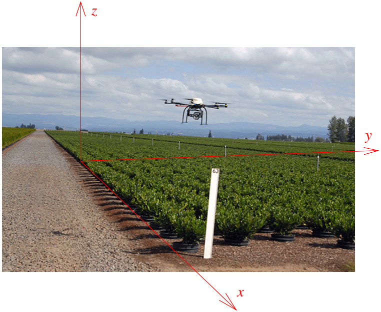
GIS in Environmental Modelling

Georeferencing

Martin Vermeer



January 11, 2019

Aalto University
School of
Engineering

Introduction (1)

It is the task of geodesy – more generally, the mapping sciences, including photogrammetry and the georeferencing of remote-sensing data – to make sure that environmental data is geolocated correctly, and to the precision level that the application requires.

This may be done

- through *general mapping technologies*, by providing precise maps on which the environmental data collected can be precisely geolocated
- by geolocating the *instruments* that collect environmental data; typically this would today be done by a satellite positioning receiver (GNSS)
- ...

Introduction (2)

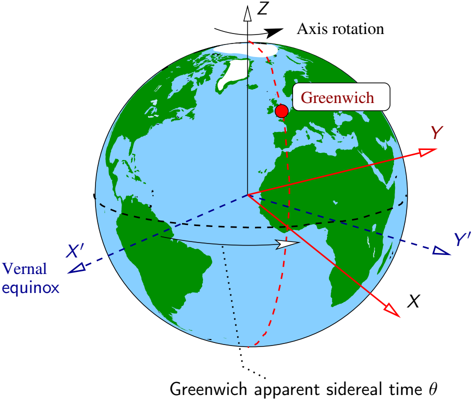
- by *geolocating the airborne or orbital platform* collecting environmental data by remote sensing. Like
  1. Photogrammetric data collected by camera, either in the visible light or in other spectral bands (e.g., false-colour photography)
  2. Multi- or hyperspectral data collected in visible light or near or far infrared, including thermal infrared
  3. Laser scanning data: geodetically precise 3-D point clouds for mapping, terrain modelling, and general remote sensing
  4. Synthetic-aperture radar data for terrain mapping, remote sensing, including terrain modelling and Earth crustal deformation detection
  5. Airborne or satellite gravimetry, gravity field recovery, geoid modelling, including change monitoring
  6. Satellite radar altimetry of the sea surface. Precise mapping for ocean current recovery; change monitoring, including global sea-level rise; ice studies.

Introduction (3)

To be discussed:

- Technologies for precise geolocation, both classical (terrestrial surveying) and modern (GNSS, Global Navigation Satellite Systems)
- The concept of geodetic datum; both old and modern datums, both location and height datums
- Modern geodetic co-ordinate reference systems and their realization, reference frames
- Map projections and their role in visualizing precisely geolocated data
- Transformation between geodetic datums; Helmert (similarity) transformation, affine transformation
- Application to Finland: users of geospatial information, municipalities

Co-ordinate systems on the Earth



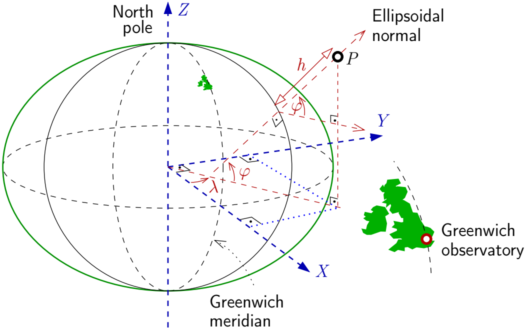
Geocentric co-ordinate systems have (1) the Earth centre of mass at the origin, and (2) the Z axis pointing along the rotation axis.

Technicalities:

- The Earth *rotates*. We distinguish inertial and ECEF (Earth centred, Earth fixed) co-ordinate systems. Between them, *Greenwich sidereal time*.
- Earth rotation is irregular: length-of-day (LOD) variations, polar motion.

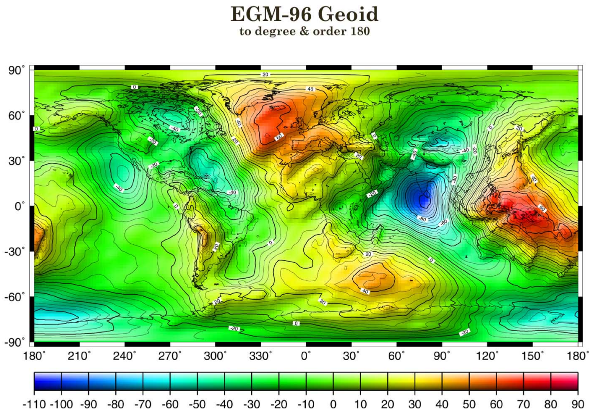
The reference ellipsoid...

