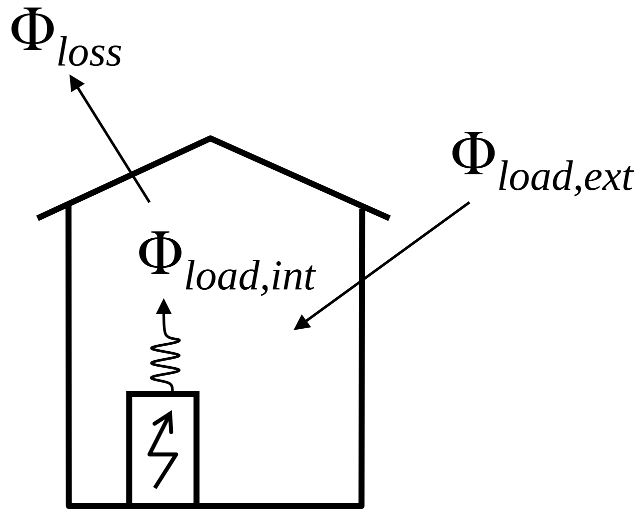
$$= 15.1\,°\text{C}$$

3. Air and surface temperatures after 15 min:
   - $T_{i,15\,\text{min}}$ = <u>19.1ºC</u>
   - $T_{w,15\,\text{min}}$ = <u>16.3 ºC</u>

Self-learning: Familiarize yourself with the concept *<u>operative temperature</u>* and find out how it is calculated from the air and surface temperatures.

Time constant vs. heat utilization





Degree of heat utilization :

$$\eta = \frac{\Phi_{gain}}{\Phi_{load}}$$

Ratio of heat loads and heat losses :

$$\gamma = \frac{\Phi_{load,\mathrm{int}} + \Phi_{load,ext}}{\Phi_{loss}}$$

Other methods to evaluate cooling load

- TEMPO
  - Norwegian method to determine maximum temperature and cooling power
  - simplified heat balance is used to evaluate the temperature rise due to intermittent heat loads
  - several assumptions: e.g. constant proportion of convection and radiation
- ASHRAE-methods (CLTD/CLF/SCL and transfer function)
  - general principle: heat loads are converted to cooling load using average temperature differences and cooling load factors
- Correlation models
  - experimental maximum room temperature is found from a diagram as a function of building type, room area, window area, supply air flow and heat gain