Exercise problems of topic 2: Smith chart and impedance matching

Write your answers clearly, so that the answer proceeds logically and includes necessary intermediate steps and sufficient explanations. Your answer should be understandable without oral explanations, too. See further instructions for systematic problems solving in MyCourses.

The exercise problem answers are to be returned during the contact sessions to the course teachers either handwritten (on paper) or typescripted (shown on screen). For other return methods, contact the teachers.

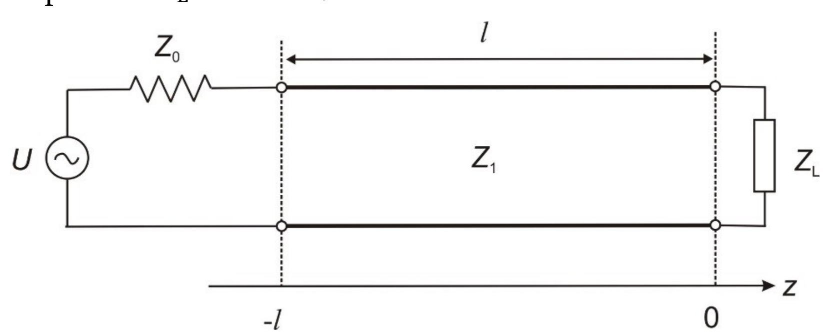
Return your answers one by one when a teacher is free. You may also ask help and instruction.

Be prepared to explain and justify your answer to the teacher. The purpose of this returning method is to enhance your learning through two-way communication and constructive feedback given by the teacher. The teacher will grade your answer in the scale of 0-3 points.

Note that at least two (2) of the problems must be returned latest on Thu 31 January and two (2) more latest on Thu 7 February. If you cannot meet this, you lose a chance to earn those points. However, if you have a good reason not the meet the DL, contact the teachers well in advance. The optimal return rate is about three (3) returned problems per week ☺

Exercise problem 2.1. Solve and answer the following small problems.

A generator (with voltage U and source impedance Z_0) is connected to a lossless ($\gamma = \mathrm{j}\beta$, β = phase constant) transmission line (length = l, characteristic impedance Z_1) and further to the load impedance Z_L as shown below.



a. Let us define the voltage reflection coefficient: $\rho(z=0) = \rho_L = \dfrac{U^-}{U^+} = \dfrac{Z_L - Z_1}{Z_L + Z_1}$.

Show all the intermediate phases (...) of the derivation of the impedance formula:

$$Z(z=-l) = \frac{U(z=-l)}{I(z=-l)} = \frac{U^+ e^{\mathrm{j}\beta l} + U^- e^{-\mathrm{j}\beta l}}{I^+ e^{\mathrm{j}\beta l} + I^- e^{-\mathrm{j}\beta l}} = \ldots = Z_1 \frac{1+\rho_L e^{-2\mathrm{j}\beta l}}{1-\rho_L e^{-2\mathrm{j}\beta l}} = \ldots = Z_1 \frac{Z_L + \mathrm{j}Z_1 \tan(\beta l)}{Z_1 + \mathrm{j}Z_L \tan(\beta l)}.$$

The problem continues on the next page!

b. If the length of the transmission line is $l = \lambda/4$. Show based on the formula of part a. that the load impedance Z_L is fully matched to the generator (impedance Z_0) when

$$Z_1 = \sqrt{Z_L \cdot Z_0}\,.$$

What values (set of numbers?) the impedance Z_L can get in this case? Explain why.

c. If $Z_L = 0\ \Omega$ (short-circuit) or Z_L = infinite Ω (open circuit), show that the formula of part a. can be simplified as

$Z_L = 0\ \Omega$: $Z(z=-l) = \mathrm{j}Z_1 \tan(\beta l)$ (short-circuited short transmission line is inductive)

Z_L = infinite Ω: $Z(z=-l) = -\mathrm{j}Z_1 \cot(\beta l)$ (open short transmission line is capacitive)

Exercise problem 2.2. Solve and answer the following problems. Write all the intermediate phases. Return the Smith chart together with your answer.

