References are to equations appearing in the course book.

Problem 2.1

a) Show that the angular momentum $\vec{r} \times \vec{p}$ of the circulating particle with respect to the center is $mrv\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane of the circle. Here, $\hat{n}$ points in a direction given by the right-hand rule applied to the particle's motion.

b) Show that the magnetic moment associated with the motion of the point charge is $qvr/2$ and thus that the gyromagnetic ratio is given by (2.19).

c) Evaluate numerically the gyromagnetic ratio γ (2.19), choosing the same mass (1.67×10^{-27} kg) and charge (1.60×10^{-19} C) as for a proton. The difference between your answer and (2.17) is due to the more complicated motion of the proton constituents, the 'quarks.' For related reasons, a neutron has a nonvanishing magnetic moment despite its zero overall charge.

Problem 2.2

It will be useful in later discussions to have the answer (2.33) rederived as a solution to the differential equation (2.24).

a) For $\vec{B} = B_0\hat{z}$, show that the vector differential equation (2.24) decomposes into the three Cartesian equations

$$\begin{aligned}\frac{d\mu_x}{dt} &= \gamma\mu_y B_0 = \omega_0\mu_y \\ \frac{d\mu_y}{dt} &= -\gamma\mu_x B_0 = -\omega_0\mu_x \\ \frac{d\mu_z}{dt} &= 0.\end{aligned} \tag{2.34}$$

b) By taking additional derivatives, show that the first two equations in (2.34) can be decoupled to give

$$\begin{aligned}\frac{d^2\mu_x}{dt^2} &= -\omega_0^2\mu_x \\ \frac{d^2\mu_y}{dt^2} &= -\omega_0^2\mu_y\end{aligned} \tag{2.35}$$

These decoupled second-order differential equations have familiar solutions of the general form $C_1 \cos\omega_0 t + C_2 \sin\omega_0 t$.

The problems are based on those in Robert W. Brown, Y.-C. Norman Cheng, E. Mark Haacke, Michael R. Thompson, Ramesh Venkatesan. *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 2nd Edition, Wiley, 2014.