We approximate the figure of the Earth by an ellipsoid of revolution, the reference ellipsoid. Characterized by *semi-major axis* or equatorial radius, a, and *flattening*, $1/f$. Currently most used reference ellipsoid is that of GRS80, Geodetic Reference System 1980, with $a = 6378137\,\mathrm{m}$ and $1/f = 298.257222101$. Closer to true figure of the Earth than older, traditional reference ellipsoids which are slowly passing out of use.



Aalto University School of Engineering

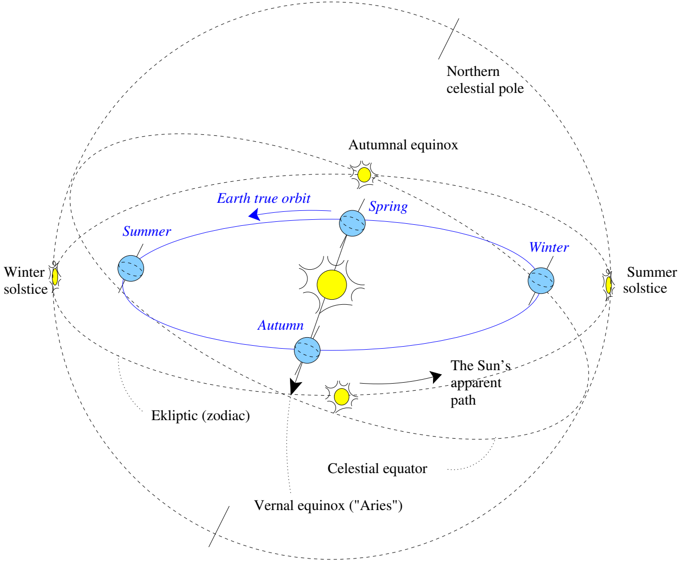
. . . and the geoid



The geoid – the mathematical figure of the Earth (Gauss), the equipotential surface of the gravity field most closely approximating mean sea level – undulates above and below the GRS80 reference ellipsoid surface by up to $\pm 100\,\mathrm{m}$ globally.

Aalto University
School of
Engineering

(Intermezzo: The Earth in space)



. . . and summer's lease hath all too short a date

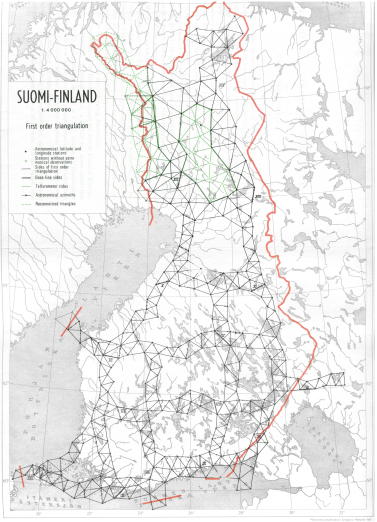
Measurement: instruments

Then. . . and now
Hildebrand theodolite (also used in Finland)
and Ashtech Z12

Aalto University
School of
Engineering

Measurement: classical techniques

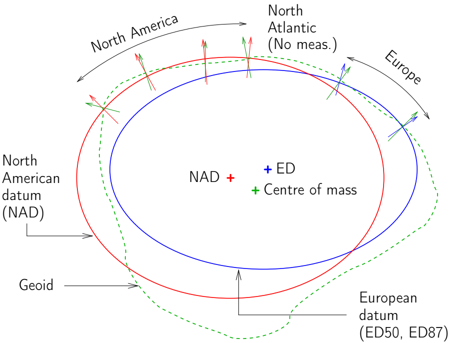


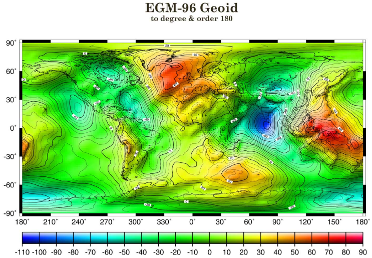
Classically, measurements were made on the Earth surface, between points on the Earth surface. National triangulation networks were connected into continental ones and jointly adjusted. Thus the classical geodetic datums of the 20th century were created.

- Europe: ED50, ED87
- North America: NAD83

The Finnish primary triangulation is part of ED50. Its adjustment was the basis for creating KKJ, the National Grid Co-ordinate System, in 1970

Classical datums (1)





Classically detemined datums are *non-geocentric*, often by several hundred metres. The reference ellipsoid is fitted to the local situation and will approximate the *geoid* locally.

