

Clicker lecture 1 of Topic 1:
Transmission line theory and
waveguides

Jan 10, 2018

Registration

Go with your mobile phone to
premo.aalto.fi/mwe1

Fill your full name into the text field for registration.

Q0: How did you prepare yourself for this clicker lecture?

Answer honestly! Your answer does **not** affect "grading".

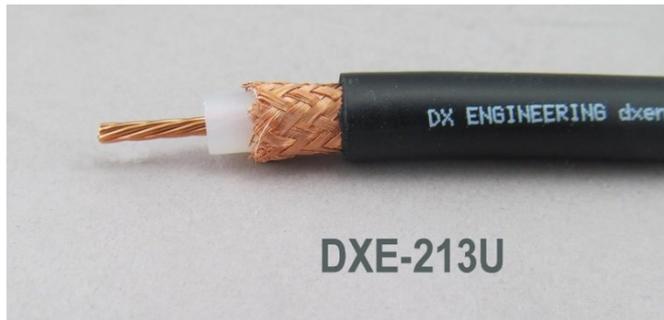
CHOOSE ONE OR MORE!

1. I got the book *and* I read the topic-related chapter in the course book
2. I answered the pre tasks
3. I supplemented my answer after reading other students' answers (or teacher's comments)
4. I started to solve the exercise problems
5. Something else
6. I did not prepaper myself at all

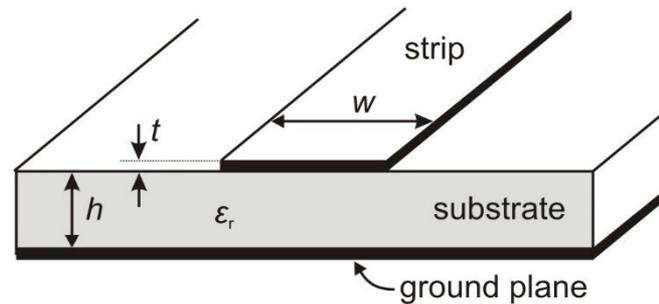
Typical transmission lines

Transmission lines are needed for transferring signals within and between components and devices.

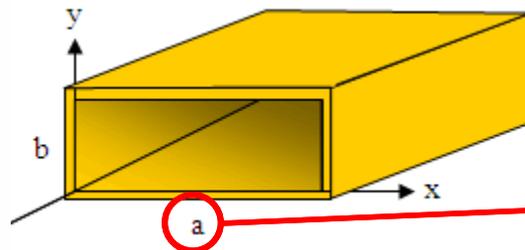
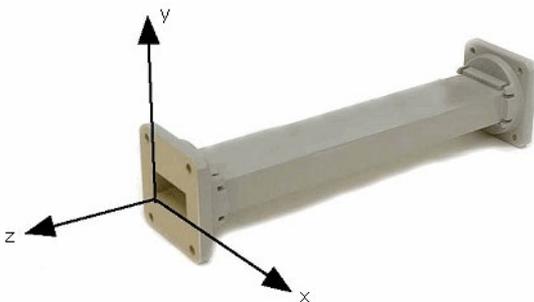
Coaxial cable



Microstrip line on printed circuit board



Rectangular waveguide



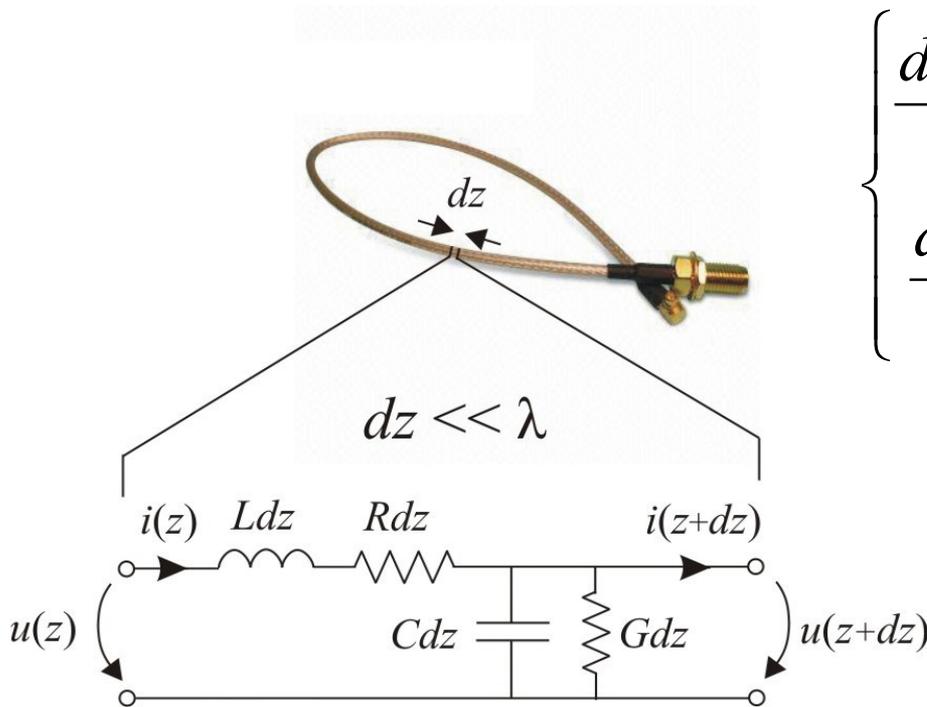
the lowest usable frequency:

$$f_{c,TE10} = \frac{1}{2a\sqrt{\mu\epsilon}}$$

→ used at at "several" GHz frequencies

Transmission line theory

Components and lines whose physical length is a “considerable” fraction of the wavelength (e.g., $> \lambda/10$) must be analyzed using the transmission line theory



$$\begin{cases} \frac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \frac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

