General SC biquad

Synthesis:

A:
1) Write a s-plane transfer function
2) Make a signal flow graph (SFG)
3) Create a RC-active equivalent topology
4) Replace the resistors with switched capacitors

B:
1) Write a s-plane transfer function
2) Use the s→z transformation to get the z-plane transfer function
3) Make the SFG for the z-plane transfer function
4) Implement the SFG by using SC integrators

C: Use the general SC biquad topology, when you know the capacitors' value's depedency on the s-plane transfer function →calculate all the capacitor values by using their frequency characteristics

Relationship between frequency parameters and capacitor values

General 2nd order transfer functions in s- and z-domain

$$H(s) = -\frac{K_2 s^2 + K_1 s + K_0}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

$$H(z) = -\frac{a_2 z^2 + a_1 z + a_0}{b_1 z^2 + b_1 z + 1}$$

Applying bilinear transformation

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

We may calculate the a_i, b_i coefficients as function of frequency parameters k_i, ω_o and Q i.e.

$$a_i = f(K_i, \omega_o, Q)$$
$$b_i = f(K_i, \omega_o, Q)$$

Using the relationship between SC biquad capacitor values and the coefficients (a_i, b_i) of 2nd order z-domain transfer function, we can calculate the relationships between capacitor values and frequency parameters.

Synthesis in z-domain

General 2nd order z-domain transfer function

$$\frac{V_{out}}{V_{in}} = \frac{a_2 z^{-2} + a_1 z^{-1} + a_0}{b_2 z^{-2} + b_1 z^{-1} + 1}$$

Multiply both sides with the nominators

$$b_2 z^{-2} V_{out} + b_1 z^{-1} V_{out} + V_{out} = a_2 z^{-2} V_{in} + a_1 z^{-1} V_{in} + a_0 V_{in} \qquad (1)$$

z-domain integrators:

$$H(z) = \frac{1}{1 - z^{-1}} \text{ or } \frac{z^{-1}}{1 - z^{-1}}$$

Thus we need to form two z-domain integrators

Let us introduce the substitution:

$$\frac{1}{p} = \frac{1}{1 - z^{-1}}$$

$$\Rightarrow p = 1 - z^{-1} \Rightarrow \begin{cases} z^{-1} = 1 - p \\ z^{-2} = (1 - p)^2 \end{cases}$$

Replace z^{-1} and z^{-2} in (1) with these relationships and organize eq(1) with powers of p.

$$b_2 (1 - p)^2 V_{out} + b_1 (1 - p) V_{out} + V_{out} = a_2 (1 - p)^2 + a_1 (1 - p) V_{in} + a_0 V_{in}$$

$$b_2 p^2 V_{out} - (b_1 + 2b_2) p V_{out} + (1 + b_1 + b_2) V_{out} = a_2 p^2 V_{in} - (a_1 + 2a_2) p V_{in} + (a_0 + a_1 + a_2) V_{in}$$

To form integrators we need state equations with $\frac{1}{p}$ - terms

$$\Rightarrow V_{out} = \frac{-1}{b_2 p^2}\left[-\left(b_1 + 2b_2\right)pV_{out} + \left(1 + b_1 + b_2\right)V_{out} - a_2 p^2 V_{in} + \left(a_1 + 2a_2\right)pV_{in} - \left(a_0 + a_1 + a_2\right)V_{in}\right]$$

$$V_{out} = \frac{-1}{b_2 p}\left[-\left(b_1 + 2b_2\right)V_{out} + \frac{1 + b_1 + b_2}{p}V_{out} - a_2 pV_{in} + \left(a_1 + 2a_2\right)V_{in} - \frac{a_0 + a_1 + a_2}{p}V_{in}\right]$$

The state equations are:

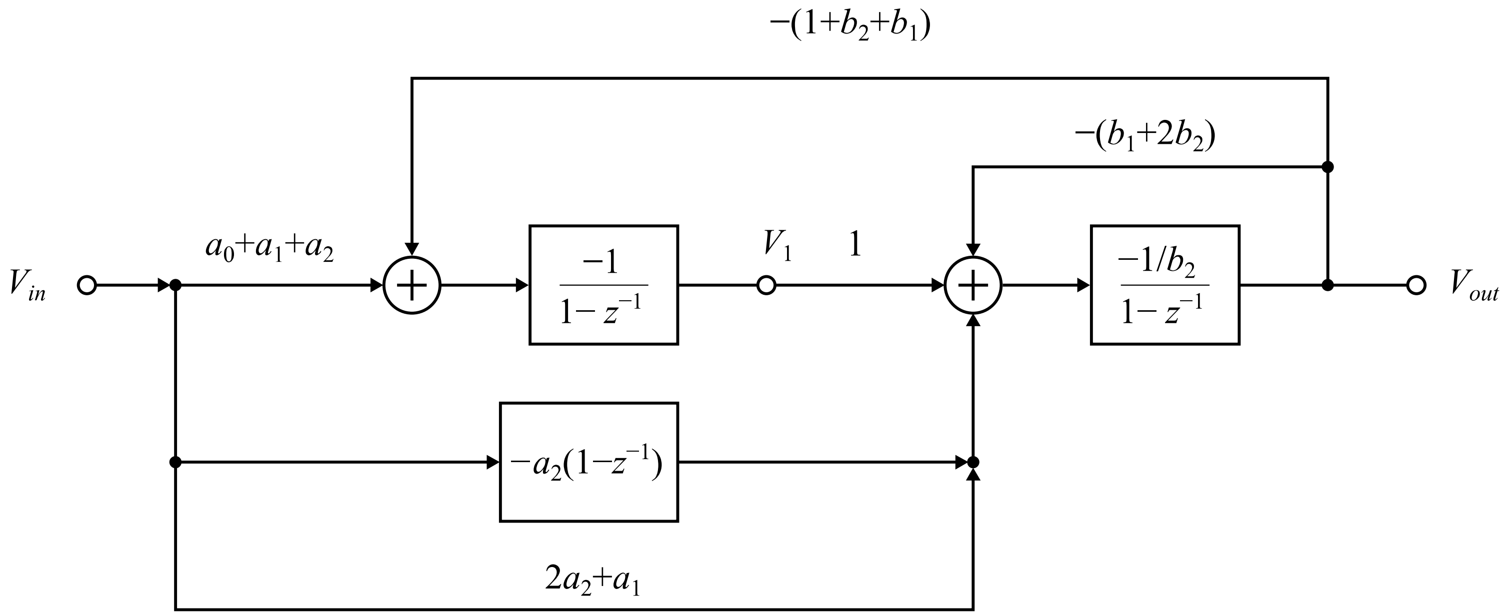
$$V_1 = \frac{-1}{p}\left[-\left(1 + b_2 + b_1\right)V_{out} + \left(a_0 + a_1 + a_2\right)V_{in}\right]$$

$$V_{out} = \frac{-1}{b_2 p}\left[-\left(b_1 + 2b_2\right)V_{out} - a_2 pV_{in} + \left(2a_2 + a_1\right)V_{in} + V_1\right]$$

