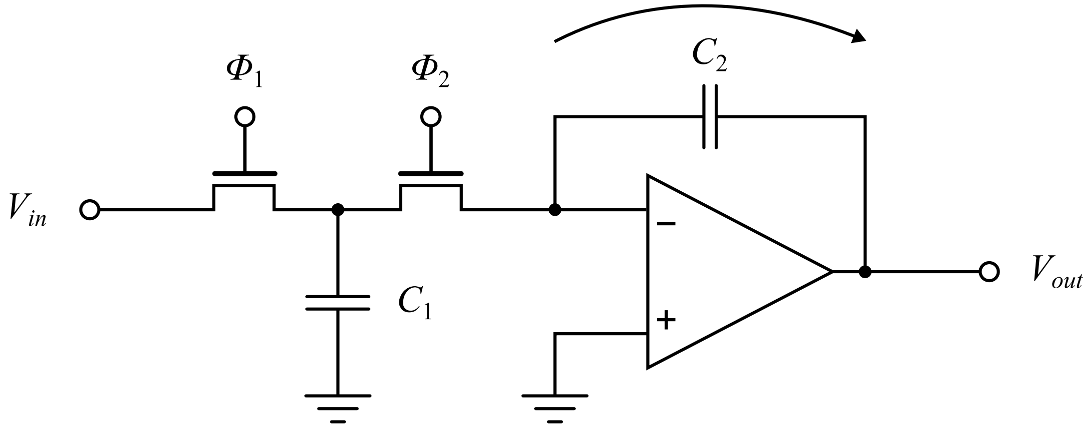
ELEC-E3530

Integrated Analog Systems L2
SC-integrators

Forward Euler SC integrator



Charge equations:

phase $\phi_1 : t = t_n :$ $\qquad \Delta Q_1(t_n) = C_1 v_{in}(t_n)$

phase $\phi_2 : t = t_n + \frac{T}{2}$ $\qquad \Delta Q_2\left(t_n + \frac{T}{2}\right) = C_1 v_{in}(t_n)$

$$Q_1 \to 0$$

Voltage change in C_2

$$\Rightarrow \Delta V_{C2} = \frac{\Delta Q_2}{C_2} = \frac{C_1}{C_2} v_{in}(t_n)$$

New output voltage value for the next clockcycle:

$$v_{out}(t_n + 1) = -\Delta V_{C2} + v_{out}(t_n) = v_{out}(t_n) - \frac{C_1}{C_2} v_{in}(t_n)$$

Apply z-transformation

$$V_{out}(z)z = V_{out}(z) - \frac{C_1}{C_2} V_{in}(z)$$

$$V_{out}(z) = -\frac{C_1}{C_2} \cdot \frac{1}{z-1} V_{in}(z) \triangleq \underline{-\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} V_{in}(z)}$$

Transfer function in z-domain:

$$H(z) = -\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}$$

Forward Euler transformation!

Analyze frequency distortion:
insert $z=e^{j\omega t}$

$$H\left(e^{j\omega T}\right) = -\frac{C_1}{C_2}\frac{1}{e^{j\omega T}-1} = -\frac{C_1}{C_2}\frac{1}{j\omega T - \omega^2\frac{T^2}{2}\ldots}$$

assume $\omega << \frac{1}{T}$

$$\Rightarrow H\left(e^{j\omega T}\right) \approx -\frac{C_1}{C_2 T}\frac{1}{j\omega} \quad \Rightarrow$$

SC integrator time-constant:

$$\underline{\tau = \frac{C_2 T}{C_1} \triangleq \frac{C_2}{f_{clk} C_1}}$$

RC integrator

$$H_{RC}(j\omega) \approx -\frac{1}{j\omega RC} \quad \Rightarrow \quad \underline{\tau = RC}$$

SC integrator time-constant depends on:

τ: a) $\alpha \frac{C_2}{C_1}$

b) $\alpha \frac{1}{f_{clk}}$

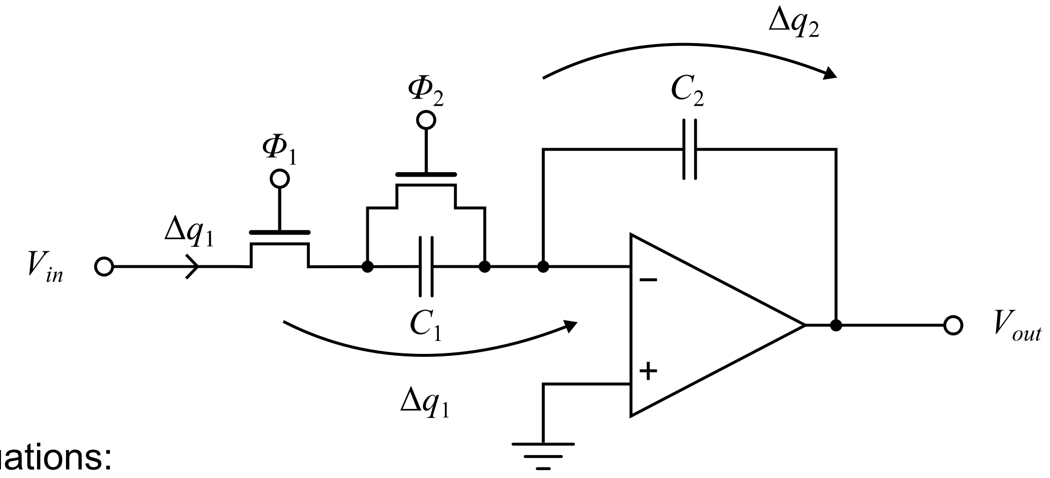
1)

$$\left.\begin{array}{l} \Delta \frac{C_2}{C_1} < 1\% \\ \Delta \frac{1}{f_{clk}} < 1\% \end{array}\right\} \Rightarrow \tau < 1\%$$

2)

Time-constant does not depend on the capacitor absolute values.
$\Rightarrow$ minimize the silicon area!

Backward Euler SC integrator



Charge equations:

phase $\phi_1 : \Delta q_1(t) = C_1 v_{in}(t_n)$

$\Delta q_2(t) = \Delta q_1(t_n)$

phase $\phi_2 : \Delta q_1 = 0$

Voltage change in C_2

$$\Rightarrow \Delta v_{C2}(t) = \frac{\Delta q_2(t)}{C_2} = \frac{C_1}{C_2} v_{in}(t_n)$$

New output voltage

$$v_{out}(t_n) = v_{out}(t_n - 1) - \Delta V_{C2}(t)$$

$$v_{out}(t_n) = v_{out}(t_n - 1) - \frac{C_1}{C_2} v_{in}(t_n)$$

Apply z-transformation:

$$V_{out}(z) = V_{out}(z) z^{-1} - \frac{C_1}{C_2} V_{in}(z)$$

Backward Euler transfer function

$$H(z) = \frac{V_{out}(z)}{V_{in}(z)} = -\frac{C_1}{C_2} \frac{1}{1 - z^{-1}}$$

Frequency response

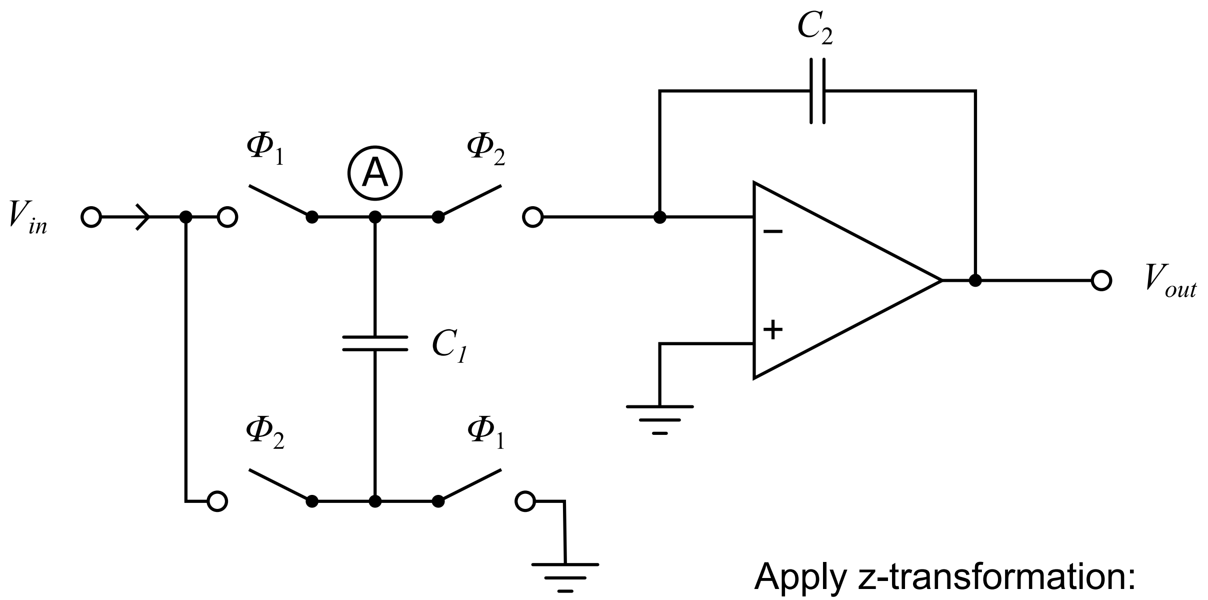
insert $z = e^{j\omega T}$

$$\Rightarrow H\left(e^{j\omega T}\right) = \frac{-\frac{C_1}{C_2}}{1 - e^{-j\omega T}} = \frac{-\frac{C_1}{C_2}}{j\omega T + \frac{\omega^2 T^2}{2} + \dots}$$

Assume $\omega << f_{clk} \Rightarrow H\left(e^{j\omega T}\right) \approx -\frac{C_1}{C_2} \frac{1}{j\omega T}$

$$\Rightarrow \tau = \frac{TC_2}{C_1} \triangleq \frac{C_2}{C_1 f_{clk}}$$

SC integrator implementing bilinear transformation



Charge equations:

phase ϕ_1: $\Delta q_1(nT-T) = C_1 V_{in}(nT-T)$

phase ϕ_2: $\Delta q_1(nT) = C_1[-V_{in}(nT) - V_{in}(nT-T)]$

$\Delta q_2(nT) = \Delta q_1(nT)$

Voltage change in C_2

$$\Rightarrow \Delta V_{C2}(nT) = \frac{\Delta q_2(nT)}{C_2}$$

New output voltage:

$$V_{out}(nT) = V_{out}(nT-T) + \Delta V_{c2}(nT)$$

$$V_{out}(nT) - V_{out}(nT-T) = -\frac{C_1}{C_2}[V_{in}(nT) + V_{in}(nT-T)]$$

Apply z-transformation:

$$V_{out}(z) - z^{-1}V_{out}(z) = -\frac{C_1}{C_2}V_{in}(z) + z^{-1}V_{in}(z)$$

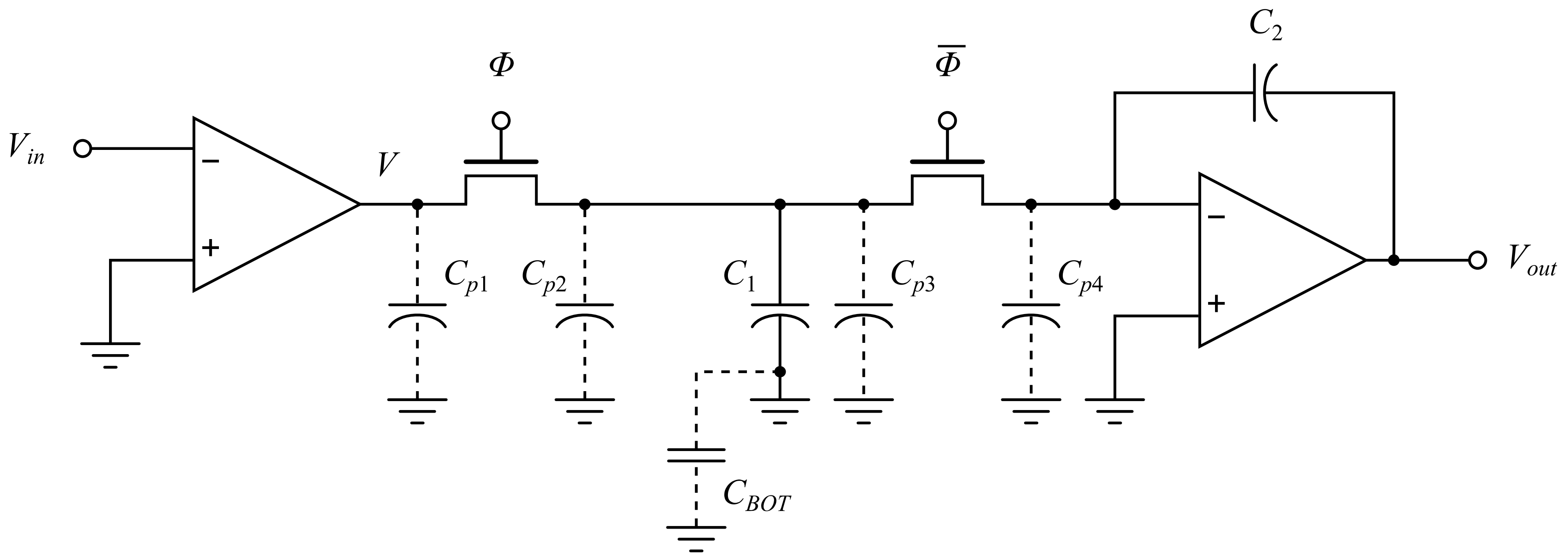
$$(1-z^{-1})V_{out}(z) = -\frac{C_1}{C_2}(1+z^{-1})V_{in}(z)$$

