

ELEC-E3530

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Integrated Analog Systems L5

Nonidealities in SC-circuits

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Non-idealities in SC integrators

- charge transfer error
- error charges from switches
- noise
- PSRR

Charge transfer error

- static error
- dynamic error

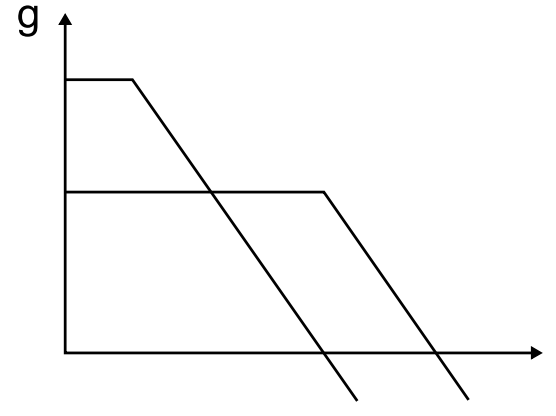
Caused by:

- amplifier gain
- amplifier bandwidth
- amplifier current drive capability (slewing)
- amplifier internal poles (doublet's)
- parasitic capacitors
- channel charge of switches
- clock feedthrough in switches

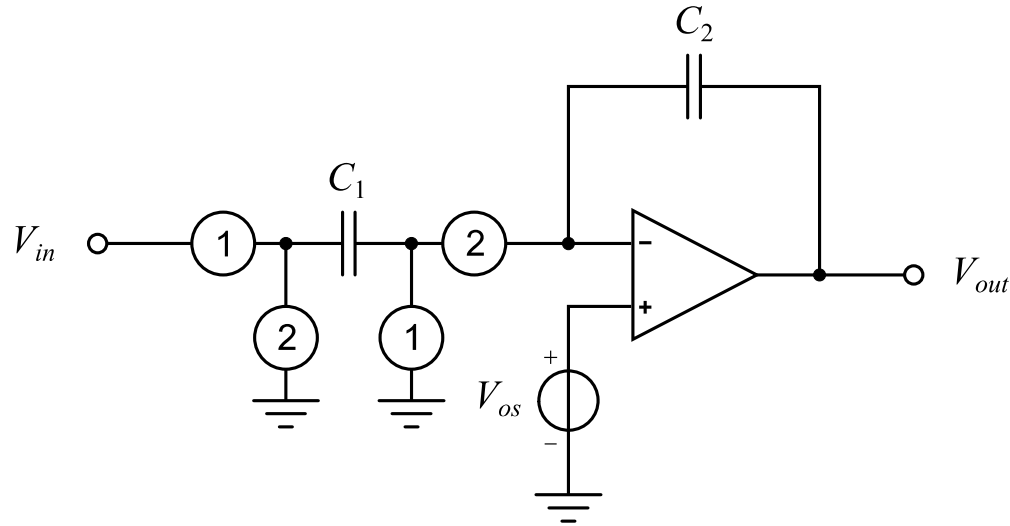
Results in:

- offset
- gain error
- distortion noise

Fully differential configuration



Amplifier offset-voltage in SC integrators

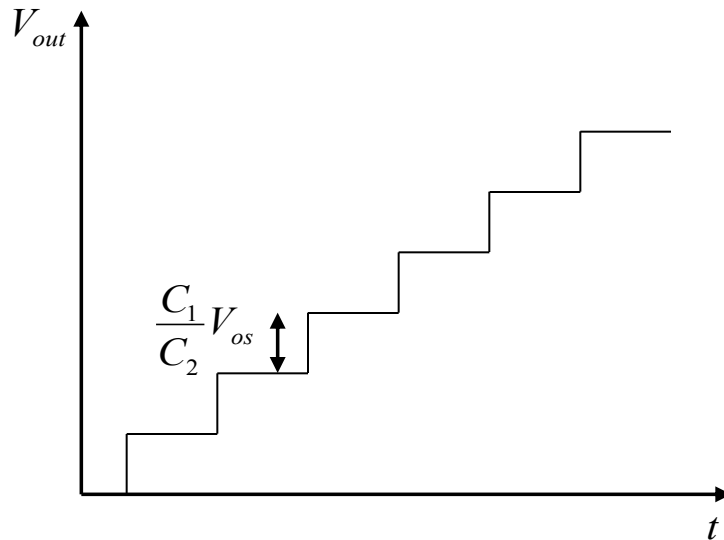


$$\Delta Q_2 = C_1(V_{in} - V_{os}) = C_1V_{in} - C_1V_{os}$$

$$\Delta V_{out} = \frac{\Delta Q_2}{C_2} = \frac{C_1}{C_2}(V_{in} - V_{os})$$

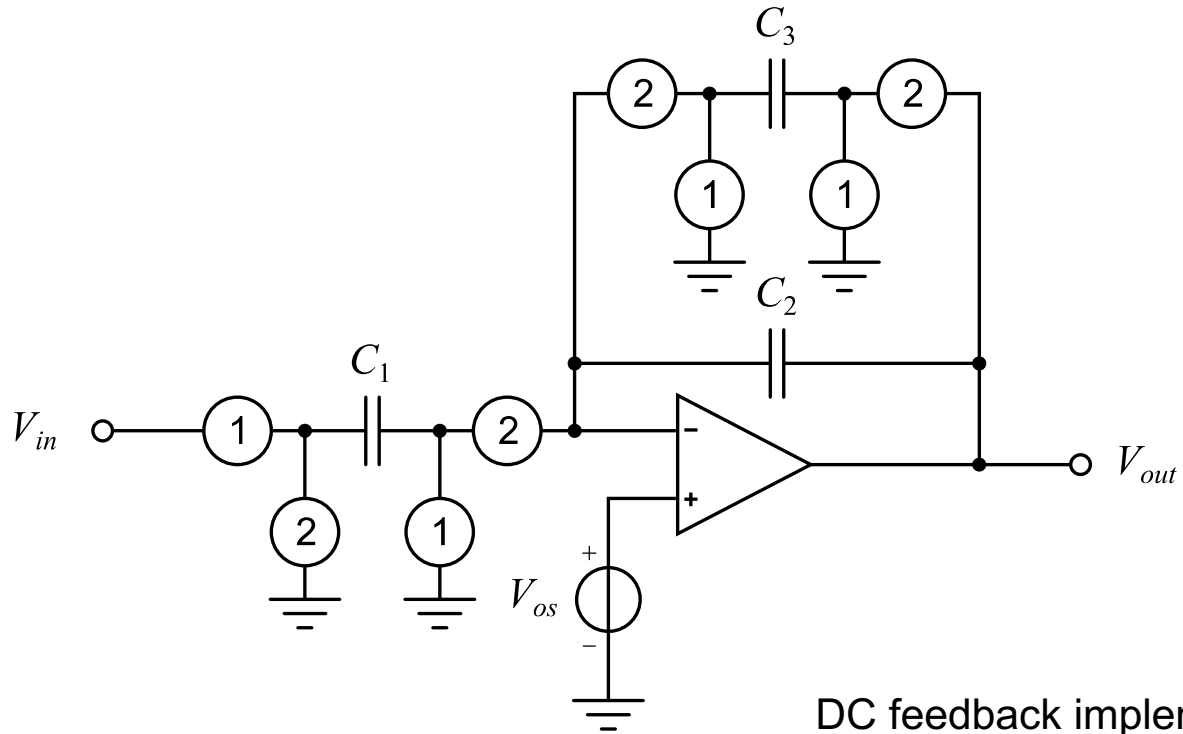
$$ol. V_{in} = 0$$

During every clock-cycle a charge packet of $\Delta Q_2 = -C_1V_{OS}$ is transferred into the integrating capacitor C_2



⇒ The accumulated charge will drive the output to either of the supplies and the amplifier saturates ⇒ DC-feedback is needed!

SC integrator with DC feedback

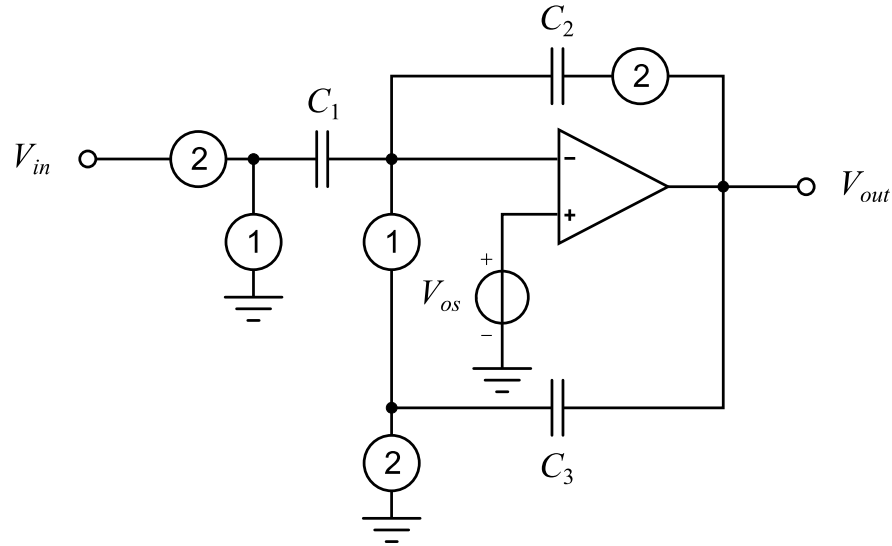


DC feedback implemented with a switched capacitor C_3 .
Now the integrator output offset is limited to