inserting p = 1-z^{-1} gives the z-domain state equations

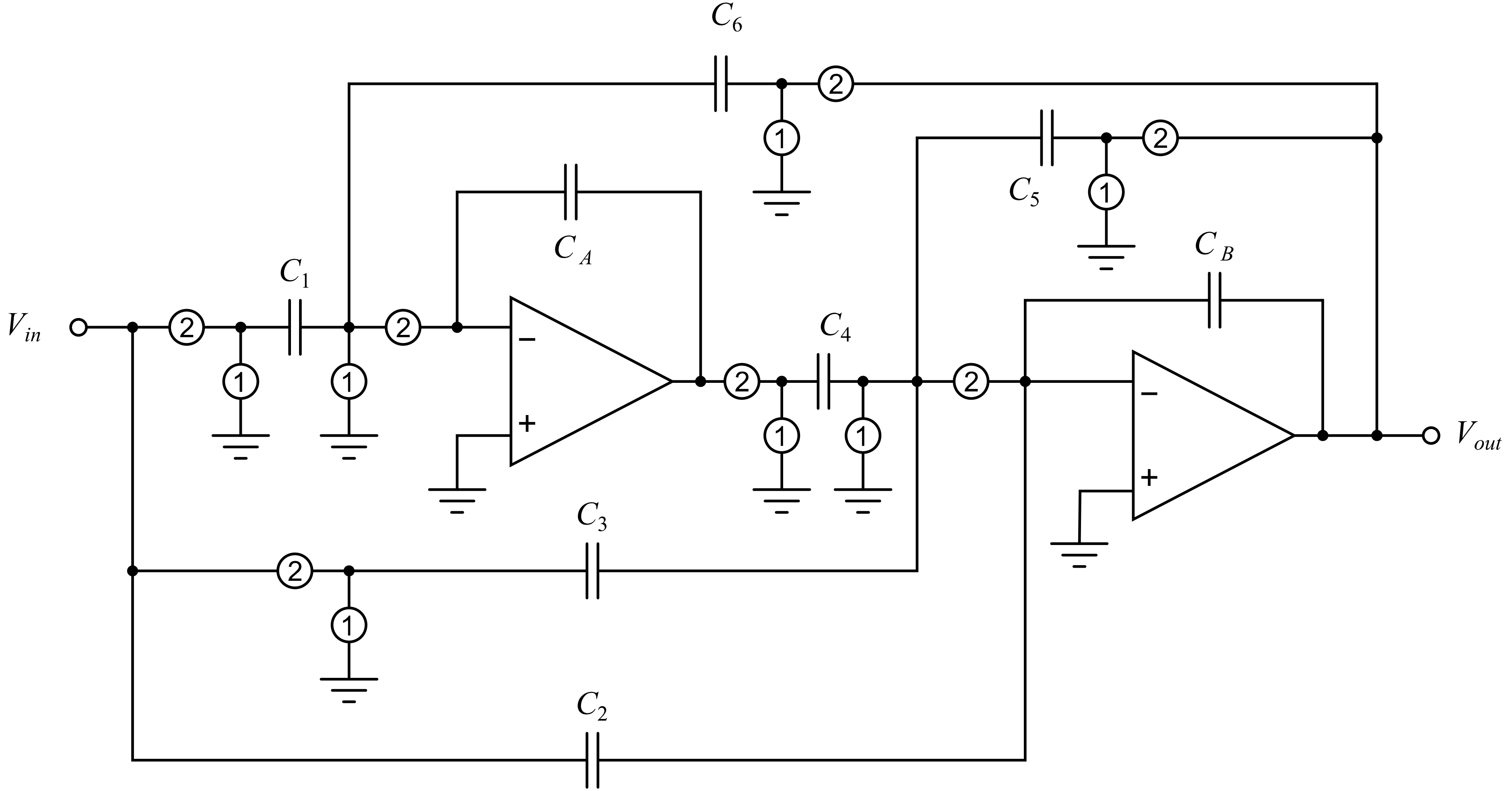
(1) $$V_1 = \frac{-1}{1 - z^{-1}}\left[-\left(1 + b_2 + b_1\right)V_{out} + \left(a_0 + a_1 + a_2\right)V_{in}\right]$$

(2) $$V_{out} = \frac{-1}{b_2\left(1 - z^{-1}\right)}\left[-\left(b_1 + 2b_2\right)V_{out} - a_2\left(1 - z^{-1}\right)V_{in} + \left(2a_2 + a_1\right)V_{in} + V_1\right]$$



$-(1+b_2+b_1)$
$-(b_1+2b_2)$
V_{in}
$a_0+a_1+a_2$
$\frac{-1}{1-z^{-1}}$
V_1
1
$\frac{-1/b_2}{1-z^{-1}}$
V_{out}
$-a_2(1-z^{-1})$
$2a_2+a_1$

SC realization of the state equations



Problem is the loop connection of the two integrators during phase 2

Let us try to synthesize this equation in another way

$$V_{out} = \frac{-1}{b_2 p}\left[-(b_1 + 2b_2)V_{out} + \frac{1+b_1+b_2}{p}V_{out} - a_2 pV_{in} + (a_1 + 2a_2)V_{in} - \frac{a_0 + a_1 + a_2}{p}V_{in}\right]$$

To avoid loop connection of the two integrators we need a delay between the integrators

The delayed integrator is given by $-\frac{z^{-1}}{1-z^{-1}}$

Thus we need to form this term for the second integrator.
This is performed as follows:

$$\Rightarrow aV_{out} - \frac{b}{1-z^{-1}}V_{out} = cV_{out} - \frac{dz^{-1}}{1-z^{-1}}V_{out}$$