Express rectangular (geocentric) co-ordinates X, Y, Z into the geodetic or geographic co-ordinates φ, λ, h of the datum:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N(\varphi) + h)\cos\varphi\cos\lambda \\ (N(\varphi) + h)\cos\varphi\sin\lambda \\ \left(\left(b^2/a^2\right)N(\varphi) + h\right)\sin\varphi \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix},$$

where

$$N(\varphi) = \frac{a^2}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}.$$

Here, $\begin{bmatrix} X_0 & Y_0 & Z_0 \end{bmatrix}^T$ is the offset of the origin (this is a *crude equation* ignoring possible rotation and scale issues). For the origin offset of ED50 we have *very roughly*
$X_0 = 87\,\mathrm{m}$, $Y_0 = 98\,\mathrm{m}$, $Z_0 = 121\,\mathrm{m}$ (European Petroleum Survey Group, `http://www.epsg.org/`). These must be *subtracted* going to geocentric ("WGS84") co-ordinates.

Aalto University
School of
Engineering

Classical datums (3)

A more precise transformation *for the Finnish territory* was derived by [Ollikainen, 1993]:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = (1+m) \begin{bmatrix} 1 & e_z & -e_y \\ -e_z & 1 & e_x \\ e_y & -e_x & 1 \end{bmatrix} \begin{bmatrix} (N(\varphi)+h)\cos\varphi\cos\lambda \\ (N(\varphi)+h)\cos\varphi\sin\lambda \\ \left(\left({}^{b^2}\!/_{a^2}\right) N(\varphi)+h\right)\sin\varphi \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix},$$

where now rotations and a scale factor are included. His values are

$$X_0 = 93,477 \pm 3,345\,\mathrm{m},\ Y_0 = 103,453 \pm 5,534\,\mathrm{m},\ Z_0 = 123,431 \pm 2,736\,\mathrm{m},$$

$$e_x = -0,246 \pm 0,16\,'',\ e_y = 0,109 \pm 0,106\,'',\ e_z = 0,068 \pm 0,112\,'',$$

$$m = -2,062 \pm 0,417\,\mathrm{ppm}.$$

Note the units! Convert angles to radians before use. Here, also uncertainty estimates are included.

Aalto University
School of
Engineering

An example of a classical datum: ED50 (1)

The European Datum 1950 ("ED50") was created by the Readjustment of the European Triangulation (RETrig) project of the International Association of Geodesy. An important motivation was military; see `http://en.wikipedia.org/wiki/ED50`.

The adjustment took place on the Hayford or International Ellipsoid of 1924 ($a = 6378388\,\mathrm{m}, 1/f = 297$), and as input were used the national triangulation networks of the participating countries.

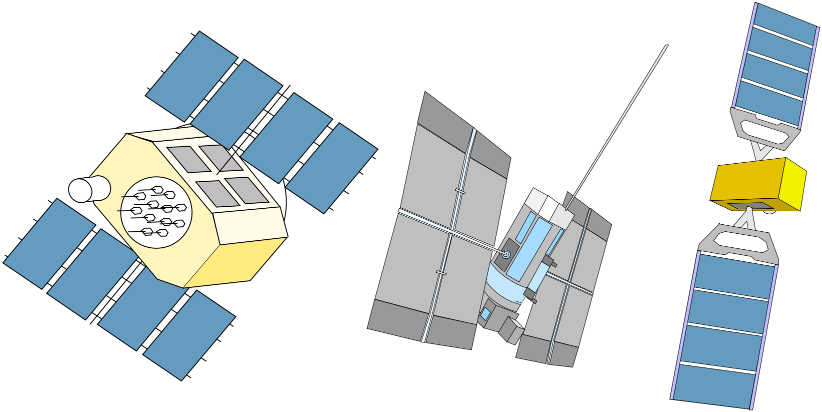
The datum point (zero point) of the network adjustment was initially the Helmert-tower on the Telegrafenberg in Potsdam; when this became impossible (located in the GDR), the Frauenkirche in Munich was chosen.

An example of a classical datum: ED50 (2)

- The adjustment of the Western European triangulation network was performed three-dimensionally, on the Hayford ellipsoid as explained. For this, besides heights above sea level of the triangulation points, also the heights *of sea level above (or below) the Hayford reference ellipsoid* in these points was needed. For this, a geoid model was developed known as the Bomford geoid, by Brigadier Guy Bomford, a British geodesist. It was computed using astronomical determinations of the direction of the plumbline at many triangulation stations throughout Europe ("astrogeodetic geoid").
- Together with the ED50 datum, also a map projection was adopted: the Universal Transverse Mercator (UTM) projection, with projection zones of 6° width. Participating countries did not typically publish their maps in this projection, but NATO did.