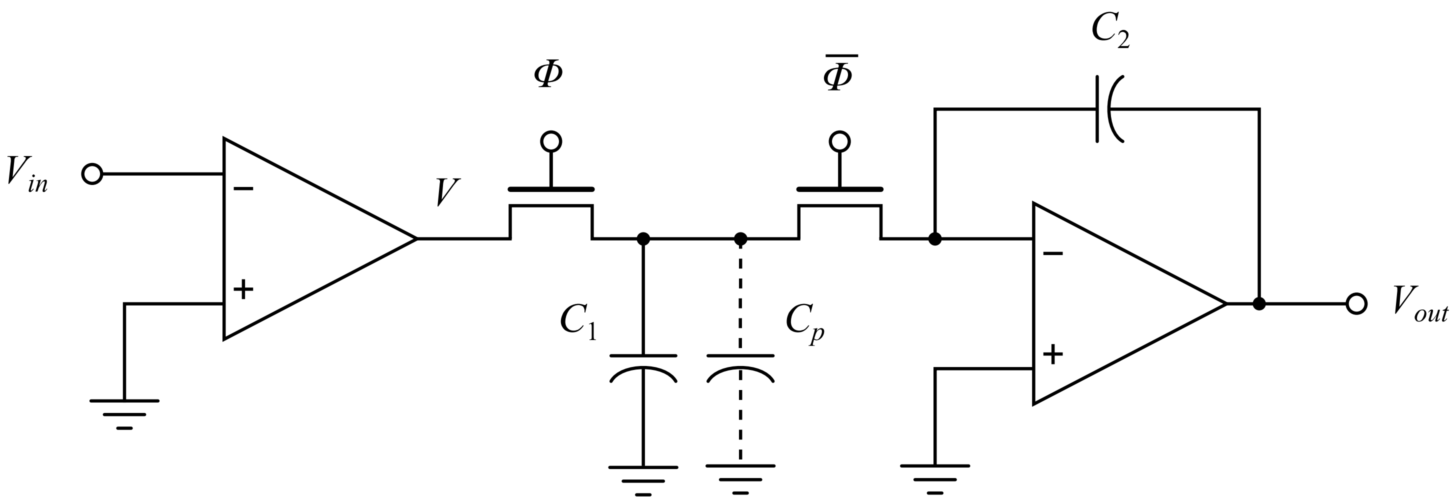
Solve the transfer function:

$$H(z) = \frac{V_{out}(z)}{V_{in}(z)} = -\frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} = -\frac{C_1}{C_2}\frac{z+1}{z-1}$$