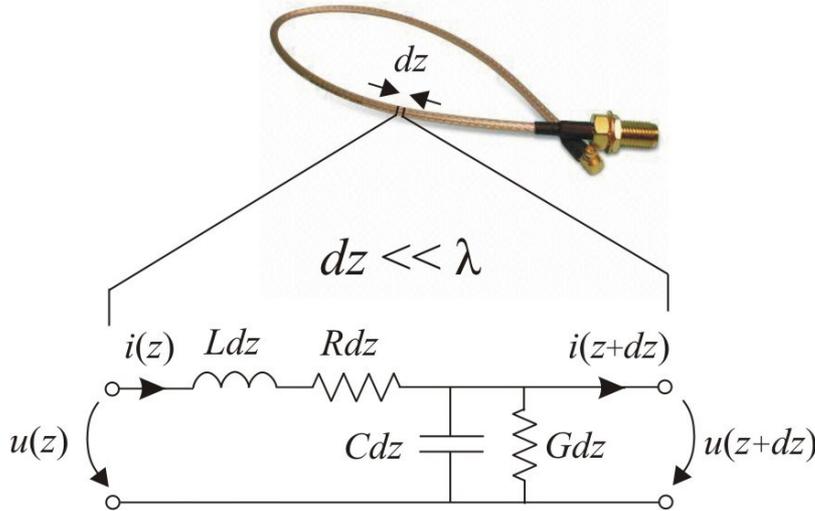
Wave equations are derived in the book – learn the derivation independently!

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta \quad \text{propagation constant} \end{aligned}$$

α = attenuation constant

β = phase constant

Q1a: One solution of the wave equations is given. What function does this solution represent in the real time domain?



1. $u(z,t) = U_0 e^{j(\omega t - \beta z)}$
2. $u(z,t) = jU_0 \cos(\omega t - \beta z)$
3. $u(z,t) = jU_0 \sin(\omega t - \beta z)$
4. $u(z,t) = U_0 \cos(\omega t - \beta z)$
5. $u(z,t) = U_0 \sin(\omega t - \beta z)$
6. I don't know

One solution:

$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

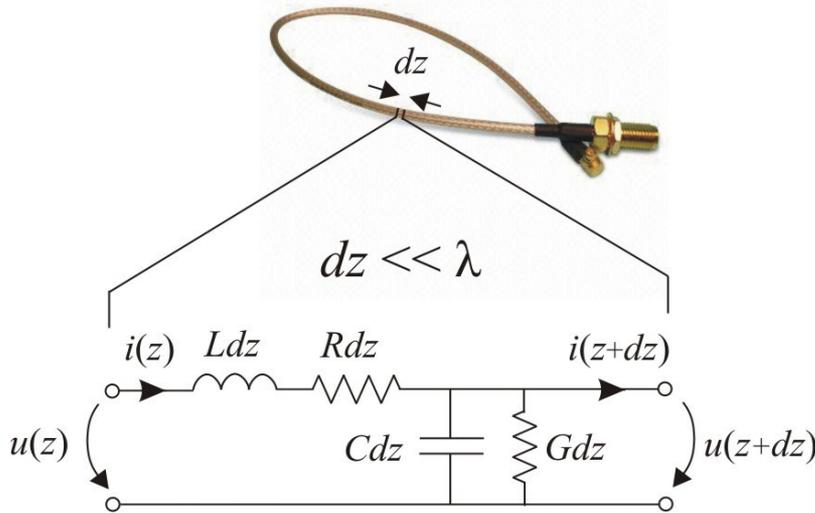
Propagation constant:

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta \end{aligned}$$

Connection between the time harmonic (complex) domain and real time domain:

$$u(z,t) = \Re \left\{ u(z) e^{j\omega t} \right\}$$

Q1b: One solution of the wave equations is given. What function does this solution represent in the real time domain?



1. $u(z,t) = U_0 e^{j(\omega t - \beta z)}$
2. $u(z,t) = jU_0 \cos(\omega t - \beta z)$
3. $u(z,t) = jU_0 \sin(\omega t - \beta z)$
4. $u(z,t) = U_0 \cos(\omega t - \beta z)$
5. $u(z,t) = U_0 \sin(\omega t - \beta z)$

One solution:

$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

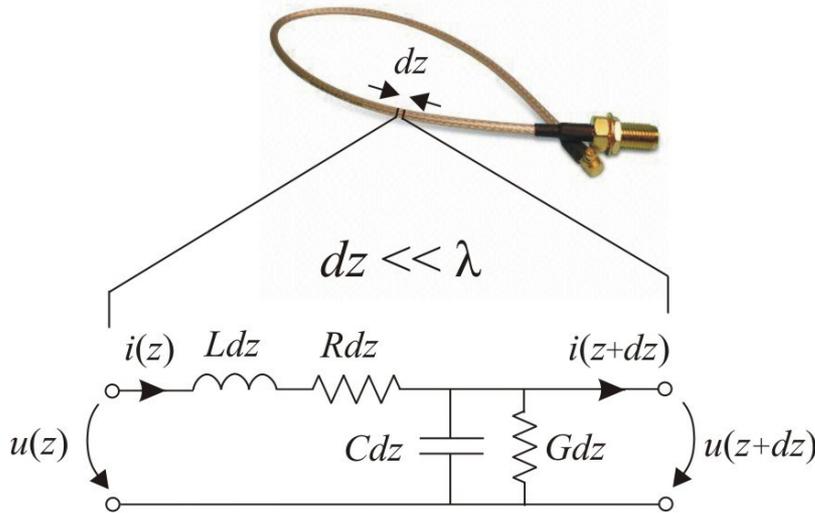
Propagation constant:

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Connection between the time harmonic (complex) domain and real time domain:

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1. $u(z,t) = U_0 e^{j(\omega t - \beta z)}$
2. $u(z,t) = jU_0 \cos(\omega t - \beta z)$
3. $u(z,t) = jU_0 \sin(\omega t - \beta z)$
4. $u(z,t) = U_0 \cos(\omega t - \beta z)$
5. $u(z,t) = U_0 \sin(\omega t - \beta z)$

One solution:

$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

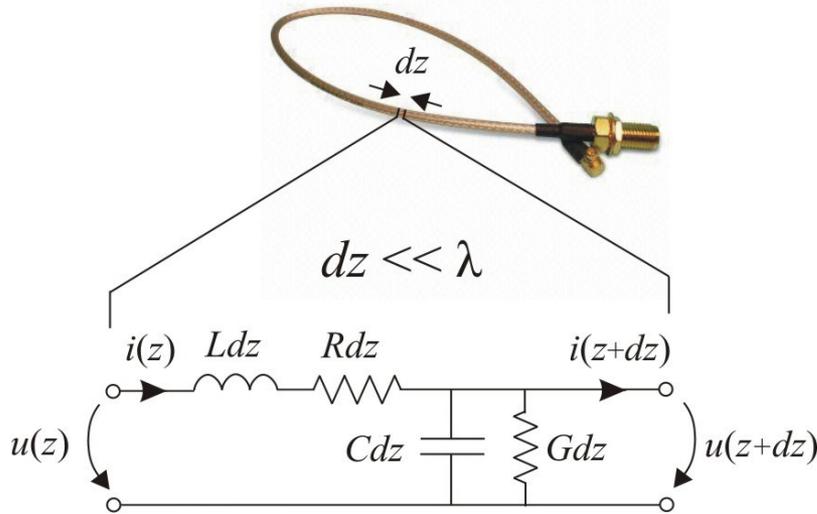
Propagation constant:

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta \end{aligned}$$

Connection between the time harmonic (complex) domain and real time domain:

$$u(z,t) = \Re \left\{ u(z) e^{j\omega t} \right\}$$

Q2a: What is the physical time-domain interpretation of this solution?



$$\begin{cases} \frac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \frac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

Propagation constant:

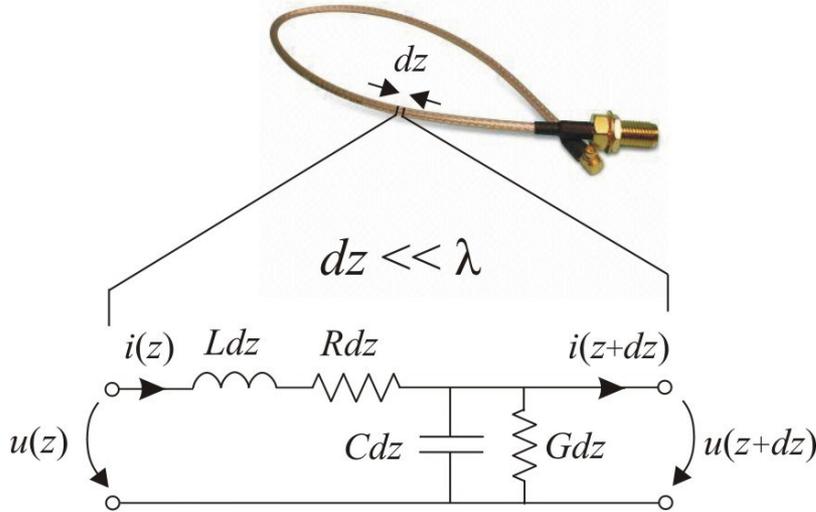
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

1. Decaying wave to **positive** z direction
2. Propagating wave (lossless) to **positive** z direction
3. Decaying wave to **negative** z direction
4. Propagating wave (lossless) to **negative** z direction
5. Propagating wave (lossless) whose source is in the location $z = 0$.
6. I don't know

$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

$$u(t, z) = U_0 \cos(\omega t - \beta z)$$

Q2b: What is the physical time-domain interpretation of this solution?



$$\begin{cases} \frac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \frac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

Propagation constant:

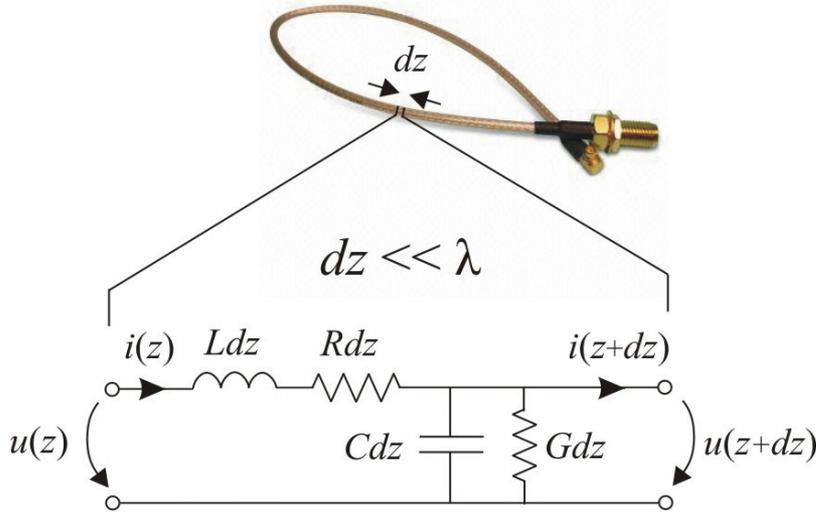
$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta \end{aligned}$$

1. Decaying wave to **positive** z direction
2. Propagating wave (lossless) to **positive** z direction
3. Decaying wave to **negative** z direction
4. Propagating wave (lossless) to **negative** z direction
5. Propagating wave (lossless) whose source is in the location $z = 0$.

$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

$$u(t, z) = U_0 \cos(\omega t - \beta z)$$

Q2: What is the physical time-domain interpretation of this solution?



$$\begin{cases} \frac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \frac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

Propagation constant:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

1. Decaying wave to **positive** z direction
2. Propagating wave (lossless) to **positive** z direction
3. Decaying wave to **negative** z direction
4. Propagating wave (lossless) to **negative** z direction
5. Propagating wave (lossless) whose source is in the location $z = 0$.