Aalto University
School of
Engineering

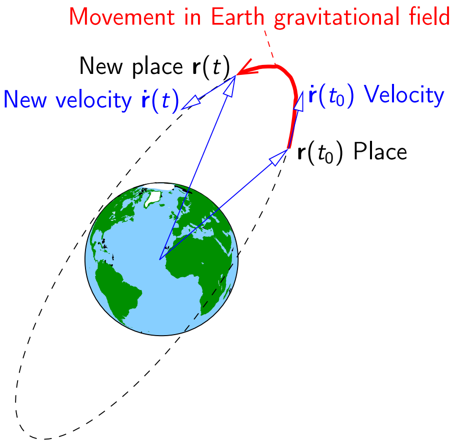
Measurement techniques: GNSS



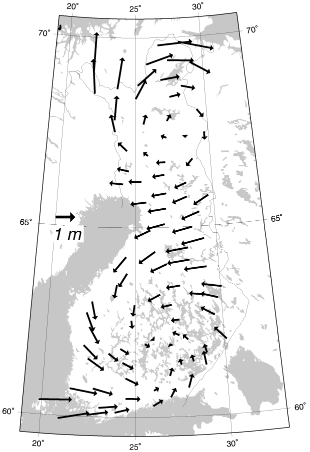
GNSS: Global Navigation Satellite Systems. Global Positioning System (GPS), Glonass, Galileo

Modern datums

Modern datums are based on space geodetic techniques, specifically GNSS. As the equations of motion used for modelling the orbits of the satellites are written in a geocentric frame – i.e., a frame with the origin at the centre of mass of the Earth – the orbits will be also in a geocentric co-ordinate frame. As these are the orbits transmitted by the satellites themselves ('broadcast ephemeris') as well as disseminated by Internet services of the international geodetic community ('precise ephemeris') this is also the reference frame geodetic network solutions will be in.



How good were the old datums?



Not very good, see picture giving deformation of old Finnish KKJ datum relative to modern EUREF-FIN datum. We know this now thanks to satellite positioning technology.

However, remember that for local applications, this kind of large-scale deformation is of little consequence: local road, bridge, harbour, industrial or residential construction projects will take their co-ordinates from local reference bench marks, which are internally consistent. The same holds for vertical reference systems.

The deformation picture shows, that the "waist" of Finland was weak in the triangulation network. In principle the use of Laplace point measurements (one in every three triangulation points) should have fixed this, but didn't.

The granite pillar (Fundamental Bench Mark) defining the N60 datum, in the yard of Helsinki Astronomical Observatory, Kaivopuisto. The height value inscribed on it reads 30.4652 m. This is the height above the zero point of the water scale attached to Katajanokka Bridge, which in 1904-1907 was 109 mm below mean sea level.

The first precise levelling of Finland used this starting point and produced a height datum called NN. In the second precise levelling of Finland, a temporary height datum N43 was created using the same starting point.

←"Love Bridge"

Height datum N60 (2)

As described in [Kääriäinen, 1966] p. 49, for defining the N60 datum, the height of the Fundamental Bench Mark was taken as 30.51376 m.

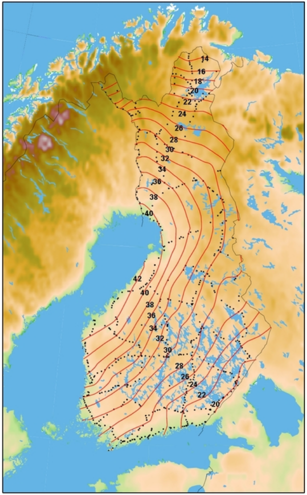
This benchmark served as starting point for the Second Precise Levelling of Finland, disseminating this height system to bench marks all over Finland, providing precise heights for infrastructure and community construction projects everywhere. In the computation of heights, post-glacial land uplift was taken into account and the 'epoch' (definition point in time) of the N60 datum is 1960.0.

Aalto University
School of
Engineering

Height datum N2000 (1)

After the Third Precise Levelling of Finland was completed, a new height datum was established for the epoch 2000.0; the old epoch 1960.0 was already so far in the past that heights in that system no longer realistically described the Finnish situation: land uplift varies from 4 mm/yr in the Helsinki area to 9 mm/yr in the neighbourhood of Oulu. Over 40 years that produced a 20 cm tilt.

The plot (© FGI) gives the differences N2000 - N60. $\longrightarrow$



Aalto University
School of Engineering

Height datum N2000 (2)

N.A.P.

The starting point is again a granite pillar, this time in Metsähovi, with the height value inscribed on it. This time, the zero level is conceptually that of *Amsterdam*, the N.A.P. (Normaal Amsterdams Peil) datum `http://en.wikipedia.org/wiki/Amsterdam_Ordnance_Datum`

- However, Amsterdam has no open connection with the sea anymore... currently realized otherwise
- The levelling connection Amsterdam-Finland is very long, and not very precise
- Sea level in Helsinki is some 30 cm above that in Amsterdam, due to *sea-surface topography* (Baltic Sea salinity related).