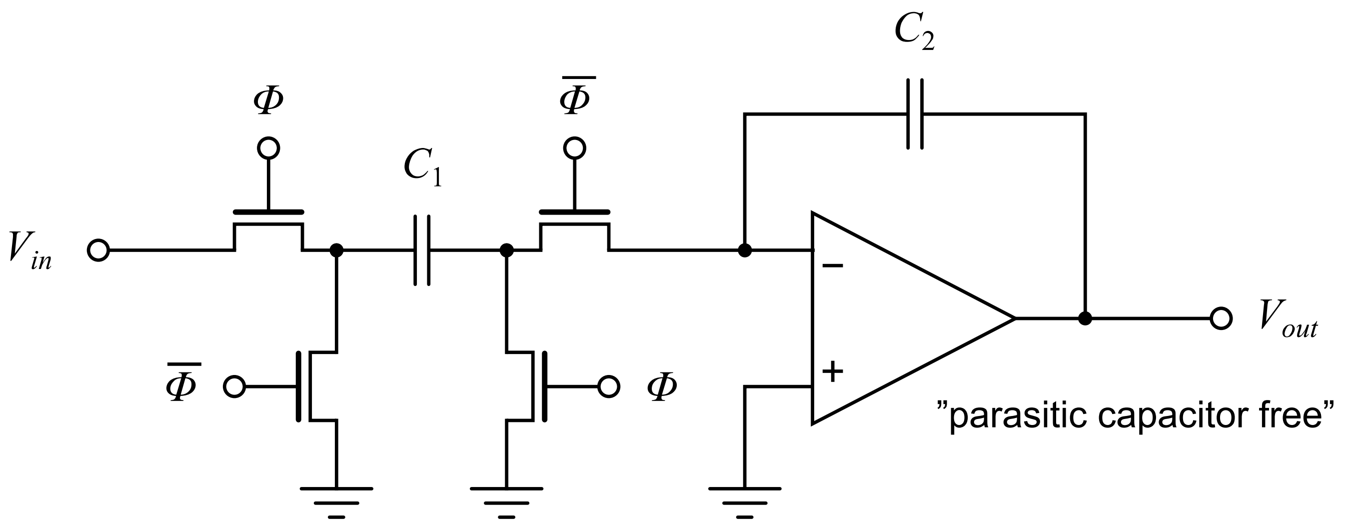
Bilinear transformation!

Parasitic capacitors in SC integrators





Switched capacitor filters



Charge equations:

phase Φ: $Q_1 = C_1 \cdot V_{in}(n-1) = Q_1(n-1)$

$Q_2 = C_2 \cdot V_{out}(n-1) = Q_2(n-1)$

phase $\overline{\Phi}$: $Q_1(n) = 0$

$Q_2(n) = Q_2(n-1) + \Delta Q_2 = C_2 V_{out}(n)$

charge $Q_1(n-1)$ transferred to C_2

$\Delta Q_2 = Q_1(n-1)$

$$\Rightarrow Q_2(n) = Q_2(n-1) + Q_1(n-1)$$

Insert charges Q_1 (n−1), Q_2 (n−1) and Q_2 (n)

$$C_2 V_{out}(n) = C_2 V_{out}(n-1) + C_1 V_{in}(n-1)$$

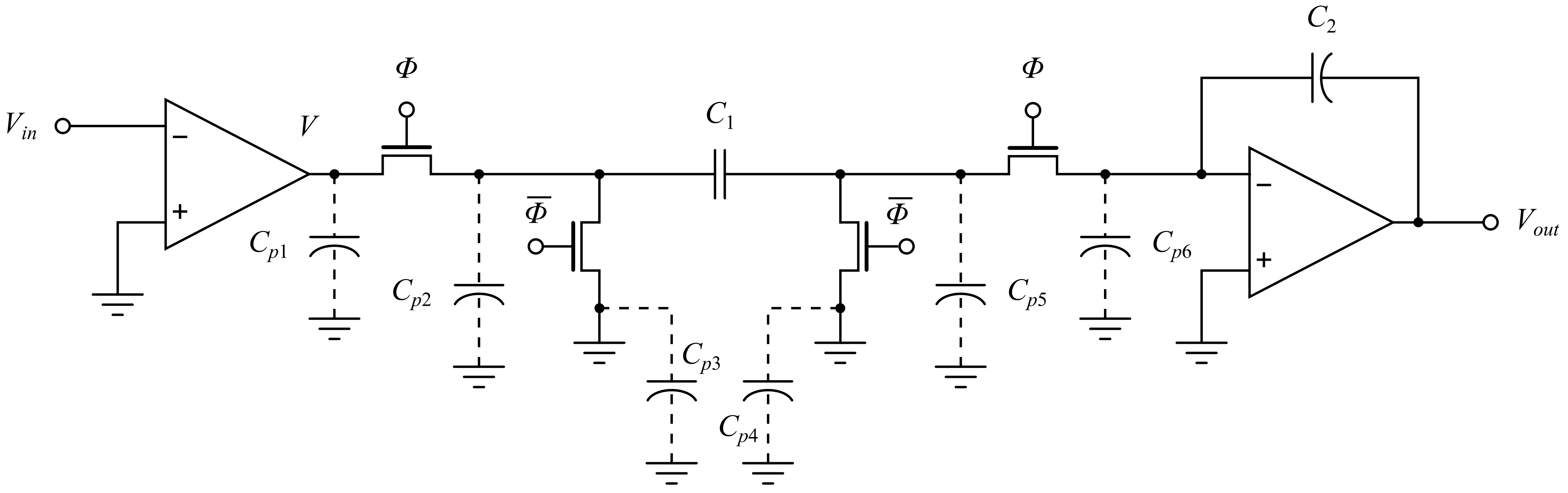
Apply z-transformation to obtain z-plane transfer function

