ELEC-E3530

Integrated Analog Systems L

Contents

RC Integrator

Current equations

$$i_1 = \frac{V_{in}}{R_1}$$

$$i_2 = -C_2 \frac{dV_{out}}{dt}$$

$$i_1 = i_2$$

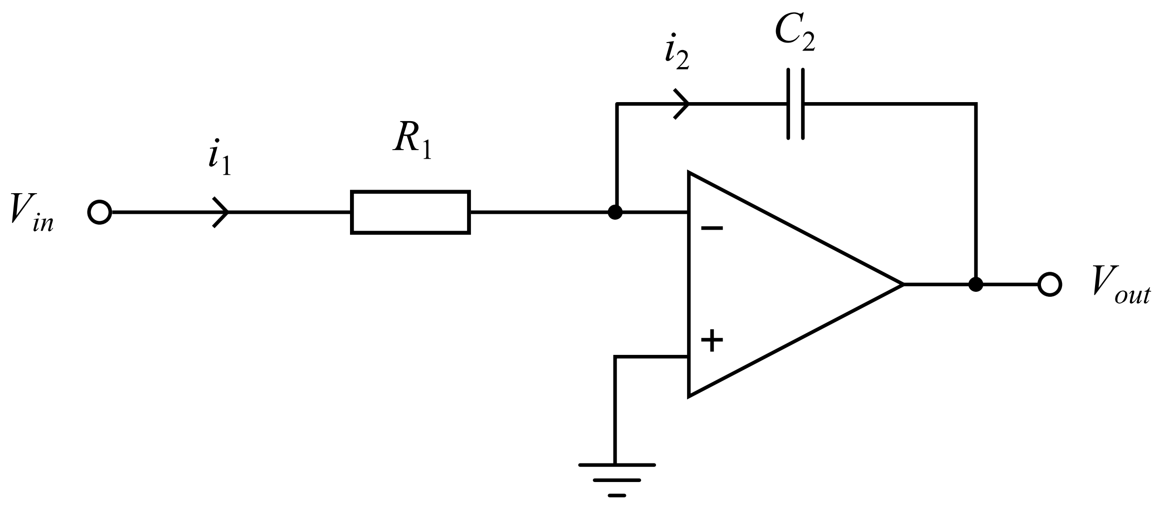
solve for V_{OUT}

$$\Rightarrow V_{OUT} = -\frac{1}{R_1 C_2} \int V_{in}(t) dt$$

Laplace transformation

$$V_{out}(s) = \frac{-V_{in}(s)}{sR_1 C_2}$$

$$\boxed{H(s) = \frac{V_{out}}{V_{in}} = -\frac{1}{sR_1 C_2}}$$



Component Type	Range of Values	Relative Accuracy	Temperature Coefficient	Voltage Coefficient	Absolute Accuracy
Poly/poly capacitor	0.3 - 0.4 fF/μ^2	0.06%	25 ppm/°C	−50 ppm/V	20%
MOS capacitor	0.35 - 0.5 fF/μ^2	0.06%	25 ppm/°C	−20 ppm/V	10%
Diffused resistor	10 - 100 ohms/sq.	2% (5 μm width)	1500 ppm/°C	220 ppm/V	35%
Poly resistor	30 - 200 ohms/sq.	2% (5 μm width)	1500 ppm/°C	100 ppm/V	30%
Ion impl. resistor	0.5 - 2k ohms/sq.	1% (5 μm width)	400 ppm/°C	800 ppm/V	5%
P-well resistor	1 - 1k ohms/sq.	2%	8000 ppm/°C	10k ppm/V	40%
Pinch resistor	5 - 20k ohms/sq.	10%	10k ppm/°C	20k ppm/V	50%

Switched capacitor principle

Charge equations

phase $\Phi_1 : Q_{C1} = V_1 C$

phase $\Phi_2 : Q_{C2} = V_2 C$

charge transfer from node ① to node ②

$\Rightarrow \Delta Q = Q_{C1} - Q_{C2} = (V_1 - V_2)C$

One charge packet per clock period

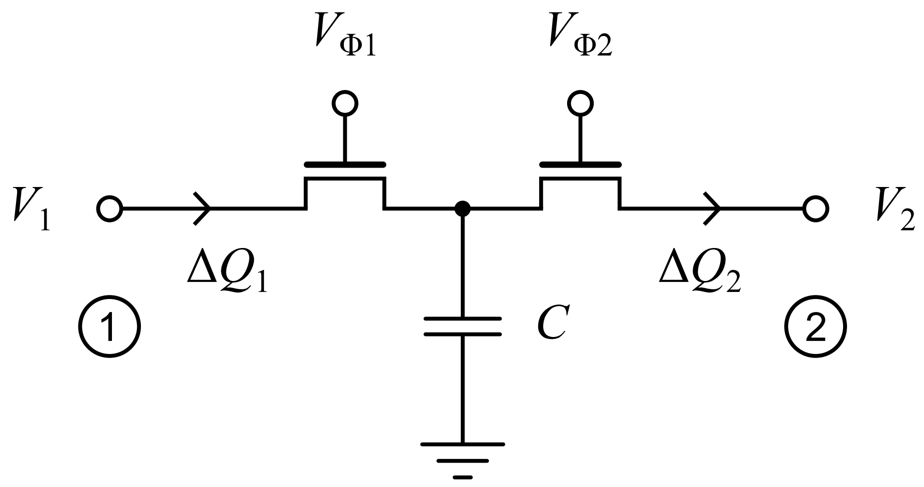
$\Rightarrow$ discrete qurrent equation

$$\Rightarrow \underline{i} = \frac{\Delta Q}{T} = \frac{C(V_1 - V_2)}{T} = \frac{C}{T}(V_1 - V_2)$$

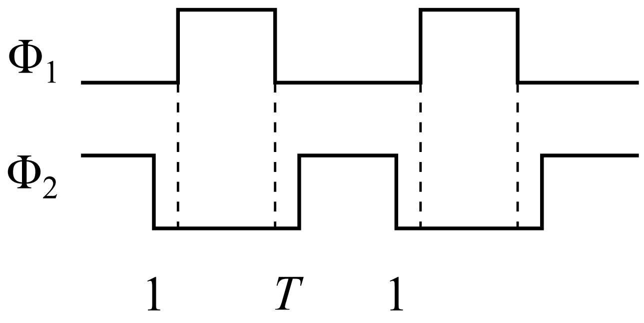
according to Ohm's law $V = RI$

equivalent discrete resistor

$$\Rightarrow R_{SC} \triangleq \frac{T}{C} = \frac{1}{f_{clk} C}$$



Non-overlapping clock signals

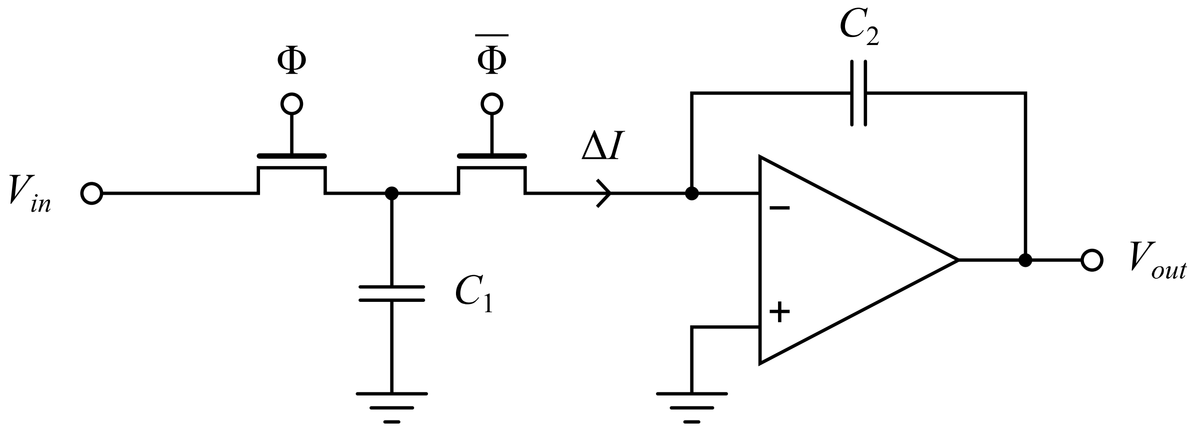


Switched capacitor filters

Charge equations:

phase Φ: $Q_1 \rightarrow C_1 \cdot V_{in}$

phase $\overline{\Phi}$: $Q_1 \rightarrow 0$ $\Delta Q_2 = Q_1 = C_1 \cdot V_{in}$