Map projections (1)

We often wish to project the Earth surface onto a flat map plane, e.g., for printing, or display on a screen. For this purpose there exist *map projections*.

It is not possible to project without deformation. We classify projections according to what they *do not deform*:

- *Conformal* projections preserve shape: little circles on the Earth surface are mapped as little circles, littles squares as little squares, and all *angles* are preserved. Mercator is a conformal projection, as is the stereographic projection. General-purpose maps such as topographic maps will use a conformal projection
- *Equivalent* (area-preserving) projections: these preserve (except for applying a nominal scale factor) the areas and area ratios. This should always be used for *thematic maps* showing population or other areal densities, as using any other projection will lead to misrepresentation. The Mercator projection is especially bad
- *Equidistant* projections: these preserve (certain) distances between points.

Map projections (2)

About the word projection: this is actually a very bad word

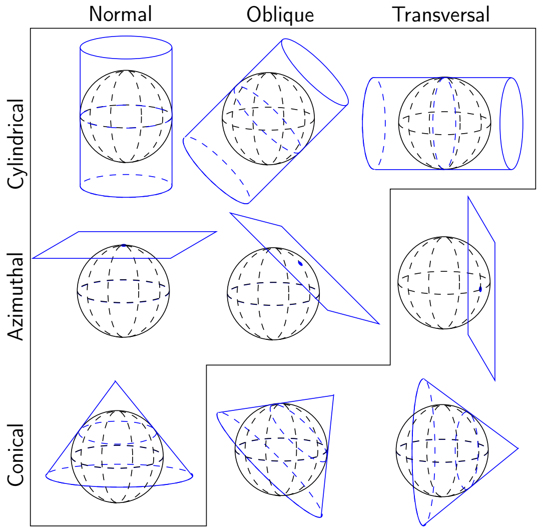
1. Hardly any of the popular cartographic projections are ‘lamp projections’; only stereographic is close
2. We speak of *cylindrical*, *conical*, and *azimuthal* (plane) projections geometrically, but this is actually not how projections are constructed in real life. On the ellipsoid, the construction is purely mathematical.

This is not how map projections work. . .

Aalto University
School of
Engineering

Map projections (3)

That being said... here is a classification. The projections outside the frame are not useful (though occasionally used).



Map projections (4)

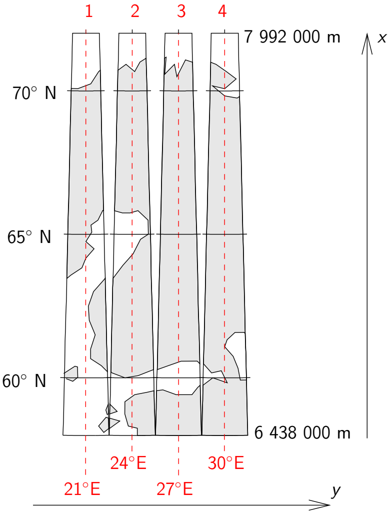
Mathematically we can use the following classification:

1. *Cylindrical projections* are projections that have a stationary scale on a *geodesic* of the ellipsoid. The geodesic can be the equator (normal orientation) or a meridian (transversal orientation)
2. *Conical projections* are projections that have a stationary scale on a *parallel*. The only sensible orientation is normal
3. *Azimuthal projections* have a stationary scale only *at a point*. The point can be a pole (normal orientation) or some other point (oblique orientation).

Practically speaking, of the conformal projections, the only useful ones are Mercator (normal cylindrical), Gauss-Krüger and UTM (both transversal cylindrical), Lambert conformal (normal conical) and stereographic (oblique, or occasionally normal, azimuthal). In Finland, of these only Gauss-Krüger and UTM are used for topographic mapping

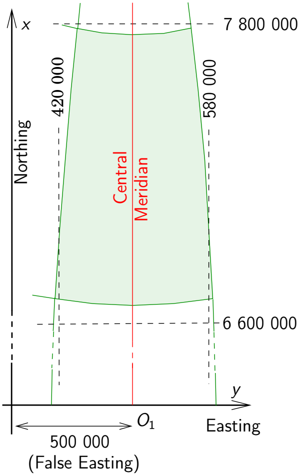
Geodetic location datums: KKJ (1)

- The KKJ system ('Map Grid Co-ordinate System') was created in 1970 [Parm, 1988] to be used in the production of topographic and other maps for the Finnish territory. The picture shows the four zones on which most of Finland is projected.
- KKJ is based in principle on the European ED50 datum, though in its construction a *trick* was used, see below.
- The map projection used is Gauss-Krüger, a transversal Mercator projection. There are six central meridians for projection used numbered 0-5 : $18°, 21°, 24°, 27°, 30°$ and $33°$ East, with the $27°$ central meridian (zone 3) also serving to map all of Finland at small scale.