A mismatched load $Z_L = 20 - \mathrm{j}\,50\ \Omega$ causes a standing wave into a transmission line. Perform the following small tasks (a-f) using graphically the Smith chart. The normalization impedance is $Z_0 = 50\ \Omega$ (if **not** otherwise informed, the reference impedance is always $50\ \Omega$!).

a. What is the length (give answer in λ) of one full round (360 degrees) on the Smith chart? Justify.

b. Mark the normalized load impedance $z_L = Z_L/Z_0$ on the Smith chart. Note that the impedance is the same as in Exercise problem 1.6 of Topic 1.

c. Define the reflection coefficient ρ_L from the Smith chart and check the result also with a calculator.

d. Define the admittance y_L which corresponds to z_L. Check your result with a calculator.

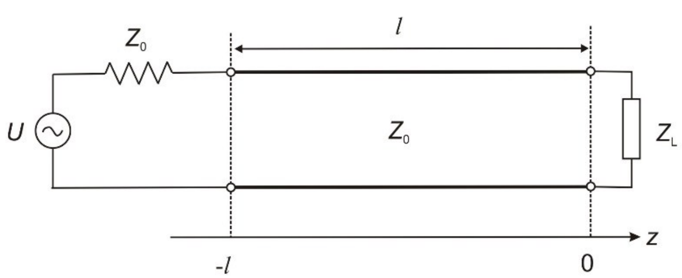
e. Z_L is attached to a 50-Ω transmission line. Define the voltage standing wave ratio, VSWR, from the Smith chart and check the result with a calculator.

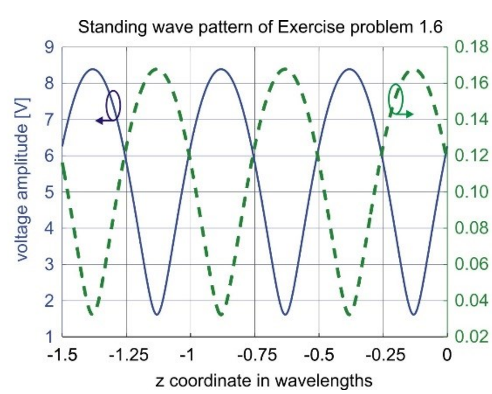
f. Define the distance of the nearest **voltage minimum** of the standing wave from the load. Compare the result with the result of Exercise problem 1.6. (see below).

g. How far is then the nearest **voltage maximum** from the load?

h. Define the distance of the nearest **current maximum** of the standing wave from the load. Compare the result with the result of Exercise problem 1.6. (see below).

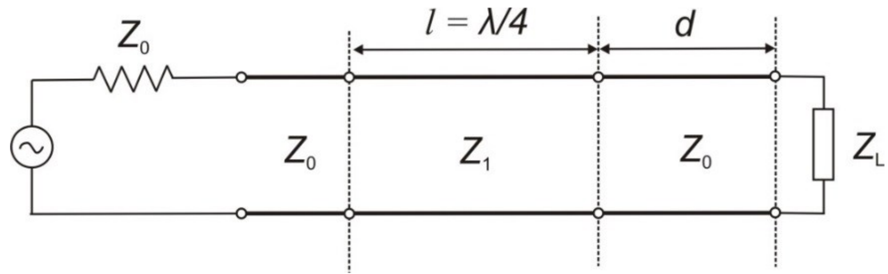
i. How far is the nearest **current minimum** from the load?



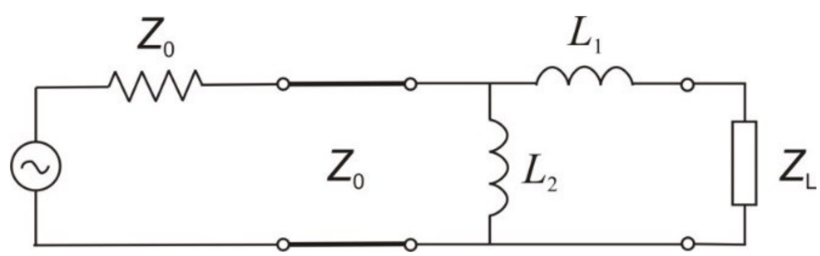


Exercise problem 2.3. Solve and answer the following problems using the Smith chart graphically. Write all the intermediate phases and good explanations to your answers. Return the Smith charts together with your answers. The load impedance is again $Z_L = 20 - j\,50\ \Omega$ and $Z_0 = 50\ \Omega$.