Charge transferred per clock cycle

$$I = \frac{\Delta Q_2}{\Delta t} = \frac{C_1 V_{in}}{T_0}$$

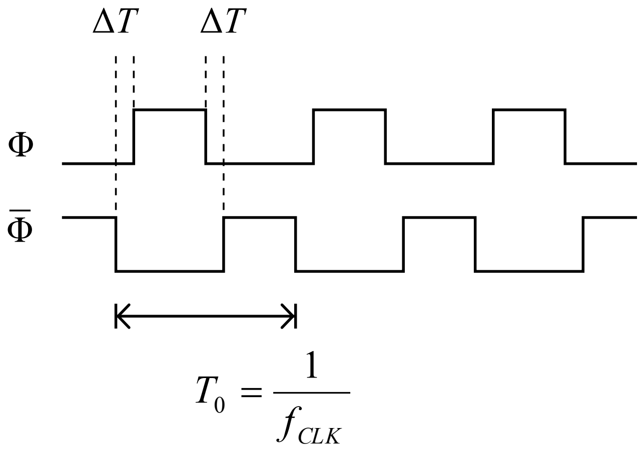
equivalent resistor

$$\Rightarrow R_{SW} = \frac{V_{in}}{I} = \frac{T_0}{C_1} = \frac{1}{C_1 f_{CLK}}$$

For RC integrator:

$$H(s) = \frac{V_{out}}{V_{in}} = -\frac{1}{sR_{in}C_2}$$

Non-overlapping clocks

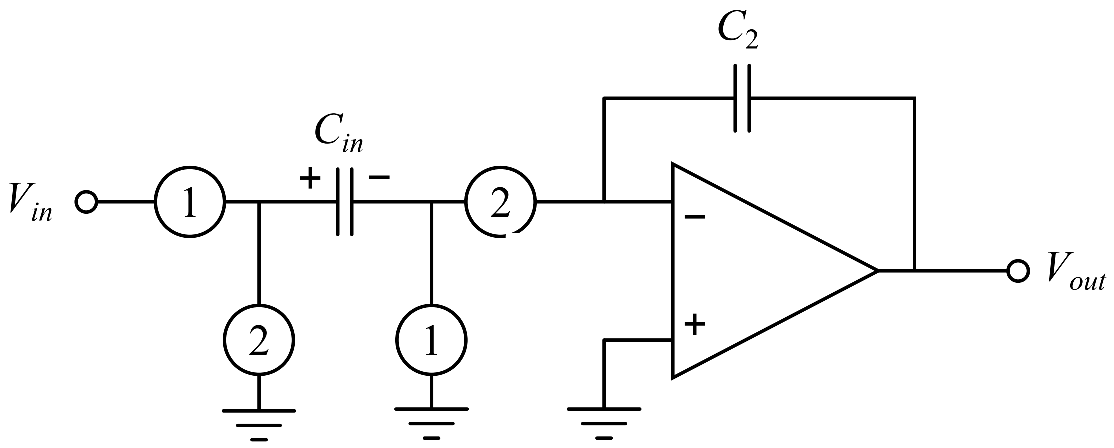


Replace R_{in} with R_{sw} for SC integrator:

$$H(s) = \frac{V_{out}}{V_{in}} = -\frac{1}{sR_{sw}C_2} = -\frac{1}{S\frac{C_2}{C_1 f_{CLK}}}$$

$$\Rightarrow \boxed{\tau_{SC} = \frac{C_2}{C_1 f_{CLK}}}$$

SC Integrator

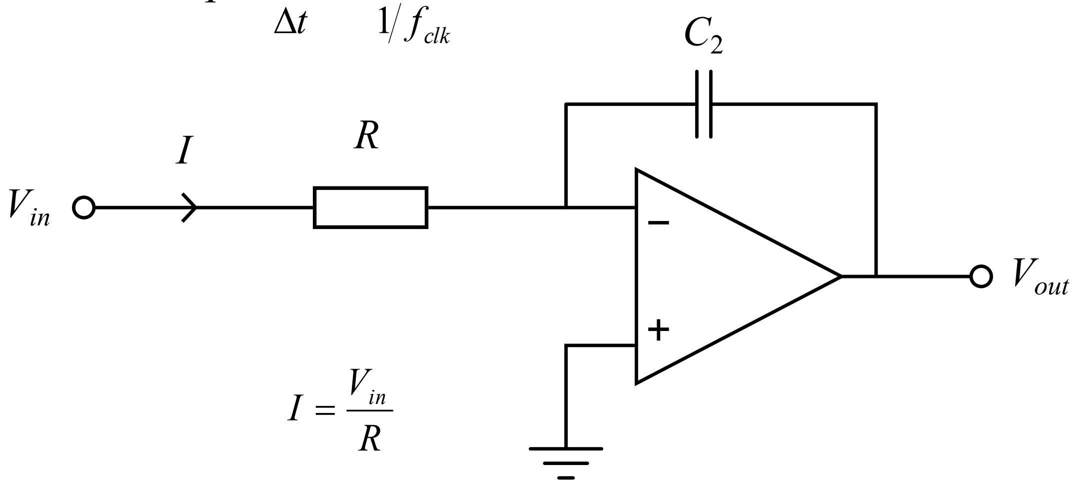


SC resistor:

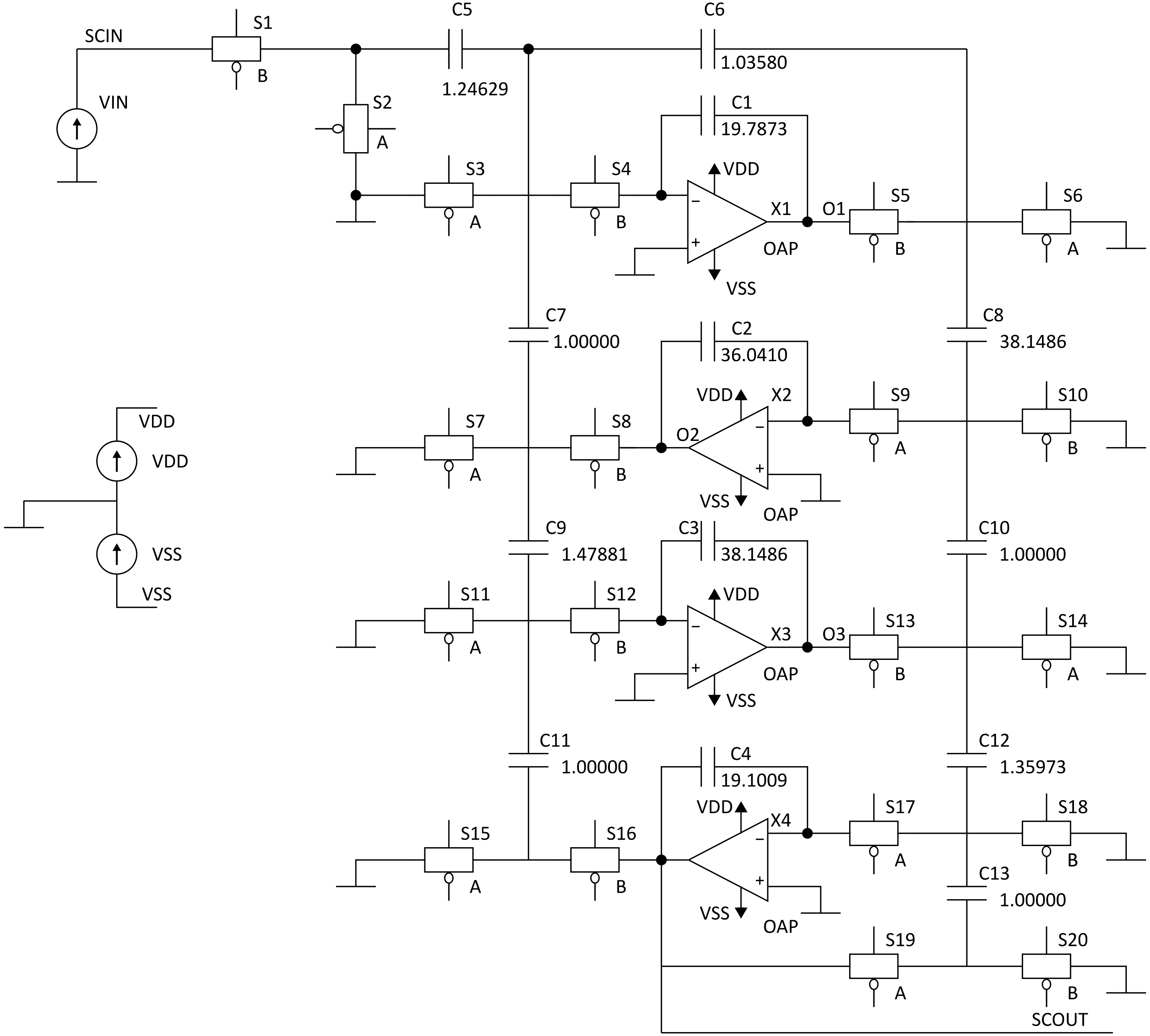
$$R_{SC} = \frac{1}{fC_{in}}$$

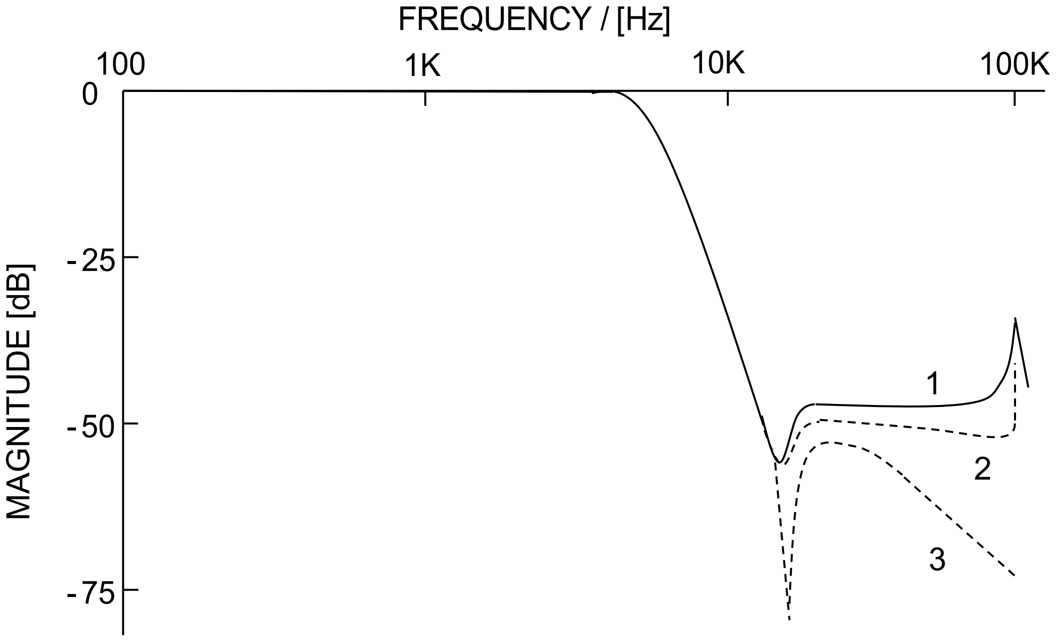
$$\boxed{\tau = RC_2 = \frac{C_2}{f_{clk}C_{in}}}$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{C_{in} \cdot V_{in}}{1/f_{clk}}$$



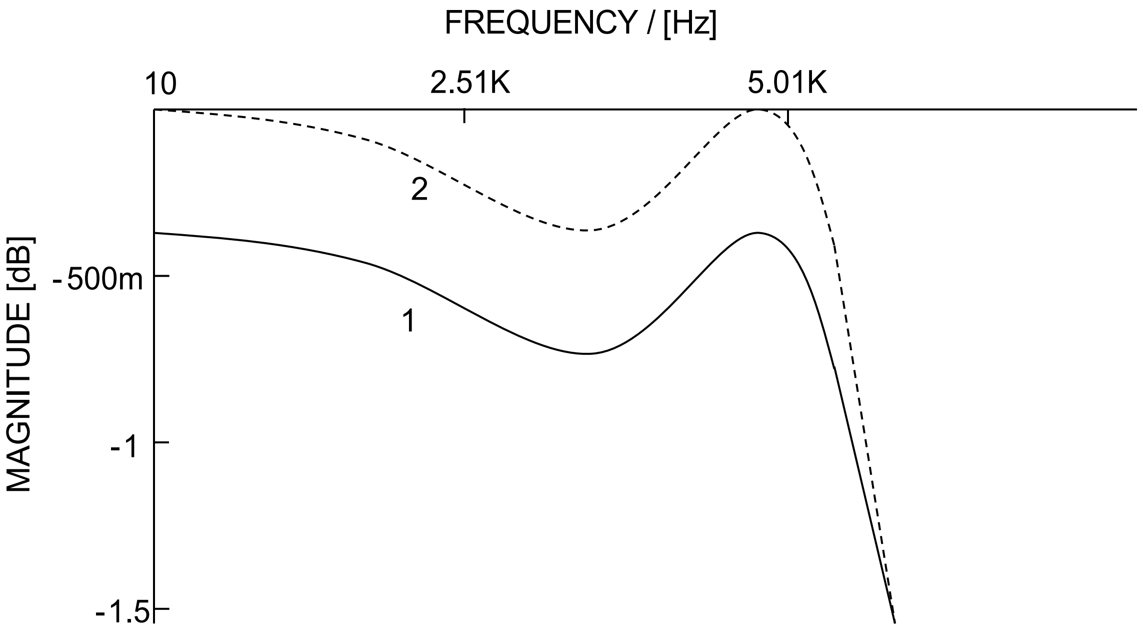
$$I = \frac{V_{in}}{R}$$


SCIN
VIN
S1
B
C5
1.24629
C6
1.03580
S2
A
C1
19.7873
S3
A
S4
B
VDD
X1
OAP
VSS
O1
S5
B
S6
A
C7
1.00000
C8
38.1486
C2
36.0410
VDD
X2
O2
S7
A
S8
B
S9
A
S10
B
VSS
OAP
VDD
VDD
VSS
VSS
C9
1.47881
C3
38.1486
C10
1.00000
S11
A
S12
B
VDD
X3
O3
OAP
VSS
S13
B
S14
A
C11
1.00000
C4
19.1009
C12
1.35973
VDD
X4
S15
A
S16
B
S17
A
S18
B
VSS
OAP
C13
1.00000
S19
A
S20
B
SCOUT