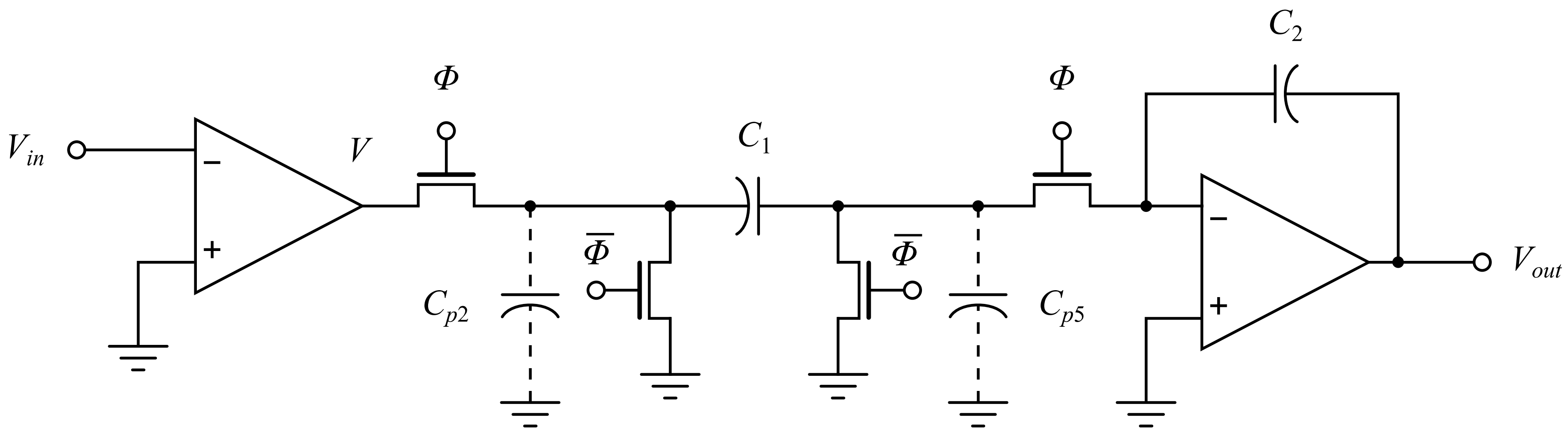
$$C_2 V_{out}(z) = C_2 z^{-1} V_{out}(z) + C_1 z^{-1} V_{in}(z)$$

$$\Rightarrow H(z) = \frac{V_{out}(z)}{V_{in}(z)} = \frac{z^{-1}}{1 - z^{-1}} \cdot \frac{C_1}{C_2}$$