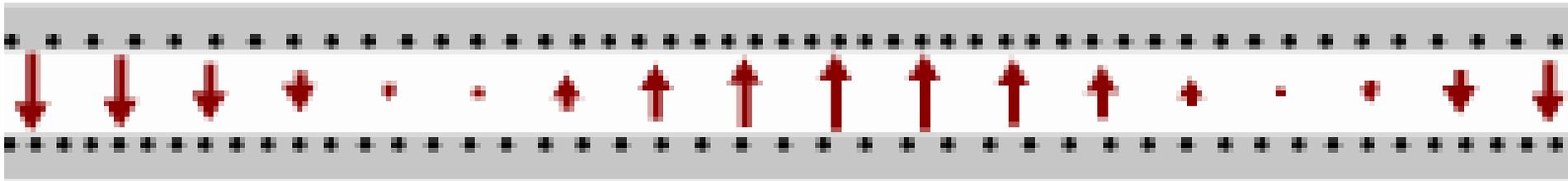
$$u(z) = U_0 e^{-j\beta \cdot z}, \alpha = 0$$

$$u(t, z) = U_0 \cos(\omega t - \beta z)$$

Propagating wave in the time domain



= electric field vector / voltage $u(z,t)$ ● = electron



→ +z direction

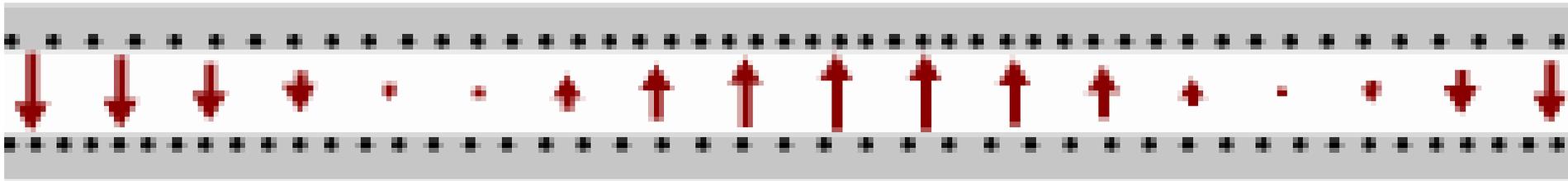
Animation Source: en.wikipedia.org

1. Look at a constant E field / voltage wave front, how does it behave?
2. Look at constant z location, how E field / voltage behaves in that location?
3. What is roughly estimated length of the shown line in wavelengths?

Propagating wave in time domain



= electric field vector / voltage $u(z,t)$ ● = electric charge



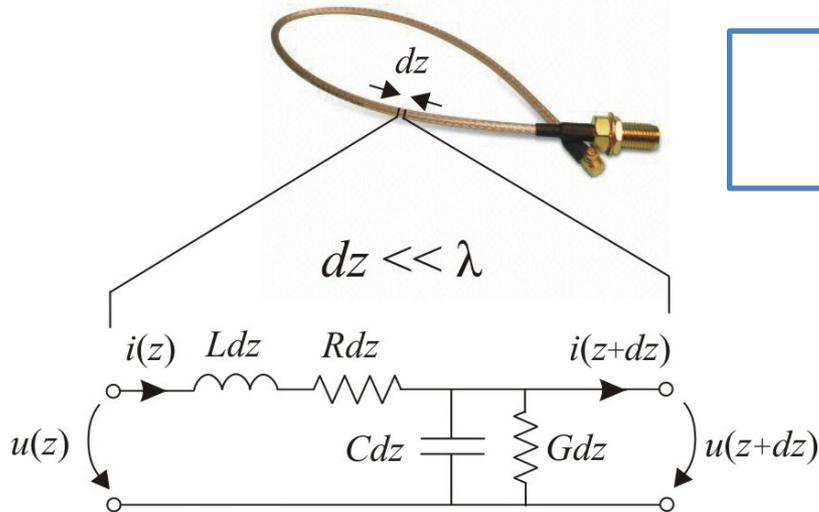
→ +z direction

Animation Source: en.wikipedia.org

$$u(t, z) = U_0 \cos(\omega t - \beta z)$$

U_0 is the peak voltage!

Q3a: What is the physical interpretation of this solution?



$$u(z) = U_0 e^{+j\gamma z} = U_0 e^{+\alpha z} e^{+j\beta z},$$

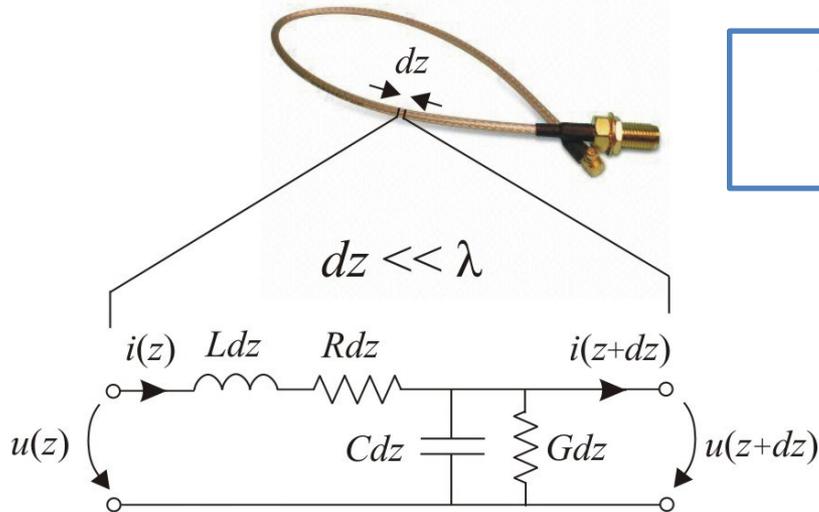
$$\alpha, \beta \neq 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

1. Decaying wave to **positive** z direction
2. Amplifying wave to **positive** z direction
3. Decaying wave to **negative** z direction
4. Amplifying wave to **negative** z direction
5. None of above
6. I don't know

Q3b: What is the physical interpretation of this solution?