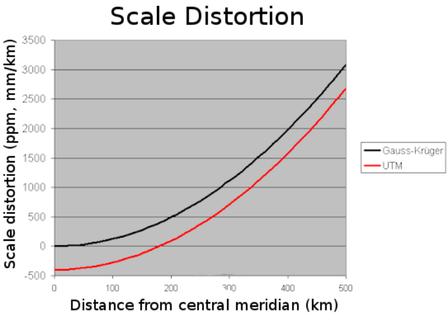
KKJ (2)

- The North co-ordinate x is the distance along the Hayford ellipsoid; the East co-ordinate y is the distance from the central meridian, augmented with a false Easting of 500,000 m, to avoid negative co-ordinate numbers. The projection zone number is prepended to y.
- After computing (x, y) from the ED50 co-ordinates (φ, λ) using the Gauss-Krüger projection formulas [Hirvonen, 1972] on the Hayford ellipsoid, *a further two-dimensional Helmert transformation was applied* to make the KKJ co-ordinates close to those of the pre-existing VVJ ("Old State System") map co-ordinates that were already in widespread use. This is the "trick" referred to above. Compatibility between KKJ and VVJ is at the few-metres level.



Scale distortion

The scale distortion for Gauss-Krüger is given by $m = 12.29 \cdot 10^{-15}\mathrm{m}^{-2} \cdot (y - y_0)^2$, where y is the Easting (to be given in metres) and y_0 the false Easting.
For the UTM projection, $400 \cdot 10^{-6}$ must be subtracted from this.



At Finnish latitudes a degree in longitude corresponds to about 50 km.

(Wikipedia)

Aalto University School of Engineering

Geodetic location datums: EUREF-FIN (1)

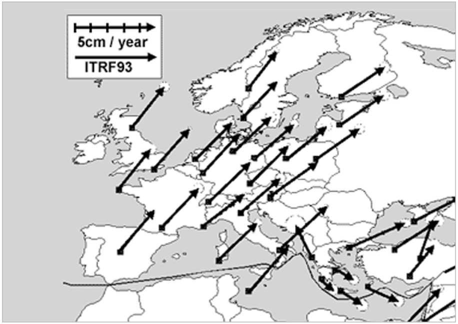
EUREF-FIN is an outflow of the work of the EUREF subcommission of IAG (International Association of Geodesy).

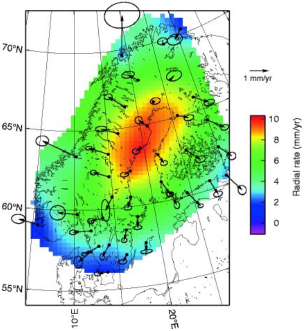
In 1989, the European co-ordinate reference system ETRS89 was defined thusly:

- At the start of 1989 (i.e., 1989.0), it coincides with the ITRS (International Terrestrial Reference System), which has been repeatedly realized using large sets of space geodetic measurements
- After 1989, the reference system is assumed to move along with the rigid part of the Eurasian tectonic plate.

EUREF-FIN (2)

This is done to make the co-ordinates in the realizations of ETRS89 independent of time.





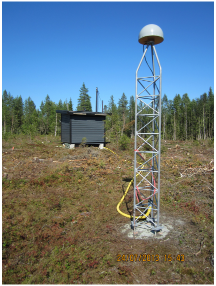
However, the post-glacial land uplift (Glacial Isostatic Adjustment, GIA) in mainly the Fennoscandian area is not taken into account in the definition, and therefore these co-ordinates continue to be time dependent.

Aalto University
School of
Engineering

EUREF-FIN (3)

Since then, many *realizations* of both ITRS and ETRS89 have been produced, by adjusting observational data sets connecting the reference system to the physical Earth. These realizations, or *reference frames*, have typical names like ITRF96, ITRF2000, EUREF89 etc.

EUREF-FIN is the Finnish national realization of the co-ordinate reference system ETRS89. Produced originally by Finnish Geodetic Institute 1996-1999; includes Finnish permanent geodetic network FinnRef, then 12 stations.