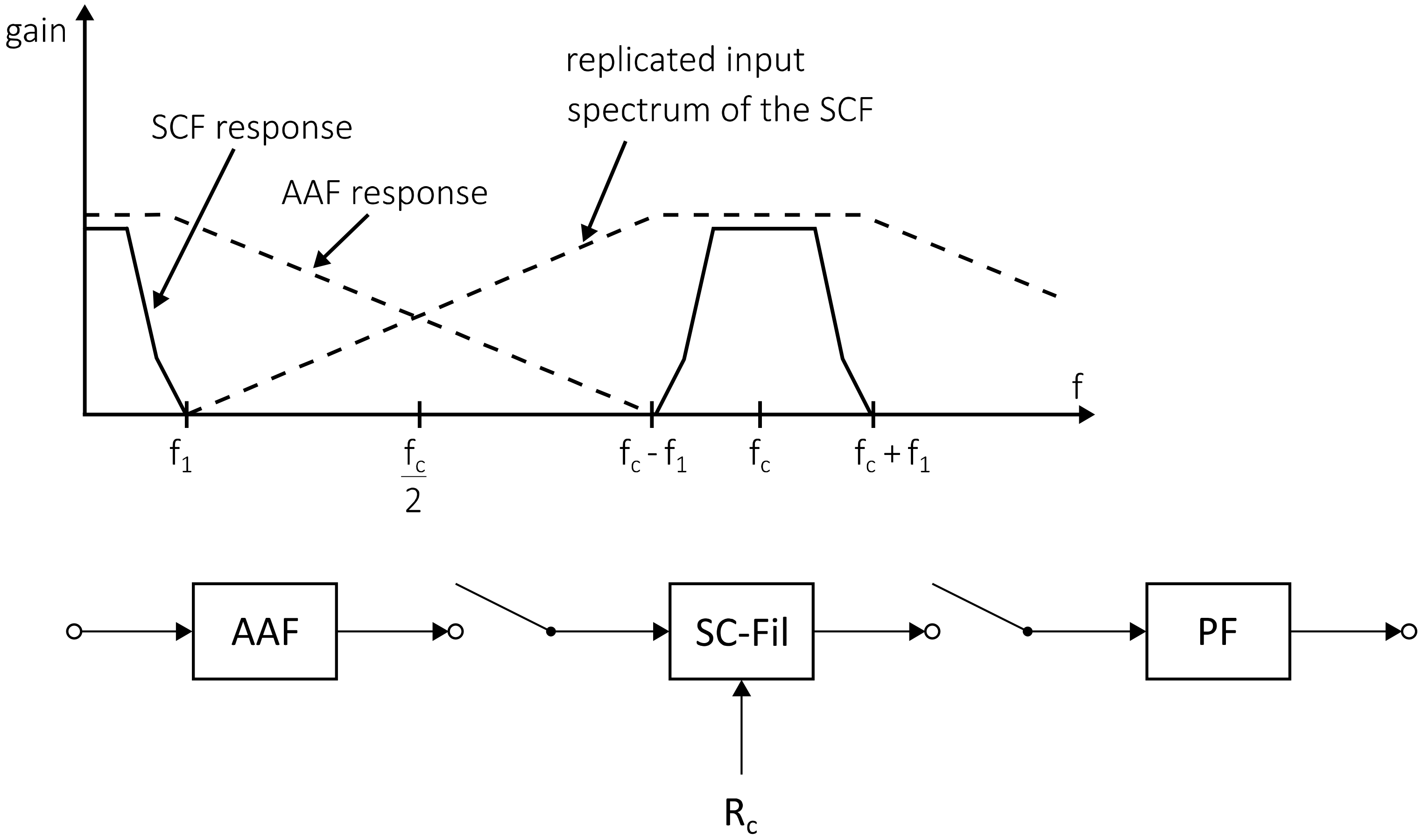
The frequency response of 4th order SC low-pass filter

(1. measured, 2. simulated (SWAP), 3. LC prototype(calc.))

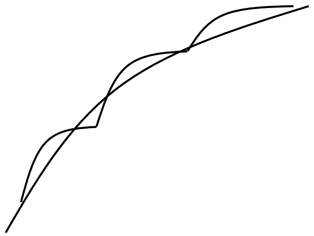


The pass-band frequency response

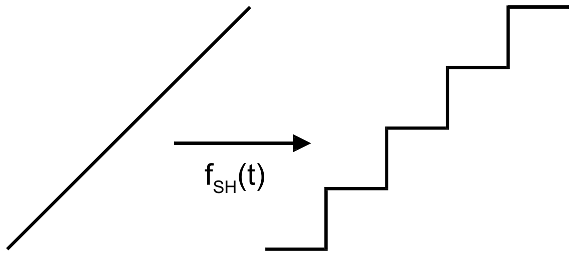
(1. measured, 2. LC prototype(calc.))



The block diagram of the sampled data system and the relative frequency responses of anti-aliasing (AAF), SC and smoothing filters (SCF)



Sample and Hold



Sample and hold

$$f_{SH}(t) = \sum_{n=0}^{\infty} f(nT)\left[u(t-nT) - u(t-nT-T)\right]$$

Laplace transformation

$$F_{SH}(s) = \frac{1-e^{-sT}}{s} \sum_{n=0}^{\infty} f(nT) e^{-snT} \qquad z^{-1} = e^{-sT}$$

Sample and hold function:

$$\Rightarrow H_{SH}(s) = \frac{1-e^{-sT}}{s}$$

Insert s = jω

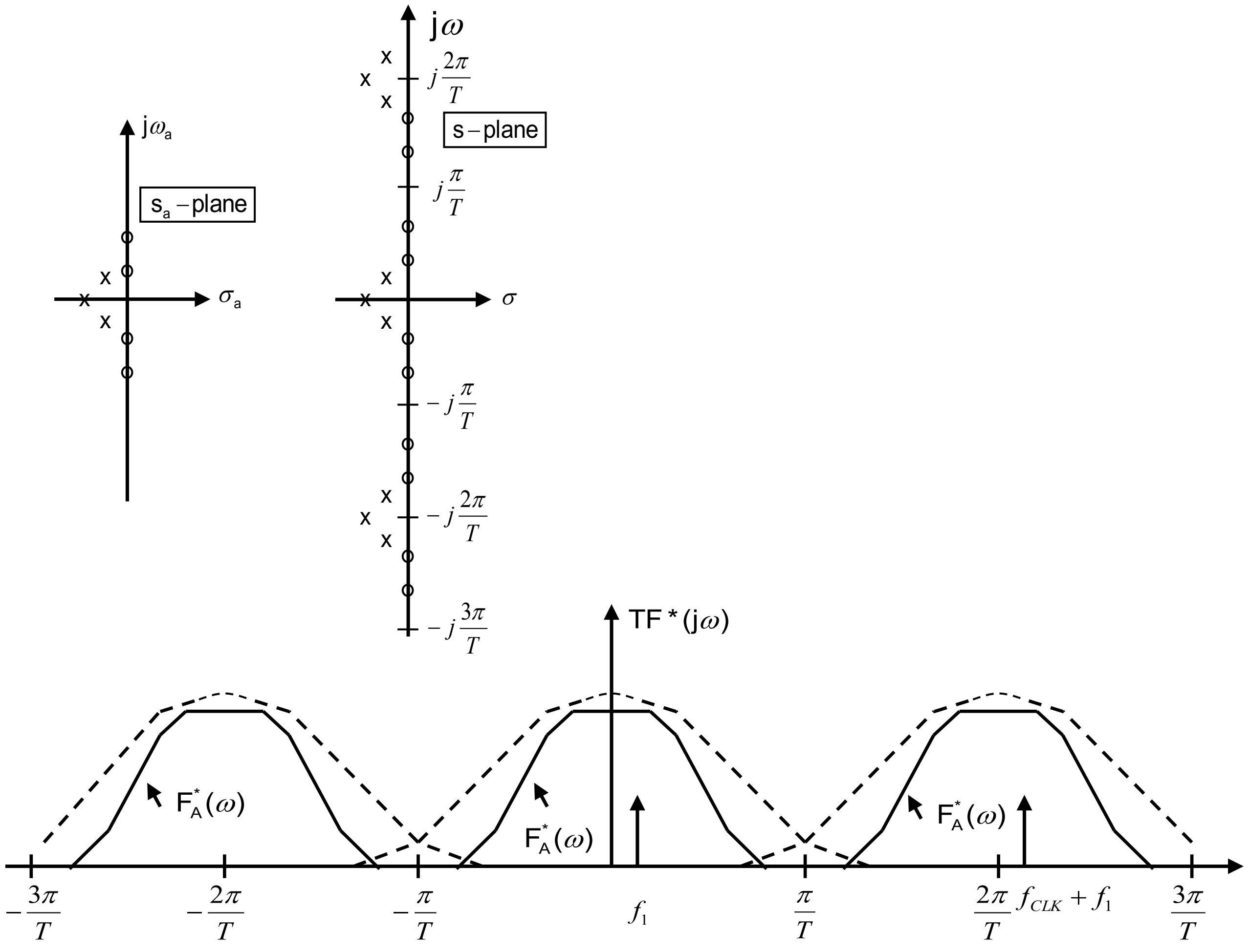
$$H_{SH}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} = Te^{-j\omega T/2} \frac{\sin(\omega T/2)}{\omega T/2}$$

$$F_{SH} = e^{-j\omega T/2} \frac{\sin(\omega T/2)}{\omega T/2} \sum_{k=-\infty}^{\infty} F\left(j\omega - jk\frac{2\pi}{T}\right)$$