$$u(z) = U_0 e^{+j\gamma z} = U_0 e^{+\alpha z} e^{+j\beta z},$$

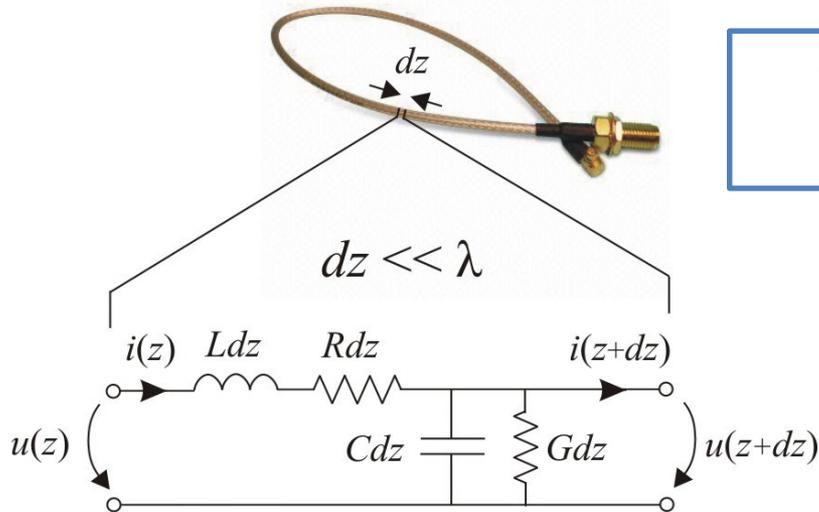
$$\alpha, \beta \neq 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

1. Decaying wave to **positive** z direction
2. Amplifying wave to **positive** z direction
3. Decaying wave to **negative** z direction
4. Amplifying wave to **negative** z direction
5. None of above

Q3: What is the physical interpretation of this solution?



$$u(z) = U_0 e^{+j\gamma z} = U_0 e^{+\alpha z} e^{+j\beta z},$$
$$\alpha, \beta \neq 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \alpha + j\beta$$

1. Decaying wave to **positive** z direction
2. Amplifying wave to **positive** z direction
3. Decaying wave to **negative** z direction
4. Amplifying wave to **negative** z direction
5. None of above

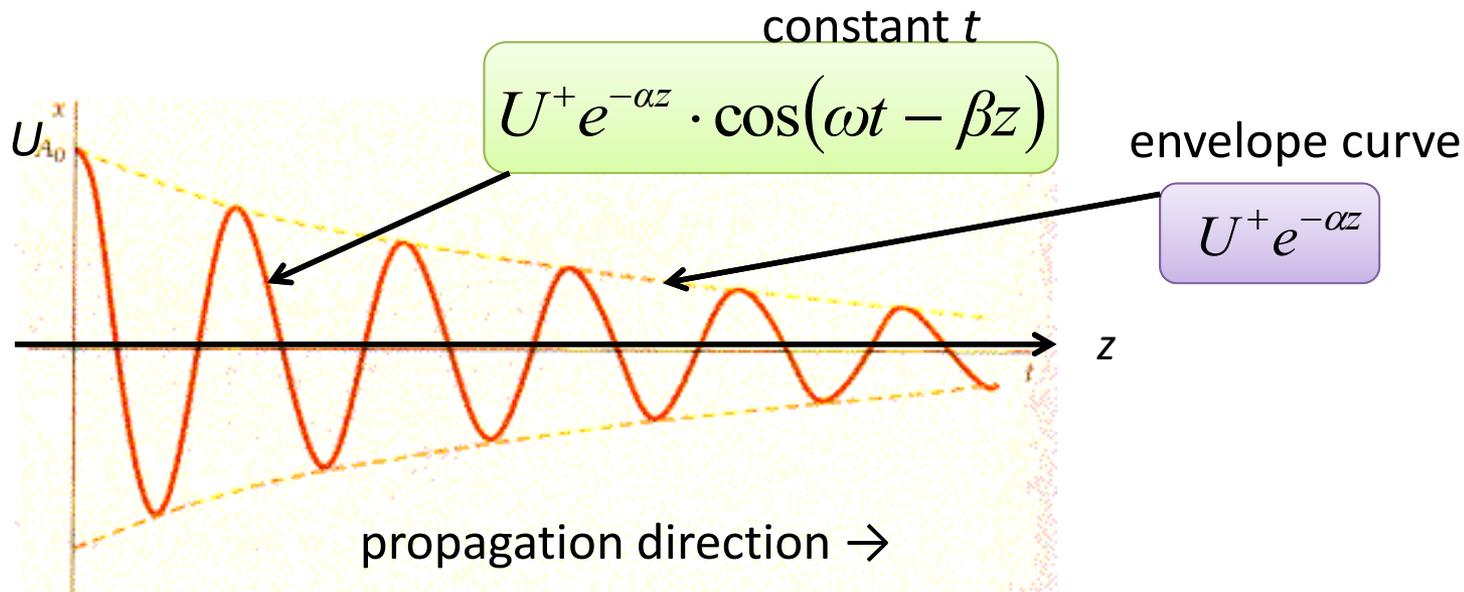
Transmission line theory

- Propagation constant γ is a complex number:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

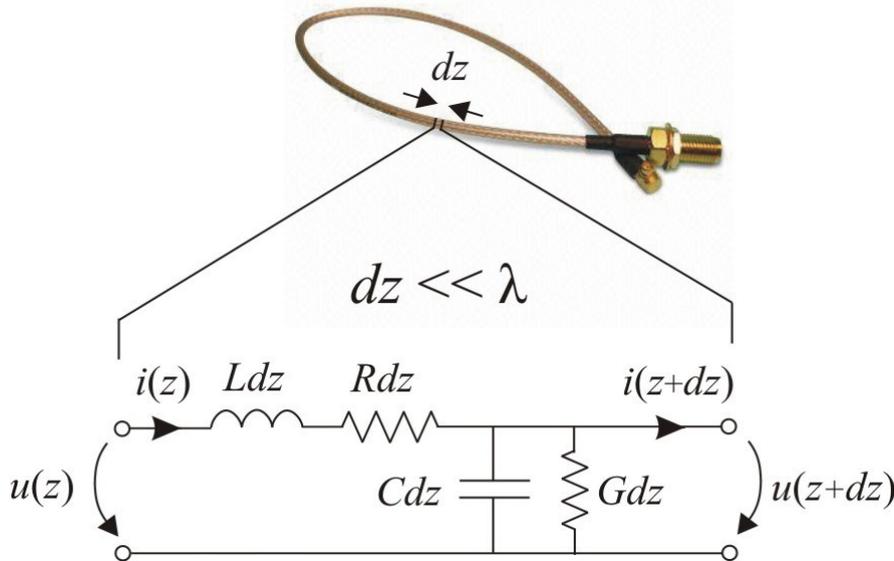
α is the attenuation constant
 β is the phase constant