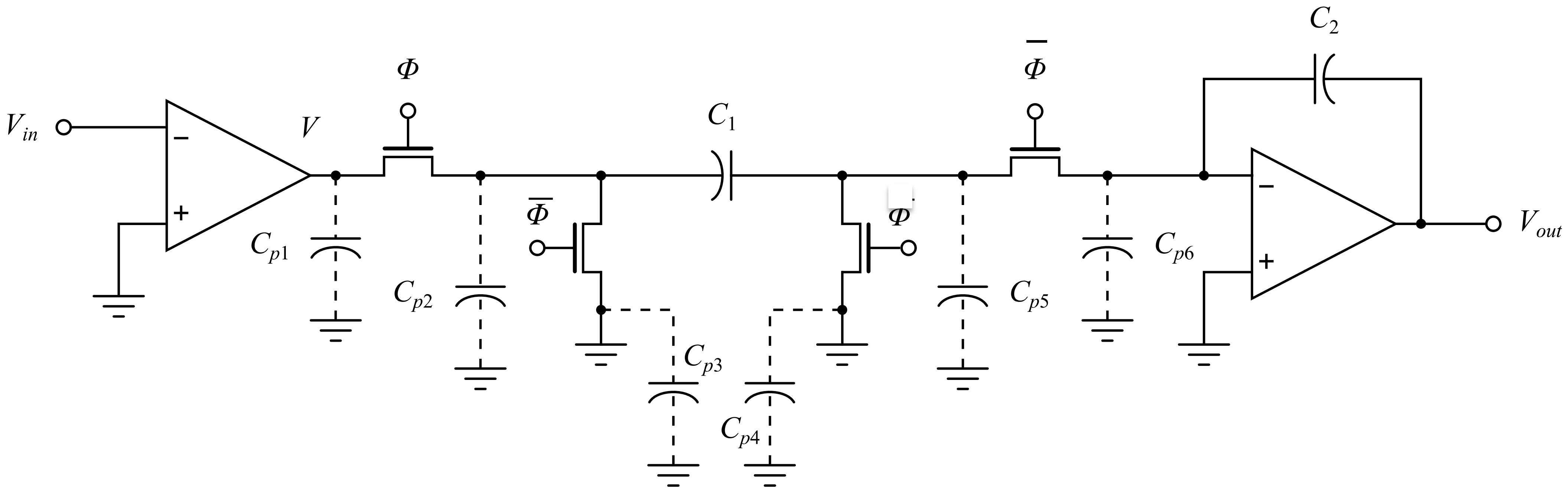
This is a non-inverting integrator.

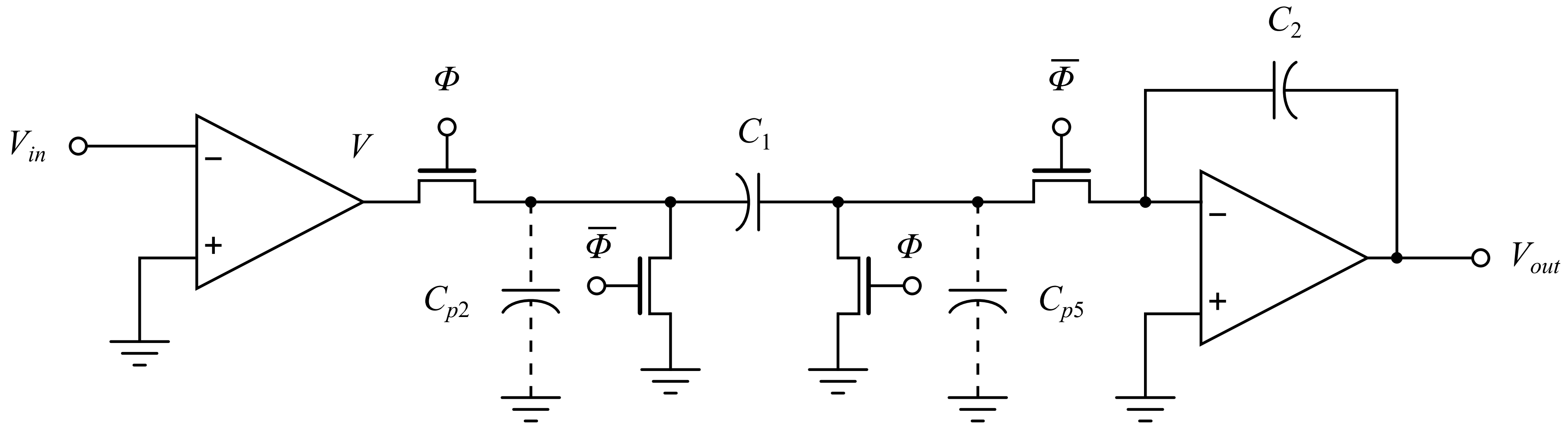
Parasitic insensitive inverting SC integrator