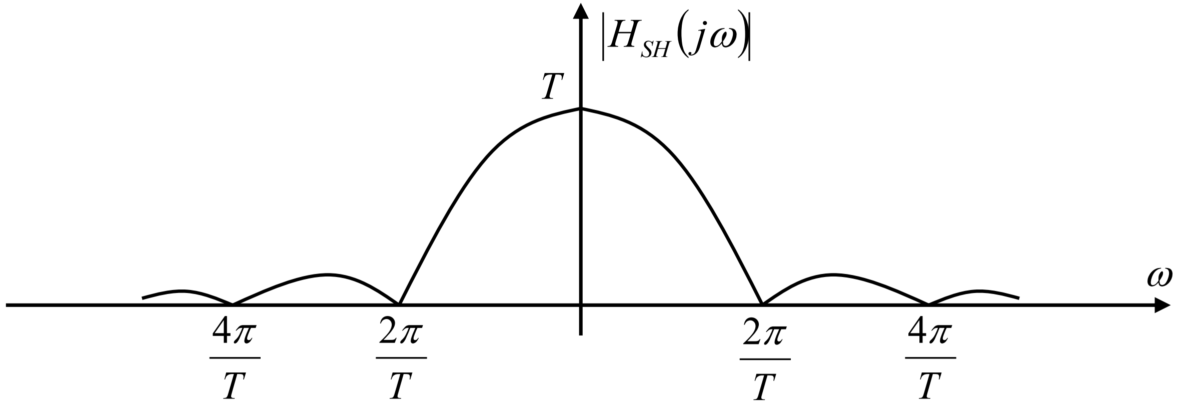
$s = j\omega \Rightarrow F_{SH}(j\omega) \cong \sum_{n=0}^{\infty} f(nT) e^{-jn\omega T}$ spectrum is periodical!

$\Rightarrow$ aliasing! $\Rightarrow \frac{\pi}{T} > \omega_A$ (Nyqvist criterium)

$\omega = \frac{\pi}{T} =$ Nyqvist frequency

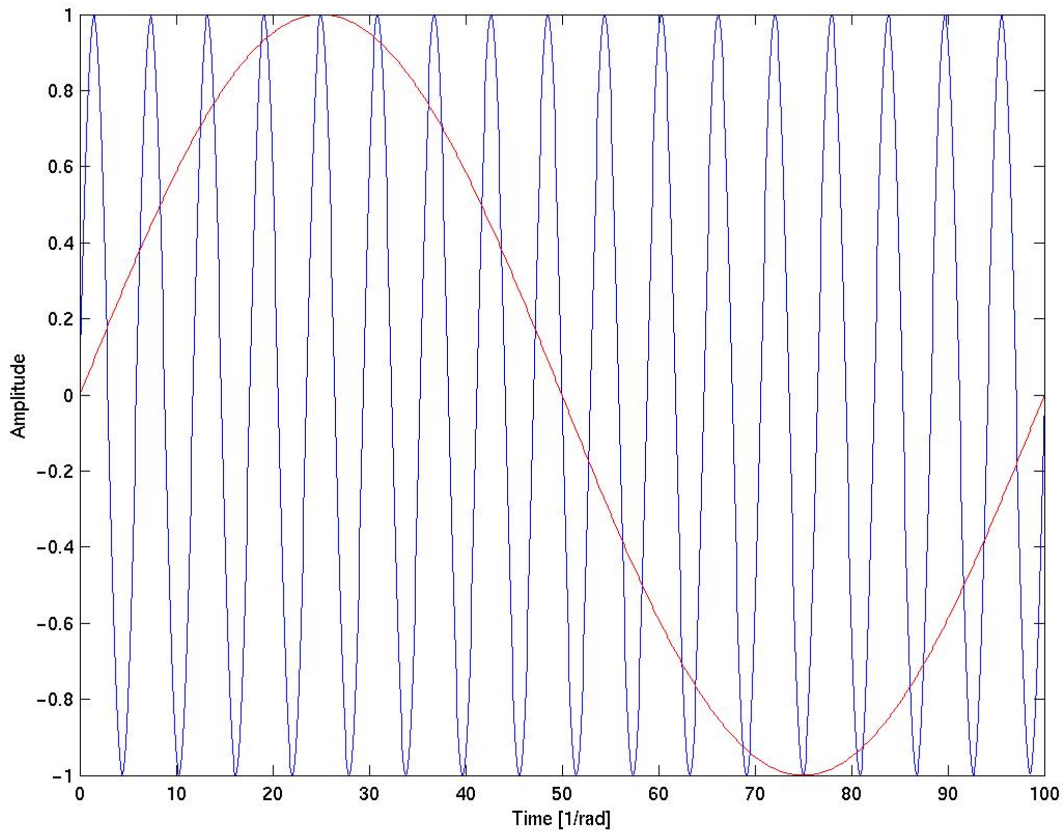


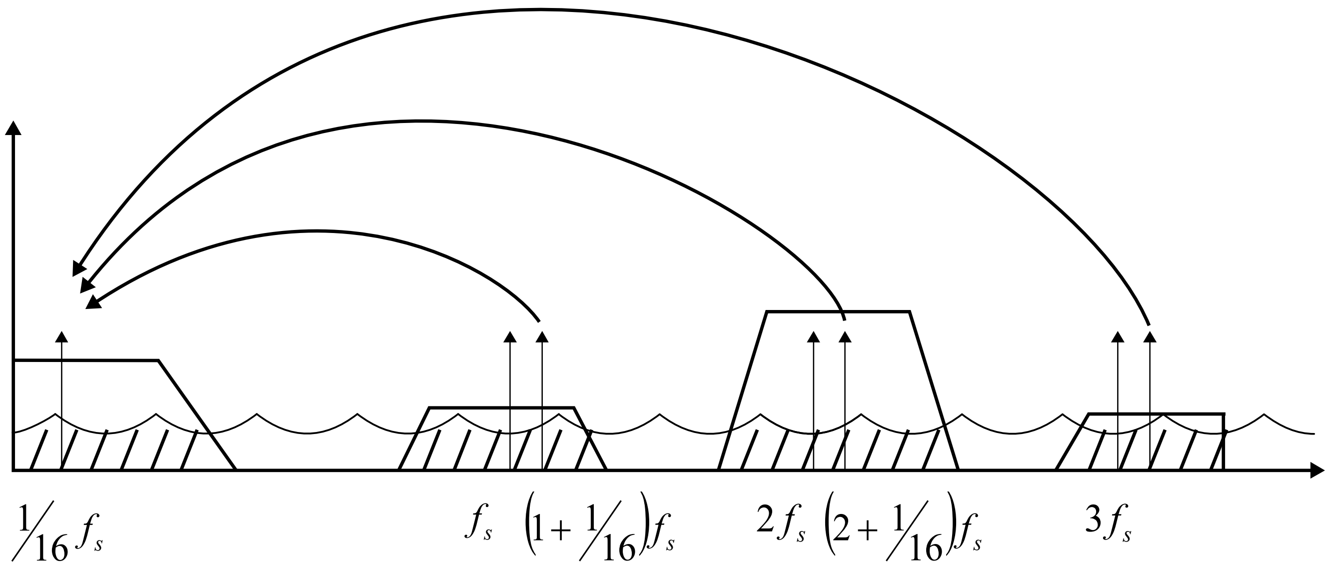
The poles of the continuous-time and sampled circuit, and the frequency response of the sampled data circuit