- Forward travelling (decaying) wave can be written



Transmission line theory

- Components and lines whose physical length is a considerable portion of the wavelength must be analyzed using the transmission line theory



$$\begin{cases} \frac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \frac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases} \quad \text{"telegraph equations"}$$

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta \quad \text{propagation constant} \end{aligned}$$

Full solutions of telegraph equations:

$$\begin{aligned} U(z) &= U^+ e^{-\gamma z} + U^- e^{\gamma z} \\ I(z) &= I^+ e^{-\gamma z} + I^- e^{\gamma z} \end{aligned}$$

what do the solutions mean – i.e., interpret the physical meaning

Transmission line theory

propagation constant:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

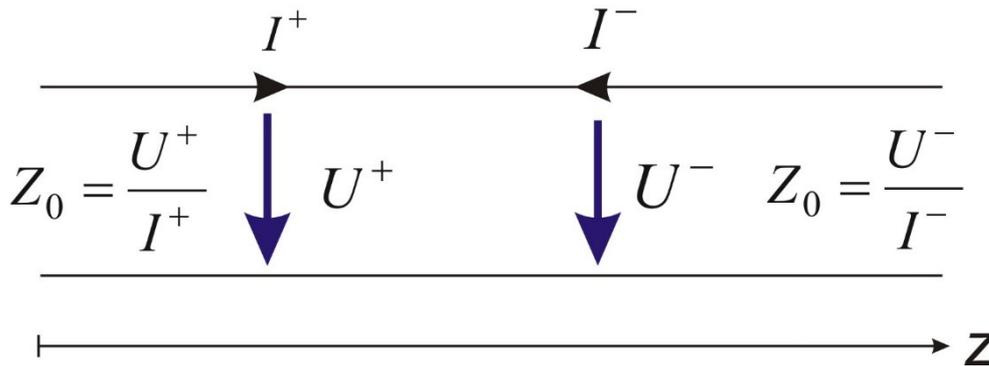
Solution:

$$U(z) = U^+ e^{-\gamma z} + U^- e^{\gamma z}$$

$$I(z) = I^+ e^{-\gamma z} + I^- e^{\gamma z}$$

forward
wave

reverse
wave



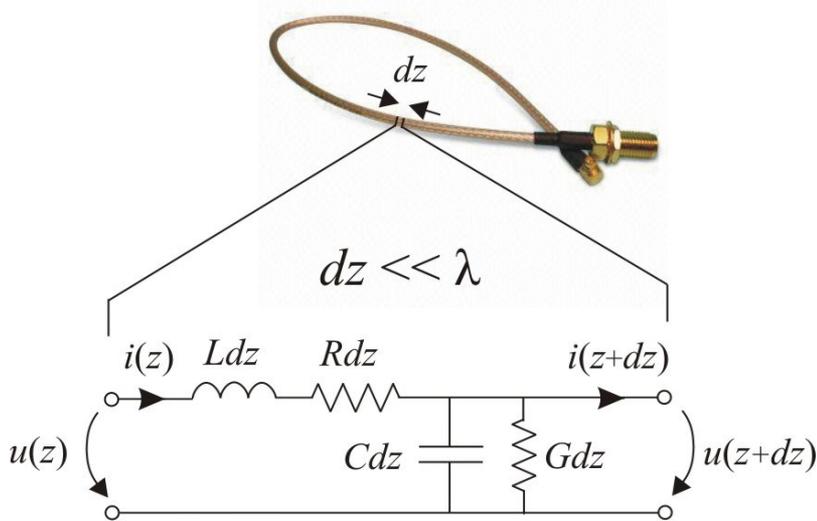
Characteristic impedance (unit Ω):

$$Z_0 = \frac{U^+}{I^+} = \frac{U^-}{-I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The current can be written
in terms of voltage and
impedance:

$$I(z) = \frac{U^+}{Z_0} e^{-\gamma z} - \frac{U^-}{Z_0} e^{\gamma z}$$

Q4a: The characteristic impedance Z_0 of a **lossless** line is

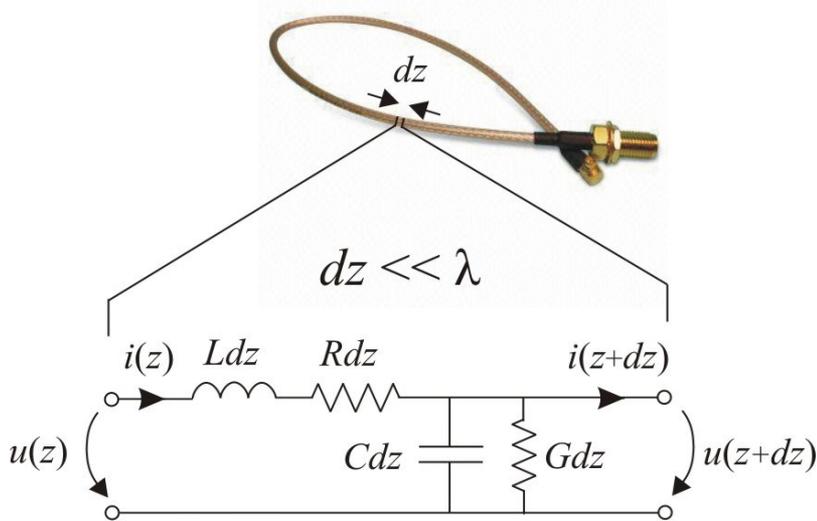


The characteristic impedance is defined as the ratio between the voltage and current:

$$Z_0 = \frac{U(z)}{I(z)} = \frac{U^+}{I^+} = \frac{U^-}{-I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