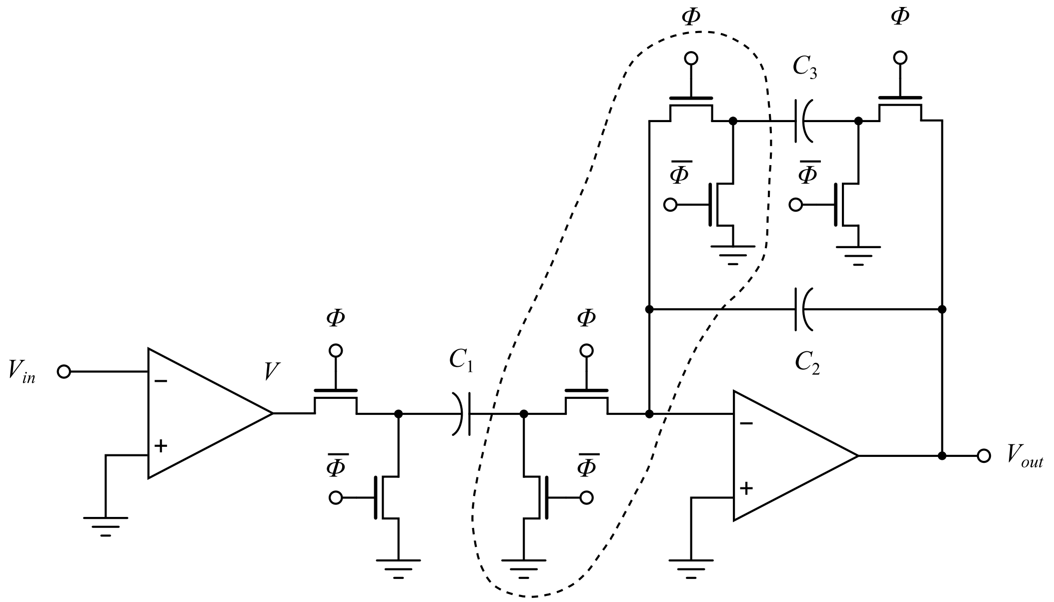
Parasitic insensitive non-inverting SC integrator

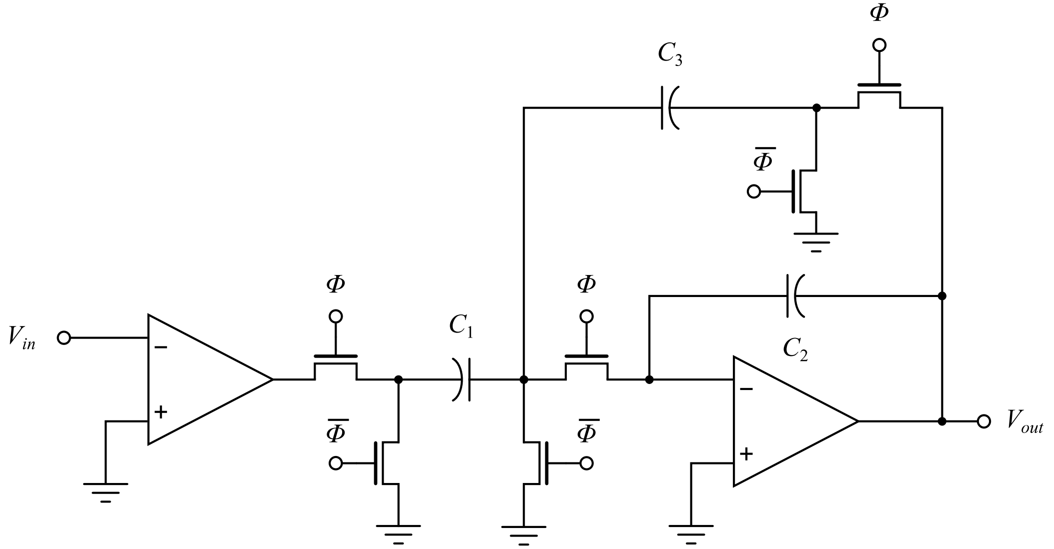




Lossy SC integrators

Inverting





Non-Inverting