The frequency response of S/H-function

Sampling a tone at (1+1/16)fs

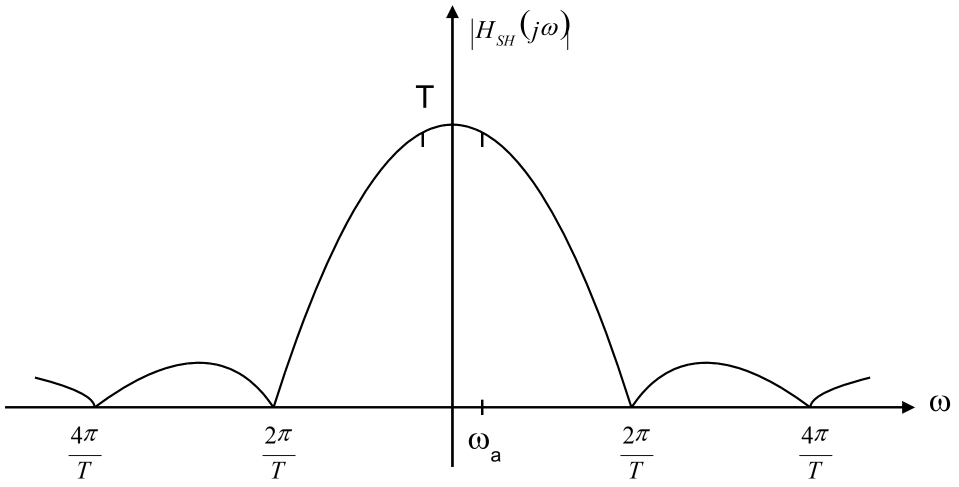




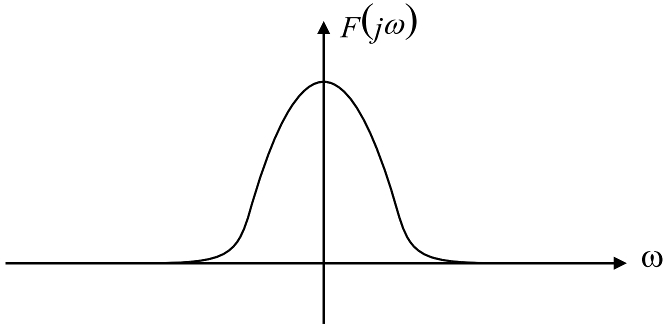
$1/16\, f_s$
f_s
$(1+1/16)f_s$
$2f_s$
$(2+1/16)f_s$
$3f_s$

Effect of sample and hold on signal spectrum

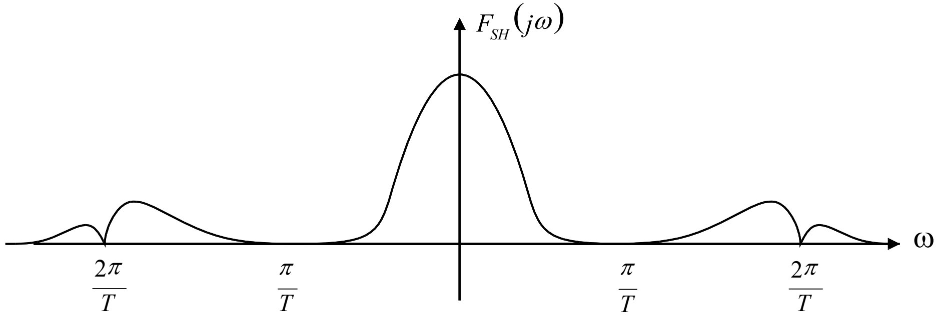
The amplitude response $|H_{SH}(j\omega)| = 2\sin(\omega T/2)/\omega$



(a) a continuous-time signal f(t)



(b) a sample and hold signal obtained from f(t)



Principle of stability

Stability in s-plane:

pole: $s_i = a_i + jb_i$

impulse response: $e^{a_i t}\cos b_i t\ ,\ e^{a_i t}\sin b_i t$

$\Rightarrow$ stable, if $a_i < 0$ i.e. the poles are in the left-hand half-plane

$a_i = 0 \Rightarrow$ poles are located at jω-axis

Stability in z-plane:

transformation from s-plane to z-plane:

$z = e^{sT}$

mapping of s-plane pole into z-plane

$\Rightarrow z_i = e^{a_1 T + jb_i T} = e^{a_i T} e^{jb_i T}$

$|z_i| = e^{a_i T}$

$\Rightarrow$ stable, if $|z_i| < 1$

i.e. the poles are inside the unit circle

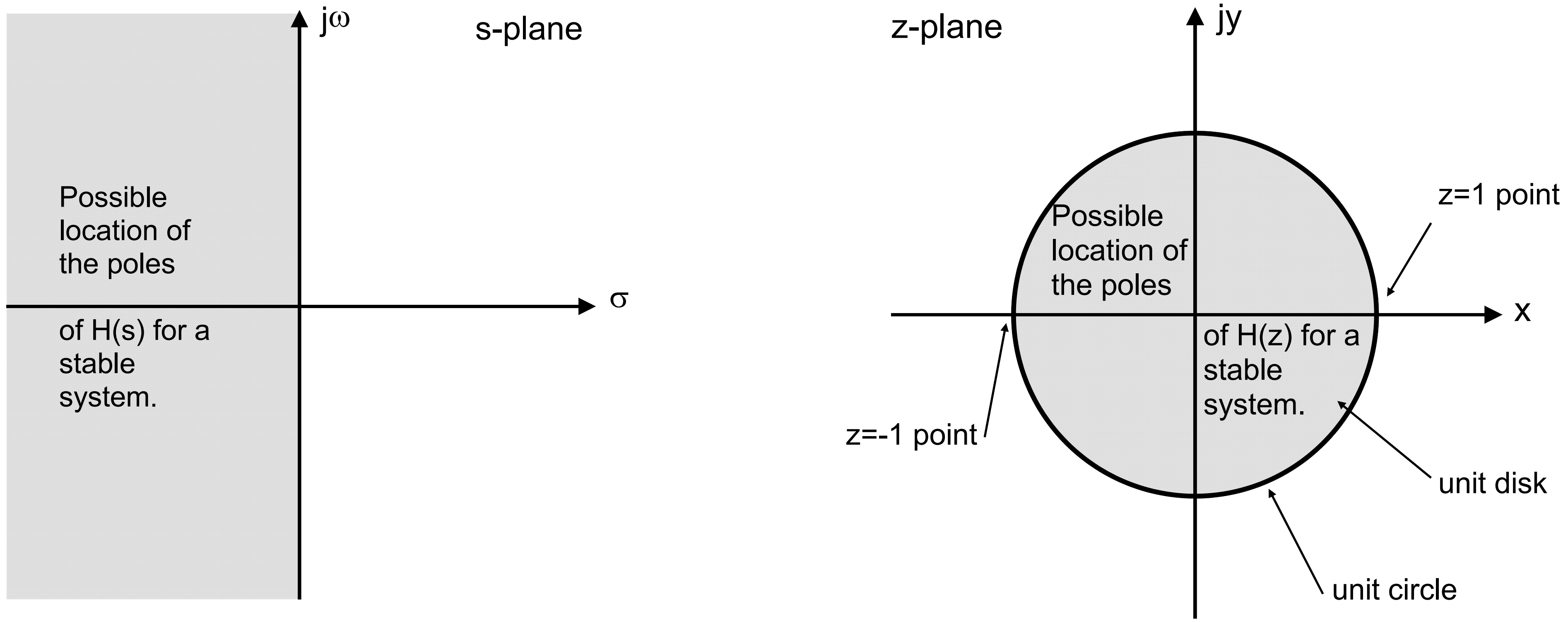
$|z_i| = 1 \Leftrightarrow a_i = 0$

i.e. jω -axis will be mapped on the unit circle

- left-hand s-plane will be mapped inside the unit circle
- right-hand s-plane will be mapped outside the unit circle

$z = e^{sT}$ is periodic!