1. Purely real **positive** number ($r + j x, r > 0, x = 0$)
2. Purely real **negative** number ($r + j x, r < 0, x = 0$)
3. Purely imaginary number ($r + j x, r = 0, x \neq 0$)
4. Complex number ($a + j b, a, b \neq 0$)
5. None of above
6. I don't know

Q4b: The characteristic impedance Z_0 of a **lossless** line is

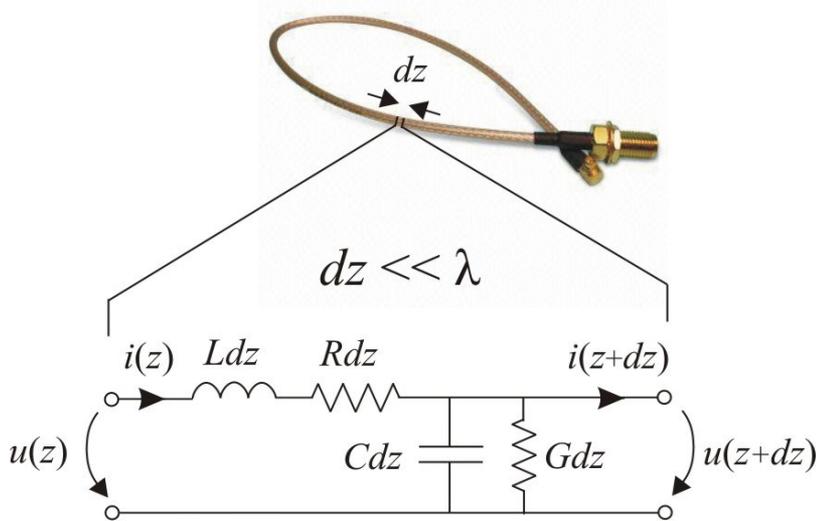


The characteristic impedance is defined as the ratio between the voltage and current:

$$Z_0 = \frac{U(z)}{I(z)} = \frac{U^+}{I^+} = \frac{U^-}{-I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

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3. Purely imaginary number ($r + j x, r = 0, x \neq 0$)
4. Complex number ($a + j b, a, b \neq 0$)
5. None of above

Q4b: The characteristic impedance Z_0 of a **lossless** line is

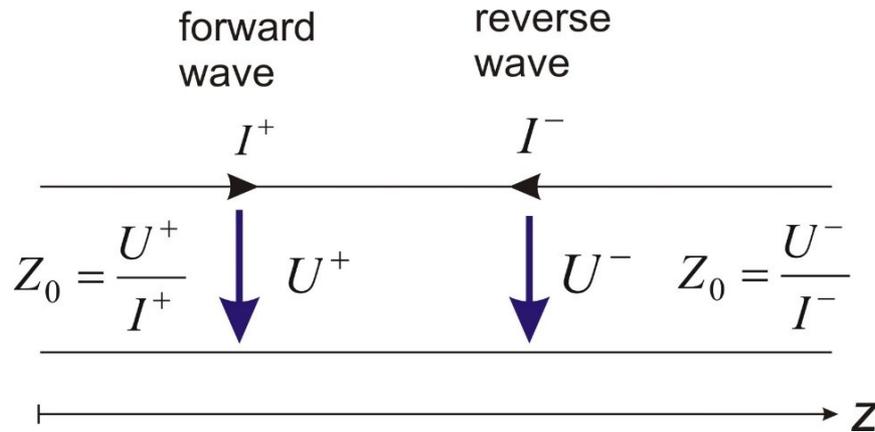


The characteristic impedance is defined as the ratio between the voltage and current:

$$Z_0 = \frac{U(z)}{I(z)} = \frac{U^+}{I^+} = \frac{U^-}{-I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

1. Purely real **positive** number ($r + jx, r > 0, x = 0$)
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5. None of above

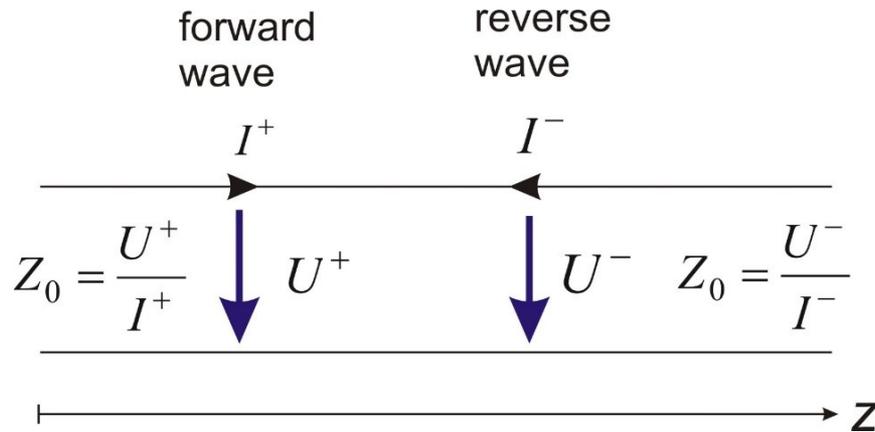
Q5a: What does power P (see formula below) physically mean?



$$P = \frac{|U^+|^2}{2Z_0}$$

1. Loss power (rms) in the line due to resistive losses
2. Peak power propagating to positive +z direction
3. Total net power (rms) in the line
4. Power (rms) propagating to positive +z direction
5. Power (rms) delivered to the load impedance $Z_L (\neq Z_0)$
6. I don't know