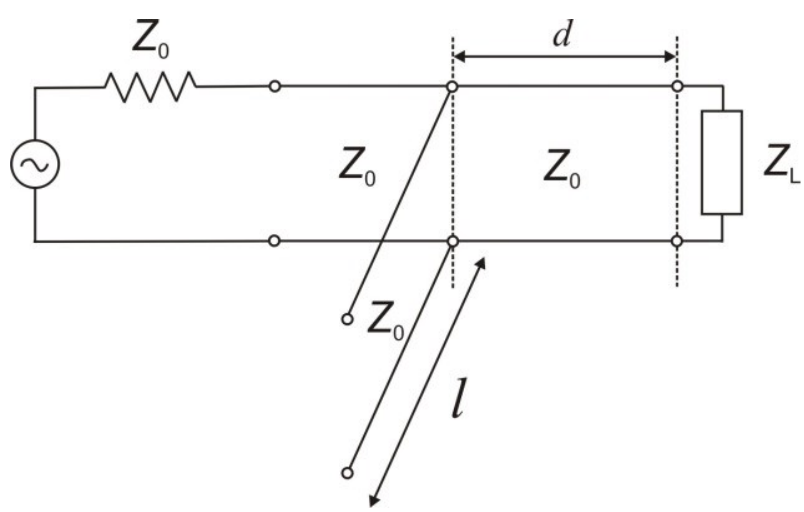
a. Match the load using the matching circuit shown below – i.e., calculate the length d in *wavelengths* and the impedance Z_1 of the quarter-wavelength transformer.



b. Match the load using an L-section inductor-inductor matching circuit (see figure below) at 1 GHz – i.e., calculate the inductor values L_1 and L_2.



c. Match the load using an open parallel stub so that **the length *l* of the stub is minimized** – i.e., calculate the lengths d and l in *wavelengths*.



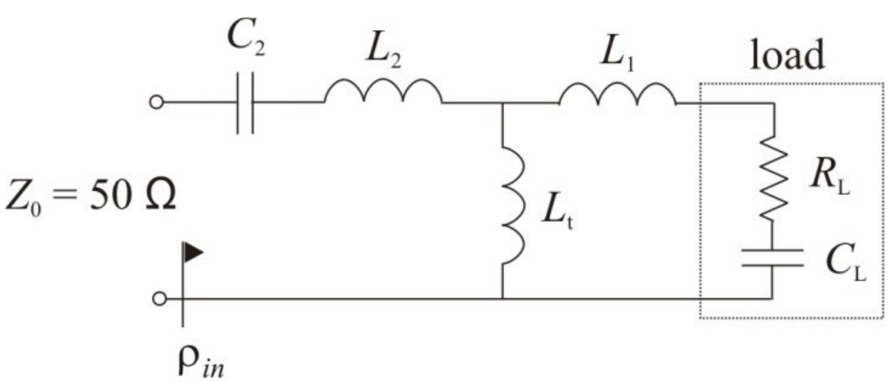
Exercise problem 2.4. The following problem is to be done with AWR. Ask computer & help in the class! AWR is also available in both the computer classes of the “Maarintie 8” building.

a. The load impedance $Z_L = 20 - j\,50\ \Omega$ is modelled with a resistor R and capacitor C in series at 1 GHz. What are the values of R [Ω] and C [pF]?

b. Simulate the matching circuits of Problems 2.3 a.-c. with the AWR circuit simulator.
   - Plot the absolute value of the reflection coefficient $|S_{11}|$ (in dB) of all three circuits to the same Cartesian coordinate system in 0.5 ... 1.5 GHz. The S_{11} is seen in the input of the matching circuit.
   - Plot the reflection coefficients S11 on the Smith chart in 0.5 ... 1.5 GHz.
   - Implement the matching circuits of Problems 2.3 a. and c. on a 1.5-mm thick FR-4 substrate for which the relative permittivity $\varepsilon_r = 4.3$, loss tangent $\tan\delta = 0.02$ and the thickness of the metal $t = 35$ μm. You can use TXLine calculator of AWR.
   - If needed, tune the component values / lengths of the transmission lines such that the load is “fully matched” at 1 GHz – i.e., the circuit “resonates” at 1 GHz.

c. Which of the matching circuits (Problem 2.3 a. = quarter-wavelength transformer, b. = lumped element L-section, c. = single tuning stub) provides the largest impedance bandwidth? Can you make a sophisticated guess, which factors affect the impedance bandwidth?