Stable pole locations in s- and z-planes



Z-transformation methods

Mapping from the s-plane to z-plane

H(s) is rational

H(z) is rational

$\Rightarrow s = f(z)$ has to be rational

$H(z) = H_a(s) \quad s = f(z)$

Requirements for f(z):

1. $f(z)$ has to be rational
2. $|z| = 1 \Rightarrow f(z)$ is pure imaginary

   i.e. $f(e^{j\omega T}) = j\omega_a$
3. $|z| < 1 \Rightarrow \mathrm{Re}\{f(z)\} < 0$

Integrator – the basic building block in filters

$$\frac{dx_C(t)}{dt} = g_i(t)$$

Laplace:

$$s_a x_i(s_a) = G_i(s_a) \Rightarrow x_i(s_a) = \frac{1}{s} G_i(s_a)$$

Difference equation:

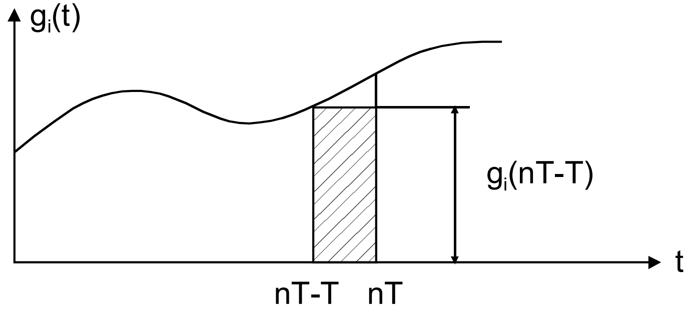
$$\int_{nT-T}^{nT} \frac{dx_i(t)}{dt} dt =$$

$$x_i(nT) - x_i(nT - T) = \int_{nT-T}^{nT} g_i(t) dt$$

Explore different numerical integration methods (Euler, trapezoidal, LDI)

- do they lead to rational transfer function
- do they lead to stable transfer function

Forward Euler



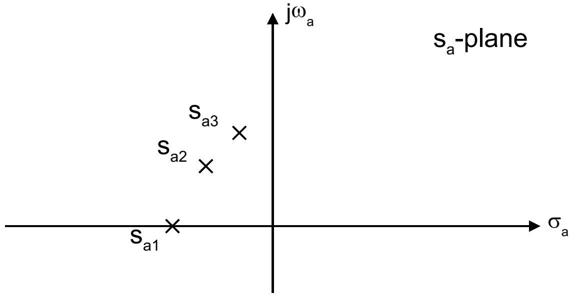
Existing function value is used to calculate the integral for the next time step:

$$x_i(nT) - x_i(nT-T) = \int_{nT-T}^{nT} g_i(t)dt \cong T g_i(nT-T)$$

$$x_i(nT) - x_i(nT-T) = T g_i(nT-T)$$

After applying z-transformation:

$$x_i(z) - z^{-1}x_i(z) = Tz^{-1}G_i(z)$$

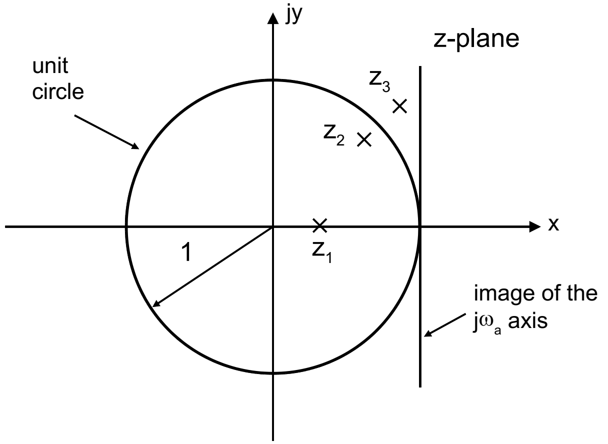


solve the transfer function

$$\frac{z-1}{T}x_i(z) = G_i(z) \Rightarrow x_i(z) = \frac{T}{z-1}G_i(z)$$

Thus Forward Euler transformation is

$$s_a = f(z) = \frac{z-1}{T}$$ "this is rational"



Check for the stability:
mapping of jω-axis into z-plane

$$z = s_aT + 1 = 1 + j\omega_aT \Rightarrow \text{unstabile!}$$

Straight - line outside unit circle

$|z| \approx 1$, when $|\omega_aT| << 1 \Leftrightarrow f_a << f_{clk}$

Backward Euler

New function value is used to calculate the integral for the next time step:

$$x_i(nT) - x_i(nT - T) = \int_{nT-T}^{nT} g_i(t)dt \cong Tg_i(nT)$$

$$x_i(nT) - x_i(nT - T) \cong Tg_i(nT)$$

Applying z-transformation:

$$\frac{1 - z^{-1}}{T} x_i(z) = G_i(z)$$

Thus Backward Euler transformation is:

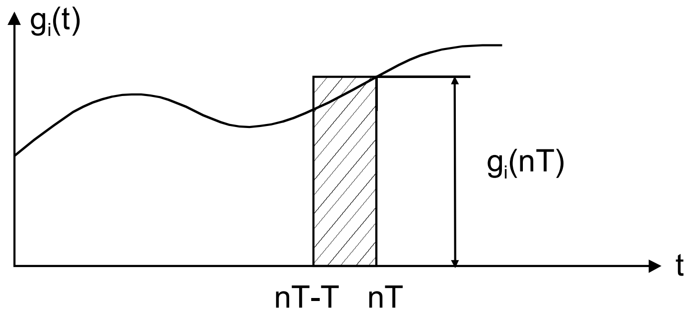
$$\Rightarrow s_a = f(z) = \frac{1 - z^{-1}}{T} \quad \text{"this is rational"}$$

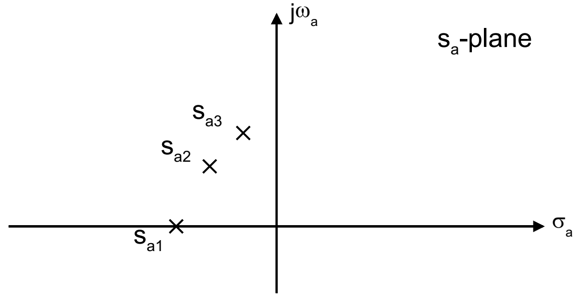
Check for stability:
mapping of jω-axis into z-plane

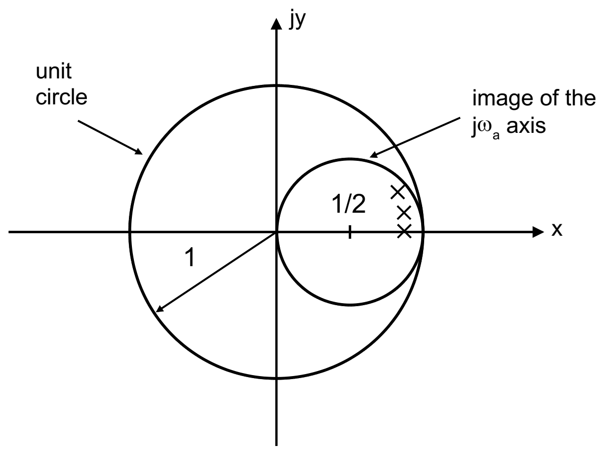
$$z = \frac{1 + j\omega_a T}{1 + \omega_a^2 T^2}$$

$$\Rightarrow \left|z\right| = \frac{1}{2}$$

half-circle inside the unit circle
⇒ stabile (distortion due to compressed pole locations)

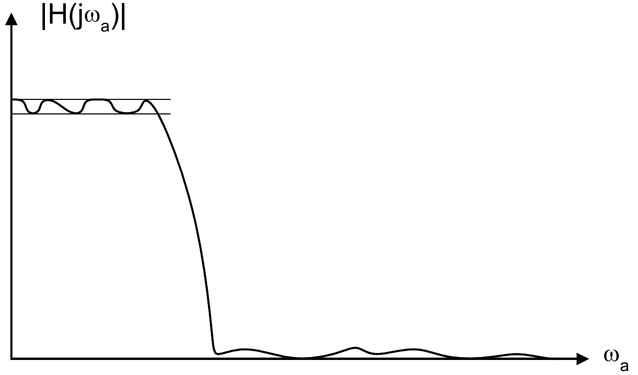




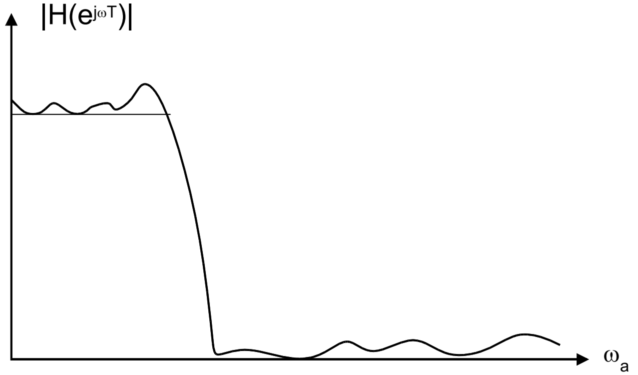


Effect of frequency distortion on low-pass filter frequency response

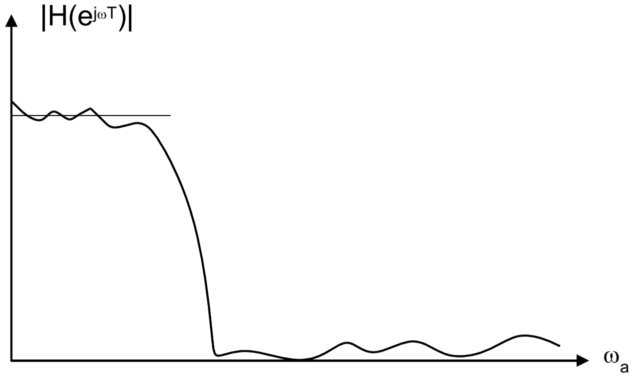
(a) continuous-time filter response with flat (equal-ripple) passband and stopband