Solving *c* and *d* and use $a = b_1+2b_2$ and $b = 1+b_1+b_2$

$$c = a - b = b_1 + 2b_2 + 1 + b_2 + b_1 = b_2 + 1$$
$$d = b = 1 + b_2 + b_1$$

The same is performed for V_{in} as well $a = -(a_1+2a_2)$ and $b = a_0+a_1+a_2$

$$c = a - b = a_2 + a_1 - (a_0 + a_1 + a_2) = -a_0$$
$$d = b = a_0 + a_1 + a_2$$

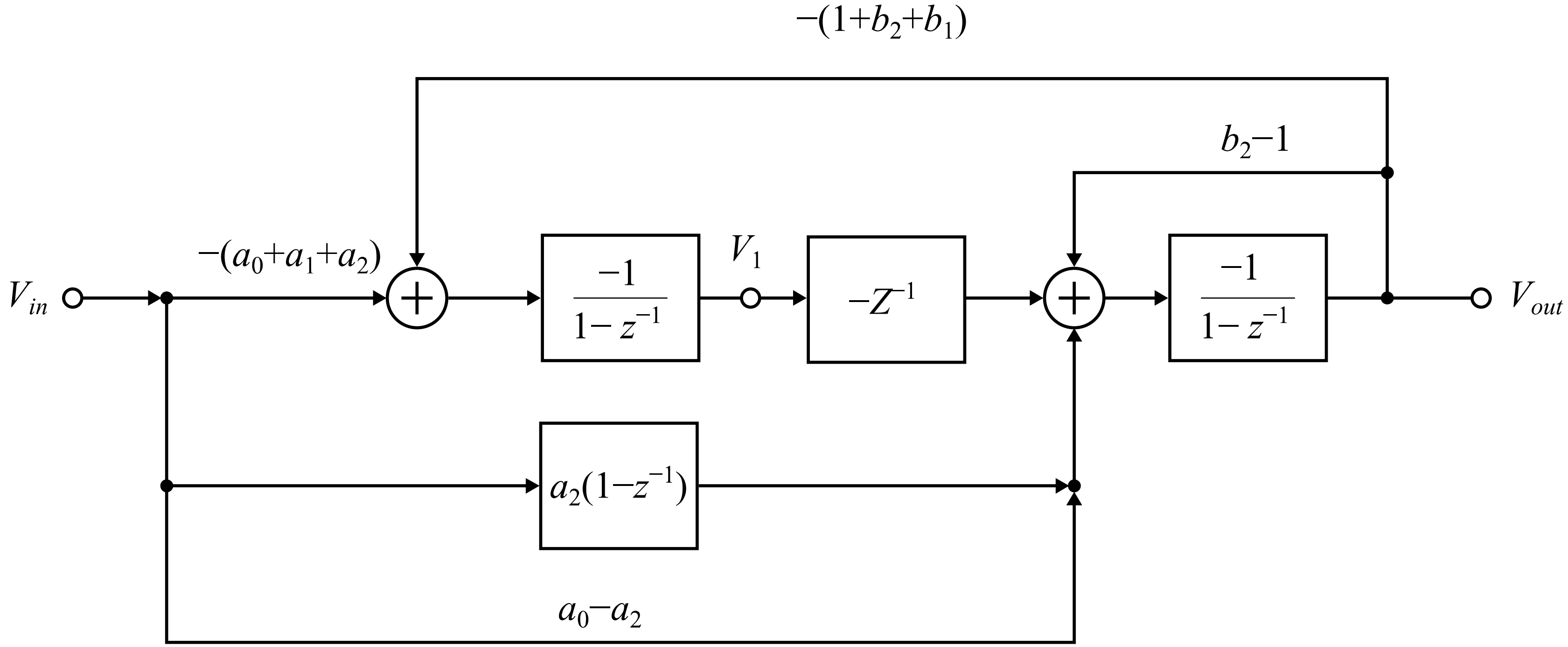
Thus V_{out} is written into the following form:

$$V_{out} = \frac{-1}{b_2(1-z^{-1})}\left[(b_2 - 1)V_{out} - \frac{(1+b_2+b_1)z^{-1}}{1-z^{-1}}V_{out} + a_2(1-z^{-1})V_{in} + (a_0 - a_2)V_{in} - \frac{(a_0+a_1+a_2)z^{-1}}{1-z^{-1}}V_{in}\right]$$