Exercise problem 2.5. Use AWR circuit simulator and match the familiar load impedance with the *dual-resonant* matching circuitry shown in the figure below. Use $R_L = 20\ \Omega$ and $C_L = 3.18$ pF. The centre frequency is 1.0 GHz, and use $|\rho_{in}| = -10$ dB as the matching criterion. Tune the component values such that you get the largest possible impedance bandwidth – i.e. the reflection coefficient has a symmetrical double loop around the centre of the Smith chart.



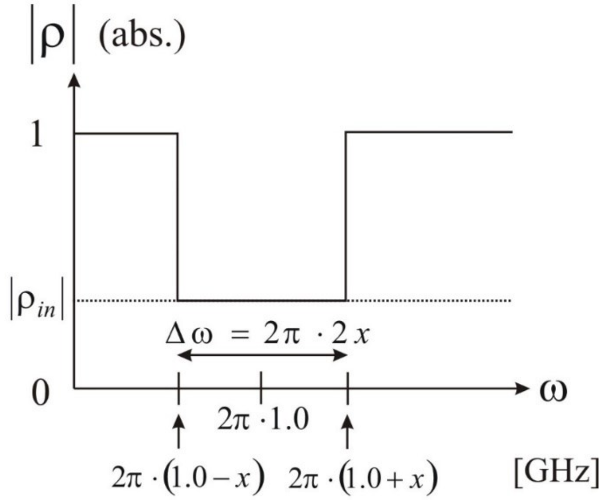
If you have time and interest, match the same load with a **triple-resonant** matching circuitry. Can you increase the bandwidth endlessly just by adding more and more resonators?

Hint: add a shunt parallel LC resonator between the feed port and the series resonator of C_2 and L_2.

Exercise problem 2.6. Solve and answer the following problems. Write all the intermediate phases and good explanations to your answers.

a. Calculate based on the *Bode-Fano criterion* the maximum impedance bandwidth (in GHz) of the load impedance of the problem 2.5 (resistor R_L = 20 Ω and a capacitor C_L = 3.18 pF in series). The maximum allowed reflection coefficient is $|\rho_{in}|$ = -10 dB = 0.316, and the centre frequency is 1.0 GHz.

b. What is the percentage fraction that can be achieved with 1) the single-resonant matching circuit (Prob. 2.3 b.) and 2) dual-resonant matching circuit (Prob. 2.5) compared to the maximum bandwidth given by the Bode-Fano criterion (part a. of this problem)?



c. Why the bandwidth given by the Bode-Fano criterion (of a. part) cannot be achieved in practice? Explain using your own words, what the purpose of the Bode-Fano criterion is.

Voluntary investigation task. If you are very familiar with basic impedance matching techniques, you may replace problems 2.1-2.3 with this investigation problem. The maximum points is 9. Inform teachers in advance for agreeing on the details of return.

Explore the Aalto-originated article [1] related to design of *dual-resonant* matching circuits. Based on the article, ponder answers to the following questions.

a. Find out and justify the topology of the dual-resonant matching circuit used in Problem 2.5.
b. Calculate analytically the component values of the dual-resonant matching circuit of Problem 2.5.
c. Is the topology derived in the article suitable for matching any load impedance?
d. Outline general principles for the design of dual-resonant matching circuits. It is recommended to use also other sources of information.

[1] J. Villanen and P. Vainikainen, "Optimum dual-resonant impedance matching of coupling element based mobile terminal antenna structures," *Microwave and Optical Technology Letters*, vol. 49, no. 10, pp. 2472-2477, October 2007.