(b) sampled-data response with peaking passband and deteriorated stopband, obtained by forward-Euler mapping



(c) response of filter obtained by the backward-Euler mapping

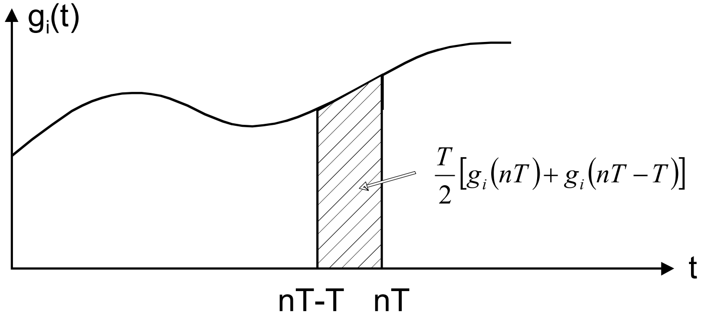


Bilinear (trapezoidal)

Both existing and new function value are used to calculate the integral for the next time step:

$$x_i(nT) - x_i(nT-T) = \int_{nT-T}^{nT} g_i(t)dt = \frac{T}{2}\left[g_i(nT-T) + g_i(nT)\right]$$

$$x_i(nT) - x_i(nT-T) \cong \frac{T}{2}\left[g_i(nT-T) + g_i(nT)\right]$$

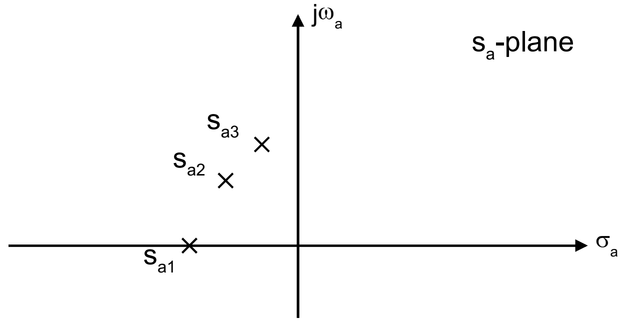


Apply z-transformation:

$$\frac{2}{T}\frac{z-1}{z+1}x_1(z) = G_i(z)$$

This gives bilinear transformation as:

$$s_a = \frac{2}{T}\frac{z-1}{z+1}$$ 'this is rational'



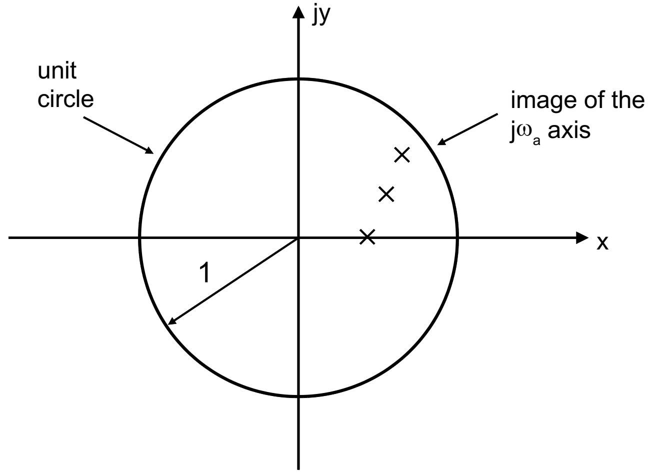
Check for stability: mapping of jω-axis into z-plane:

$$z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}} \Rightarrow |z| = 1 \qquad (s = j\omega_a)$$

jω-axis is mapped on the unit circle

$$s_a = \sigma_a + j\omega_a, \sigma_a < 0 \Rightarrow |z| < 1$$

-left-plane s-pole is mapped inside the unit circle
⇒ stabile!



Frequency distortion occurs due to the rational approximation:

Insert $z = e^{j\omega T}$ into bilinear transformation

$$\Rightarrow j\omega_a = \frac{2}{T}\frac{e^{j\omega T}-1}{e^{j\omega T}+1} \Rightarrow \omega_a = \frac{2}{T}\tan\frac{\omega T}{2}$$

Poles and zeros are moved ⇒ frequency distortion
Distortion can be compensated with prewarping

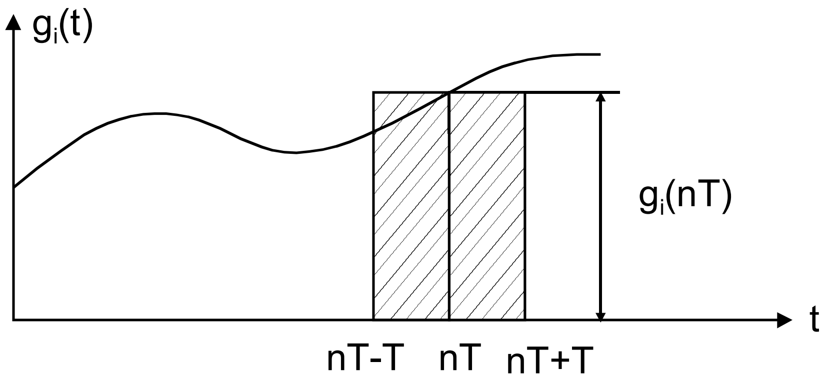
$$\omega = \frac{2}{T}\tan^{-1}\frac{\omega_a T}{2} \qquad \omega << \frac{2}{T} = f_{clk} \Rightarrow \omega_a \approx \omega$$

LDI (midpoint)

Existing function value is used to calculate the integral over past and new time step:

$$\int_{nT-T}^{nT+T} g_i(t)\,dt \cong 2Tg_i(nT)$$

$$x_i(nT+T) - x_i(nT-T) = 2Tg_i(nT)$$



Apply z-transformation

$$zx_i(z) - z^{-1}x_i(z) = 2TG_i(z)$$

$$\frac{z^2-1}{2Tz}x_i(z) = G_i(z)$$

LDI transformation:

$$s_a = f(z) = \frac{z^2-1}{2Tz}$$

Check for stability:
mapping of the jω-axis into z-plane

$$\Rightarrow z_{1,2} = s_aT \pm \sqrt{(s_aT)^2+1}$$

$$s = j\omega_a \Rightarrow |z| = 1$$

Frequency distortion:

$$z = e^{j\omega T} \Rightarrow \omega_a = \frac{1}{T}\sin\omega T$$

$$\omega T << 1 \Rightarrow f << f_{CLK}$$

prewarping:

$$\Rightarrow \omega_a = \frac{1}{T}\sin\omega T \approx \frac{1}{T}\sin\omega T = \omega$$

jω-axis is mapped on the unit circle!
Two poles: one inside the unit circle and the other is outside the unit circle!
The inside one is selected for the implementation, because it quarantees stability.

LDI transformation

$$s \to \frac{1}{T}\left(z^{\frac{1}{2}} - z^{-\frac{1}{2}}\right)$$

$$H(z) = \pm \frac{1}{Rx_i}\frac{z^{-\frac{1}{2}}}{1 - z^{-1}}$$

$$z = e^{j\omega T}$$

$$H\left(e^{j\omega T}\right) = \pm \frac{1}{j\omega Rx_i}\frac{\frac{\omega T}{2}}{\sin\frac{\omega T}{2}}$$

$$\Rightarrow m(\omega) = \frac{\frac{\omega T}{2}}{\sin\frac{\omega T}{2}} - 1$$

$$p(\omega) = 0$$

Prewarping :

$$\omega' = \frac{2}{T}\sin\frac{\omega T}{2}$$