Thus the state equations are:

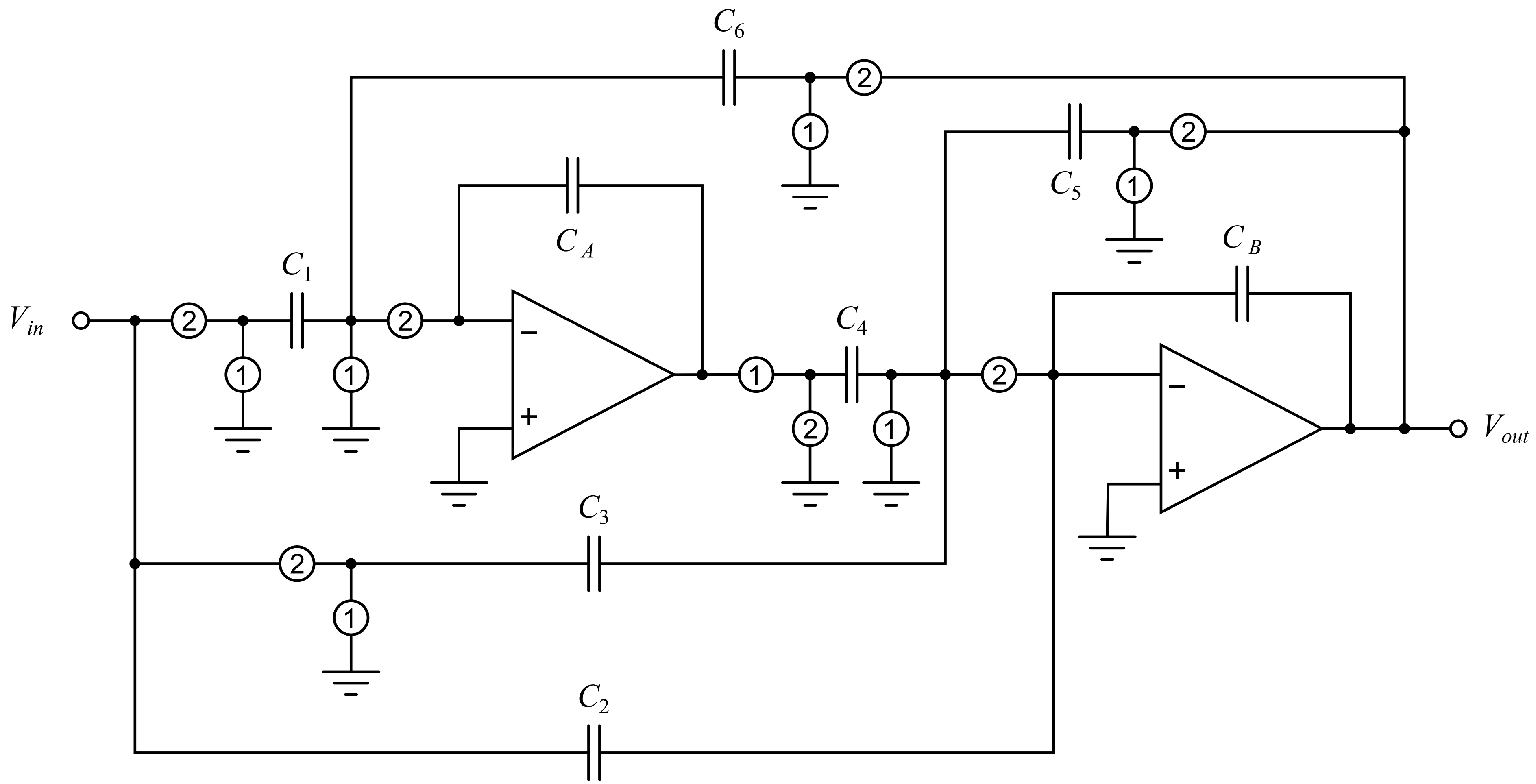
$$V_{out} = \frac{-1}{b_2\left(1 - z^{-1}\right)}\left[\left(b_2 - 1\right)V_{out} + a_2\left(1 - z^{-1}\right)V_{in} + \left(a_0 - a_2\right)V_{in} - z^{-1}V_1\right]$$

$$V_1 = \frac{-1}{1 - z^{-1}}\left[-\left(1 + b_2 + b_1\right)V_{out} - \left(a_0 + a_1 + a_2\right)V_{in}\right]$$