New FinnRef™ stations

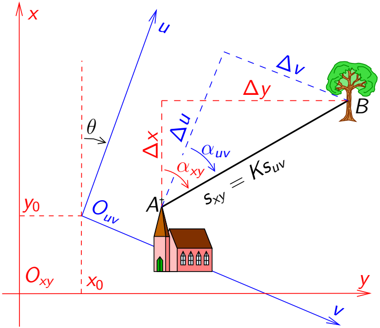
Aalto University
School of Engineering

Map projections for EUREF-FIN

Contrary to KKJ, which used only the Gauss-Krüger projection in zones three degrees apart, the modern EUREF-FIN based map projections are

- UTM, (Universal Transverse Mercator) with zones six degrees apart, used for small-scale maps; central meridian is 27° East (UTM zone 35; the count starts from the date line in the Pacific). The scale at the central meridian is 0.9996 times nominal, the projection ellipsoid used is a *geocentric* GRS80 ellipsoid. The system is called ETRS-TM35FIN. UTM projection grid lines from zones 34 and 36 are overprinted in red where relevant.
- Gauss-Krüger in zones one degree apart, for large-scale maps. Designation: ETRS-GK*nn*, with *nn* the Eastern longitude. Scale on central meridian is precisely nominal, projection ellipsoid geocentric GRS80. The scale is close to correct everywhere, allowing transfer of co-ordinates to and use in CAD/CAM systems for construction projects.

Datum transformations: Helmert



General Helmert or similarity transformation: $x, y \rightarrow u, v$

Four parameters: rotation, scaling, translation vector

Two common points will *fix* the transformation

$$\begin{bmatrix} x \\ y \end{bmatrix} = K \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}.$$

Aalto University School of Engineering

Helmert in three dimensions (1)

In 3D, Helmert similarity transformation is (seven parameters):

$$\begin{bmatrix} x^{(2)} \\ y^{(2)} \\ z^{(2)} \end{bmatrix} = (1+m) \begin{bmatrix} 1 & e_z & -e_y \\ -e_z & 1 & e_x \\ e_y & -e_x & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ y^{(1)} \\ z^{(1)} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}.$$

Re-write and linearize:

$$\begin{bmatrix} x^{(2)} - x^{(1)} \\ y^{(2)} - y^{(1)} \\ z^{(2)} - z^{(1)} \end{bmatrix} = \begin{bmatrix} m & e_z & -e_y \\ -e_z & m & e_x \\ e_y & -e_x & m \end{bmatrix} \begin{bmatrix} x^0 \\ y^0 \\ z^0 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix},$$

where all parameters in the matrix, m, e_x, e_y, e_z are *small*.

Aalto University
School of
Engineering

Helmert in 3D (2)

Rewriting now yields

$$\begin{bmatrix} x_i^{(2)} - x_i^{(1)} \\ y_i^{(2)} - y_i^{(1)} \\ z_i^{(2)} - z_i^{(1)} \end{bmatrix} = \begin{bmatrix} x_i^0 & 0 & -z_i^0 & +y_i^0 & 1 & 0 & 0 \\ y_i^0 & +z_i^0 & 0 & -x_i^0 & 0 & 1 & 0 \\ z_i^0 & -y_i^0 & +x_i^0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m \\ e_x \\ e_y \\ e_z \\ t_x \\ t_y \\ t_z \end{bmatrix}.$$

Here, all seven Helmert transformation parameters are in one vector on the right.

We added an index i to all co-ordinates, a point number. The above has the form of a set of *observation equations*: we can resolve the Helmert parameters if enough common points (in practice, three) between the (1) and (2) systems are given.

Solving the Helmert parameters (1)

We return to the 2D situation:

$$\begin{bmatrix} x_i^{(2)} - x_i^{(1)} \\ y_i^{(2)} - y_i^{(1)} \end{bmatrix} = \begin{bmatrix} +x_{i,0} & -y_{i,0} & 1 & 0 \\ +y_{i,0} & +x_{i,0} & 0 & 1 \end{bmatrix} \begin{bmatrix} m \\ \theta \\ x_0 \\ y_0 \end{bmatrix}.$$

Taken as a set of observation eqs., the *design matrix* is

$$A = \begin{bmatrix} +x_i & -y_i & 1 & 0 \\ +y_i & +x_i & 0 & 1 \end{bmatrix}.$$

Assuming the data points i given in both systems to be equally precise – mean error σ_0 – and uncorrelated, we may compute the *normal matrix*:

$$N = A^T A = \begin{bmatrix} \sum \left(x^2 + y^2\right) & 0 & \sum x & \sum y \\ 0 & \sum \left(x^2 + y^2\right) & -\sum y & \sum x \\ \sum x & -\sum y & n & 0 \\ \sum y & \sum x & 0 & n \end{bmatrix}.$$

Solving the Helmert parameters (2)