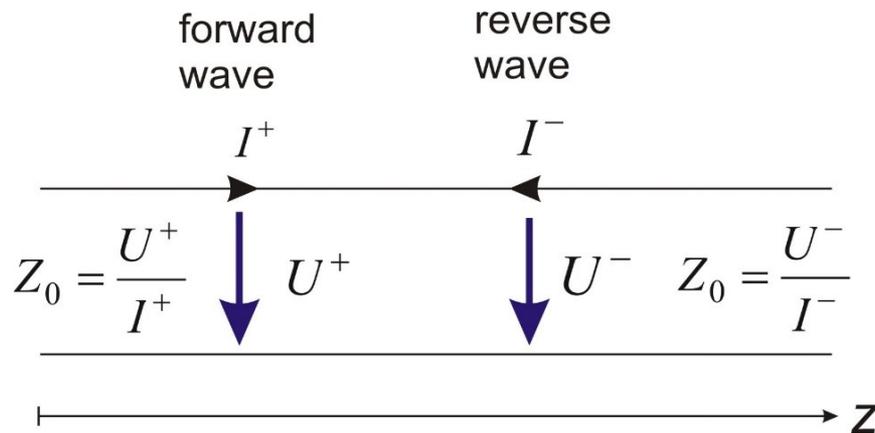
Q5b: What does power P (see formula below) physically mean?



$$P = \frac{|U^+|^2}{2Z_0}$$

1. Loss power (rms) in the line due to resistive losses
2. Peak power propagating to positive $+z$ direction
3. Total net power (rms) in the line
4. Power (rms) propagating to positive $+z$ direction
5. Power (rms) delivered to the load impedance $Z_L (\neq Z_0)$

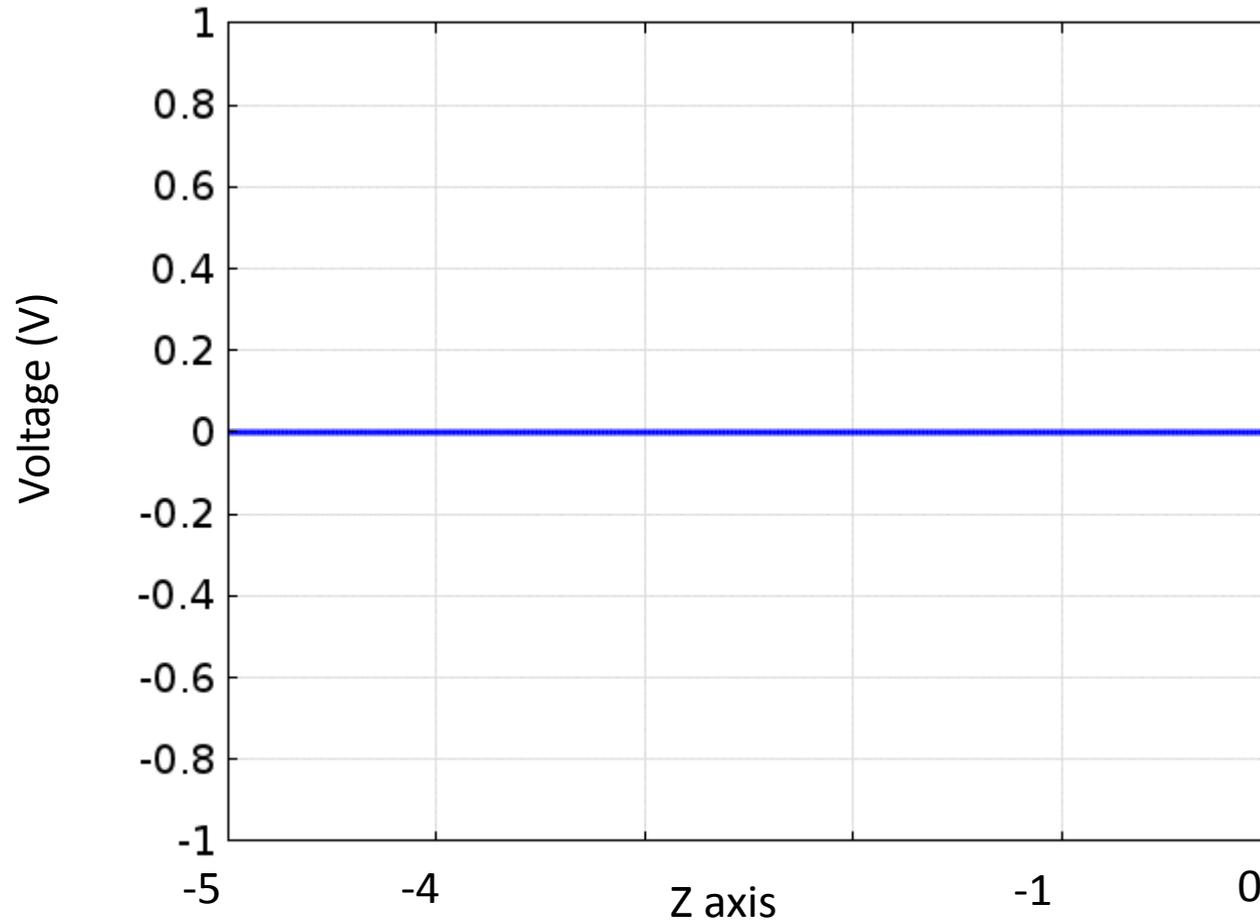
Q5: What does power P (see formula below) physically mean?



$$P = \frac{|U^+|^2}{2Z_0}$$

1. Loss power (rms) in the line due to resistive losses
2. Peak power propagating to positive $+z$ direction
3. Total net power (rms) in the line
4. Power (rms) propagating to positive $+z$ direction
5. Power (rms) delivered to the load impedance $Z_L (\neq Z_0)$

Teaser for the next week: what happens in the animation?



"Transient to standing wave" by Davidjessop - Own work. Licensed under CC BY-SA 4.0 via Commons - https://commons.wikimedia.org/wiki/File:Transient_to_standing_wave.gif#/media/File:Transient_to_standing_wave.gif