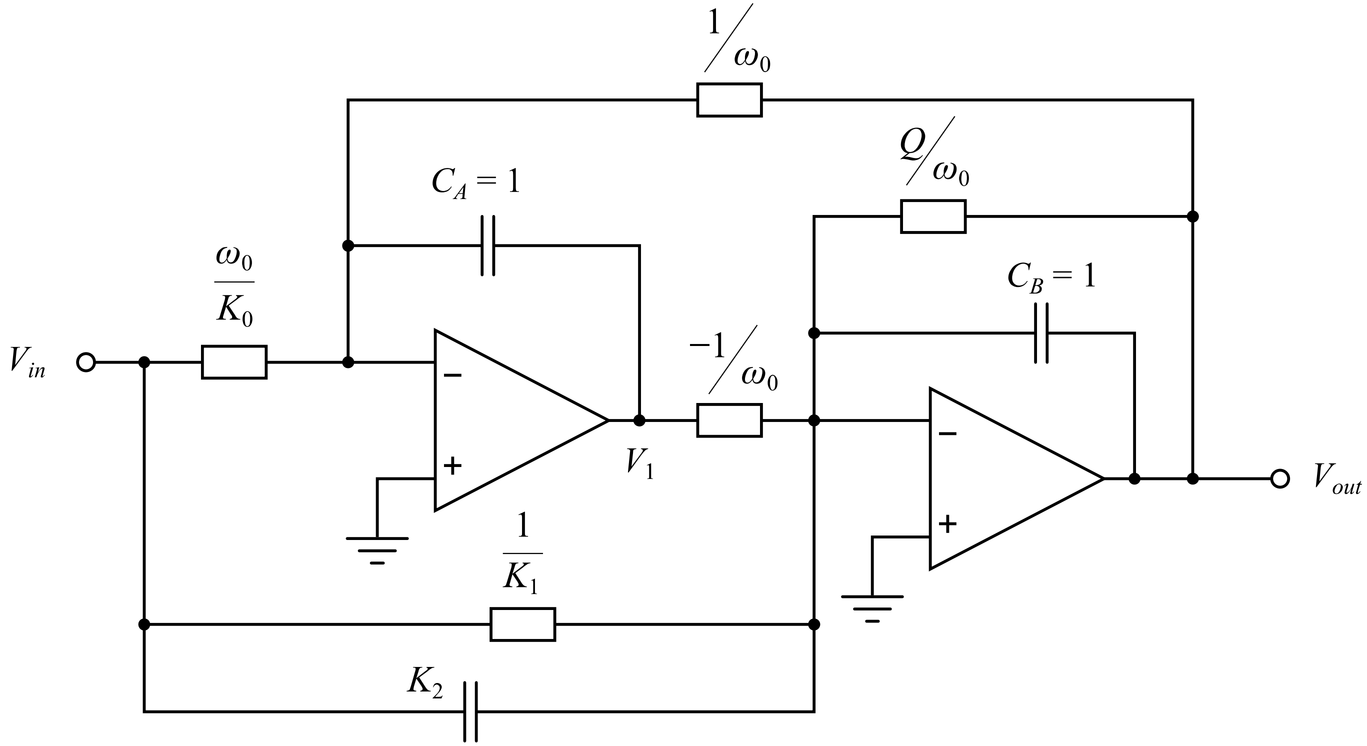
−(1+b2+b1)
b2−1
−(a0+a1+a2)
Vin
V1
Vout
−1 / 1− z−1
−Z−1
−1 / 1− z−1
a2(1−z−1)
a0−a2

