$$p(y_k \mid \theta) = N(y_k \mid \overset{\text{mean}}{H_k \theta}, \overset{\text{variance}}{\sigma^2}) \qquad p(a \mid b) = \frac{p(b \mid a)\, p(a)}{p(b)}$$

$$p(\theta) = N(\theta \mid m_0, P_0)$$

$$p(\theta \mid y_{1:T}) = \frac{\left[\prod_k p(y_k \mid \theta)\right] p(\theta)}{\left(\int \left[\prod_{k=1}^{T} p(y_k \mid \theta)\right] p(\theta)\, d\theta\right)}$$

$$\propto \left[\prod_k p(y_k \mid \theta)\right] p(\theta)$$

$$= \left[\prod_k N(y_k \mid H_k \theta, \sigma^2)\right] N(\theta \mid m_0, P_0)$$

$$= \left[\prod_k \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}(y_k - H_k\theta)^2/\sigma^2\right)\right] \cdot |2\pi P_0|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\theta - m_0)^T P_0^{-1}(\theta - m_0)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2} \underbrace{\sum_k (y_k - H_k\theta)^2} - \frac{1}{2}(\theta - m_0)^T P_0^{-1}(\theta - m_0)\right)$$

$$H = \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_T \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix}$$

$$\Rightarrow \sum_k (y_k - H_k\theta)^2 = (y - H\theta)^T (y - H\theta)$$

$$= \exp\Big(-\frac{1}{2\sigma^2}(y - H\theta)^T(y - H\theta) \underbrace{- \frac{1}{2}(\theta - m_0)^T P_0^{-1}(\theta - m_0)}\Big)$$

$$\propto N(\theta \mid m_T, P_T)$$

$$\frac{\partial}{\partial\theta}\left[\frac{1}{2\sigma^2}(y - H\theta)^T(y - H\theta) + \frac{1}{2}(\theta - m_0)P_0^{-1}\right.$$

$$(\theta - m_0)\Big] = 0$$

$$\frac{1}{\sigma^2} H^T (H\theta - y) + P_0^{-1}(\theta - m_0) = 0$$

$$\left(\frac{1}{\sigma^2} H^T H + P_0^{-1}\right)\theta = \frac{1}{\sigma^2} H^T y + P_0^{-1} m_0$$

$\vdots$

(without minus) $$\frac{\partial^2}{\partial\theta\partial\theta^T}(\cdot) = \frac{1}{\sigma^2} H^T H + P_0^{-1} = P_T^{-1}$$

$\dots$

$$p(a, b \mid c) = p(a \mid b, c)\, p(b \mid c)$$

$$p(\theta_k, \theta_{k-1} \mid y_{1:k-1})$$

$$= \underbrace{p(\theta_k \mid \theta_{k-1}, y_{1:k-1})}_{\text{BECAUSE MARKOV}}\, p(\underbrace{\theta_{k-1}}_{b} \mid \underbrace{y_{1:k-1}}_{c})$$

$$= p(\theta_k \mid \theta_{k-1})\, p(\theta_{k-1} \mid y_{1:k-1})$$

C - K:

$$\int p(\theta_k \mid \theta_{k-1})\, p(\theta_{k-1} \mid y_{1:k-1})\, d\theta_{k-1}$$

$$= \int \frac{1}{|2\pi Q|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\theta_k - \theta_{k-1}) Q^{-1} (\theta_k - \theta_{k-1})\right) \frac{1}{|2\pi P_{k-1}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\theta_{k-1} - m_{k-1})^T P_{k-1}^{-1} (\theta_{k-1} - m_{k-1})\right) d\theta_{k-1}$$

$$= \frac{1}{|2\pi Q|^{\frac{1}{2}}} \frac{1}{|2\pi P_{k-1}|^{\frac{1}{2}}}$$

$$\cdot \int \exp\left(-\frac{1}{2}(\theta_k - \theta_{k-1})^T Q^{-1} (\theta_k - Q_{k-1}) - \frac{1}{2}(\theta_{k-1} - m_{k-1})^T P_{k-1}^{-1} (\theta_{k-1} - m_{k-1})\right) d\theta_k$$

$$= \frac{1}{|2\pi(P^-_{k-1} + Q)|} \exp\left(-\frac{1}{2}(\theta_k - m_{k-1})^T (P_{k-1} + Q)^{-1} (\theta_k - m_{k-1})\right)$$

$$= N(\theta_k \mid m_{k-1}, P_{k-1} + Q)$$