If we now assume that $\sum x = \sum y = 0$, i.e., we have *centre-of-mass co-ordinates*, we obtain

$$N = \begin{bmatrix} \sum\left(x^2+y^2\right) & 0 & 0 & 0 \\ 0 & \sum\left(x^2+y^2\right) & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{bmatrix}$$

and the uncertainties (variances) of the parameters are

$$\begin{aligned} \sigma_m^2 &= \sigma_\theta^2 = \frac{\sigma_0^2}{\sum\left(x^2+y^2\right)}, \\ \sigma_{x_0}^2 &= \sigma_{y_0}^2 = \frac{\sigma_0^2}{n}. \end{aligned}$$

Solving the Helmert parameters (3)

From this again follow the co-ordinate uncertainties due to the uncertain Helmert parameters:

$$\sigma_{x_i}^2 = \sigma_{y_i}^2 = \sigma_0^2 \left(\frac{x_i^2 + y_i^2}{\sum_{\mathrm{points}} (x^2 + y^2)} + \frac{1}{n_{\mathrm{points}}} \right).$$

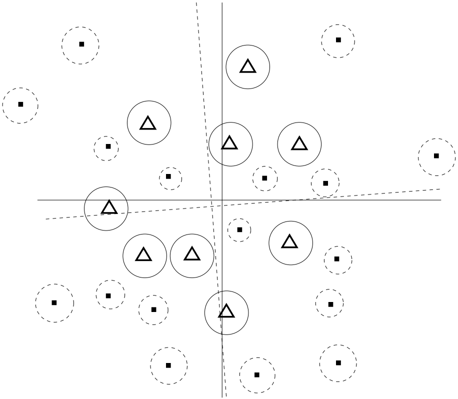
So: to get the best solution, you need to maximize the expression

$$\sum_{\mathrm{points}} \left(x^2 + y^2 \right),$$

i.e., *spread out your points over the area as much as possible*.

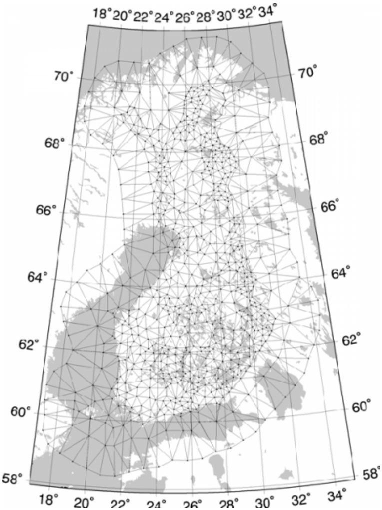
Aalto University
School of
Engineering

Solving the Helmert parameters (4)



This figure depicts the point errors for this case: drawn circles are uncertainties of common points used, dashed circles, transformation induced point uncertainties.

Datum transformations: affine transformation



The affine transformation is a six-parameter transformation that, contrary to the Helmert transformation, does not preserve shapes. In Finland, it is used together with a triangular network covering the country to realize a precise transformation from the old KKJ system to the modern ETRS-TM35FIN map projection system. Every triangle has its own affine transformation; as the transformation is bilinear, it can be made continuous over triangle boundaries. Published by the National Land Survey.

Aalto University
School of
Engineering

The Finnish situation (1)

In the municipal situation the co-ordinate issue is creating much employment and head-scratching. Not just the re-measurement of base networks, but also deriving and applying transformations to large bodies of geolocated geographic information, including parcels that owners would *hate* to see getting smaller. . .

This won't change soon

Aalto University
School of
Engineering

The Finnish situation (2)

Also, plenty of scope for science...

- Left, studying sea level and its changes, Hanko
- Right, calibrating airborne laser scanning using an area's road network, Klaukkala

Aalto University
School of
Engineering

Summary

Today we discussed the following issues:

- Technologies for precise geolocation
  - Traditional vs. GNSS
- The concept of geodetic datum
- Modern geodetic co-ordinate reference systems and their realization, reference frames
  - ETRS89, EUREF-FIN
- Map projections used in visualizing geospatial data
  - Gauss-Krüger, UTM and their properties
- Transformation between geodetic datums
  - In detail: Helmert or similarity transformation
- Geodetic co-ordinates in Finnish society
  - The co-ordinate modernization issue

Aalto University
School of
Engineering

Bibliography

Hirvonen, R. (1972).
Matemaattinen geodesia.
Teknillisen Korkeakoulun Ylioppilaskunta, Otaniemi.

Kääriäinen, E. (1966).
The Second Levelling of Finland in 1935-1955.
Publication 61, Finnish Geodetic Institute, Helsinki.

Ollikainen, M. (1993).
GPS-koordinaattien muuntaminen Kartastokoordinaateiksi.
Tiedote 8, Geodeettinen laitos, Helsinki.

Parm, T. (1988).
Kansallisen koordinaattijärjestelmän luominen Suomessa.
Maanmittaus, 63(1).