SC realization of the state equations

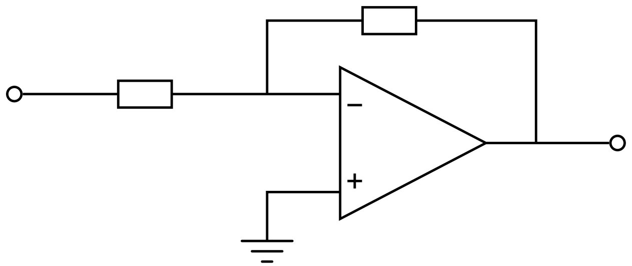


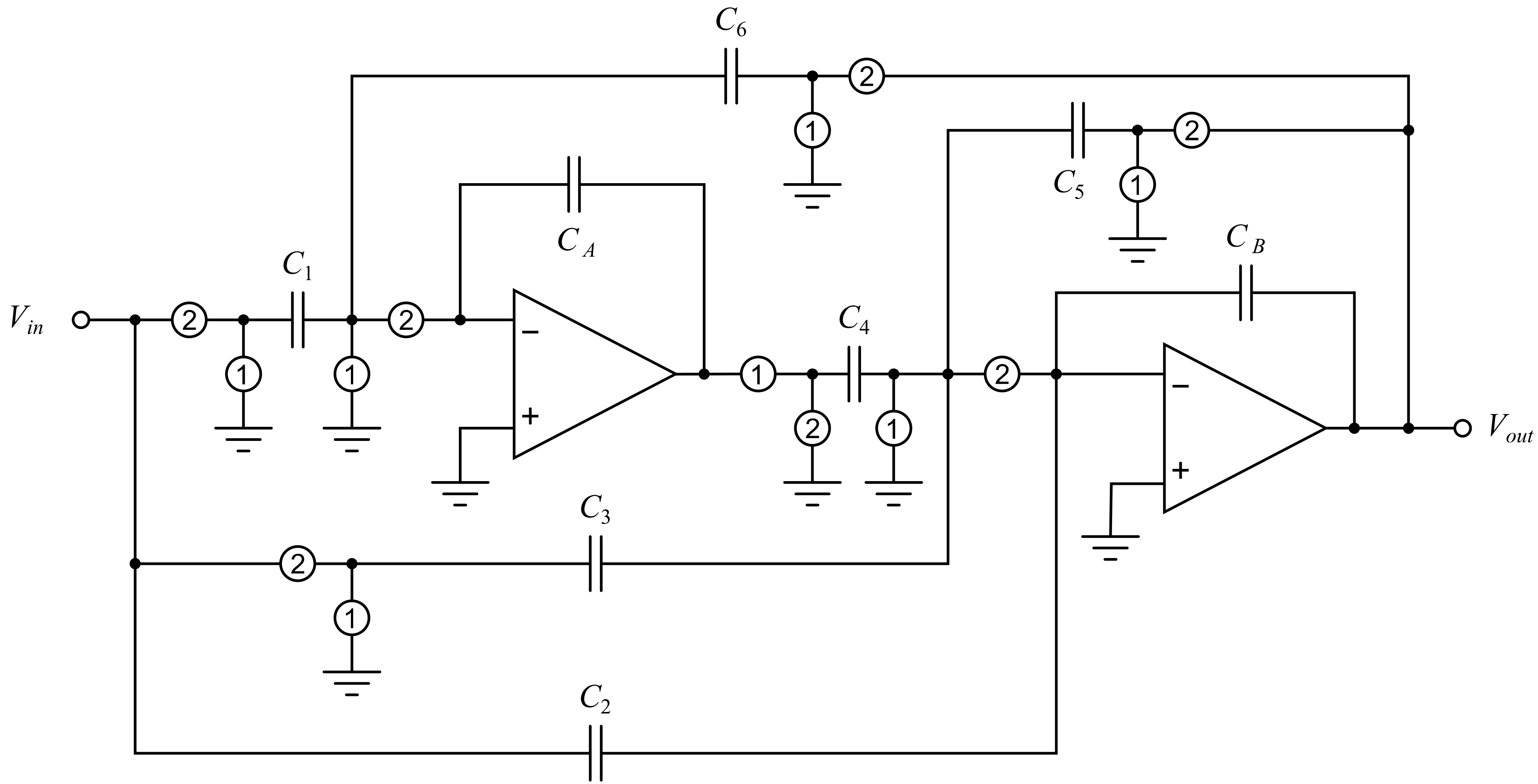
RC active filter realization of biquad

-combine the two RC integrators to form the biquad



-the negative resistor must be realized with a separate R-R amplifier





C_6
2
1
C_5
2
1
C_B
C_A
C_1
V_{in}
2
1
2
1
2
−
+
1
C_4
2
1
2
−
+
V_{out}
C_3
2
1
C_2