assume $V_{in} = 0 \Rightarrow$

$$V_{out} = \frac{C_1}{C_3} \cdot V_{os}$$

Offset free SC integrator



Phase ϕ_1 : offset is pre-sampled by C_1

Phase ϕ_2 : input voltage is sampled by C_1 and charge $C_1 V_{in}$ is transferred to C_2

Charge transfer equation:

$$\Delta Q_2 = -C_1(V_{in} - V_{os}) - \Delta Q_1$$

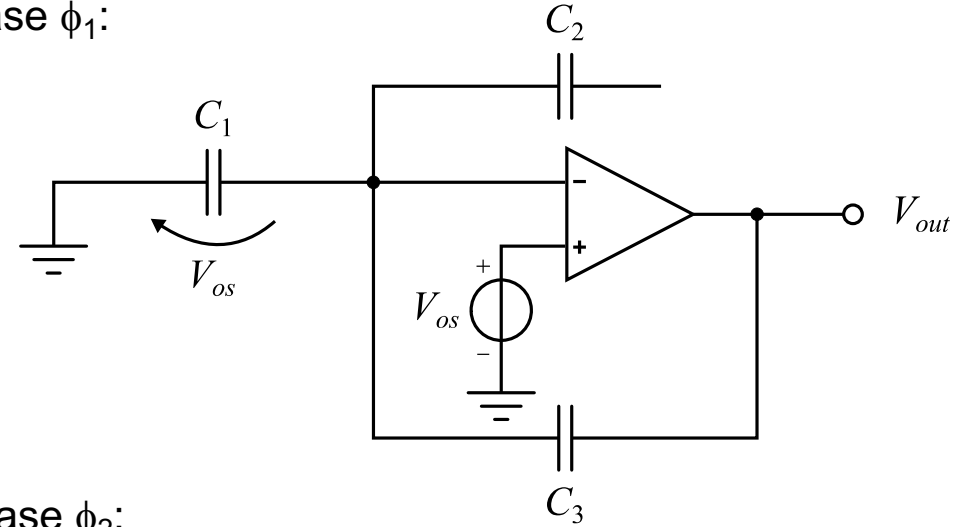
$$\Delta Q_2 = -C_1 V_{in} + \Delta Q_1 - C_1 V_{os}$$

$$= -C_1 V_{in}$$

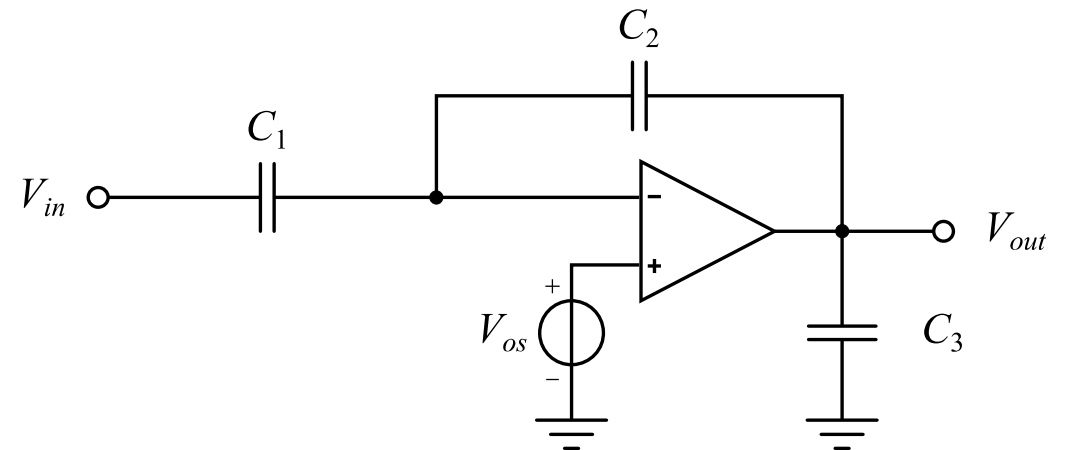
$$\Delta V_{out} = \frac{\Delta Q_2}{C_2} = -\frac{C_1}{C_2} \cdot V_{in}$$

offset charge is not transferred into C_2

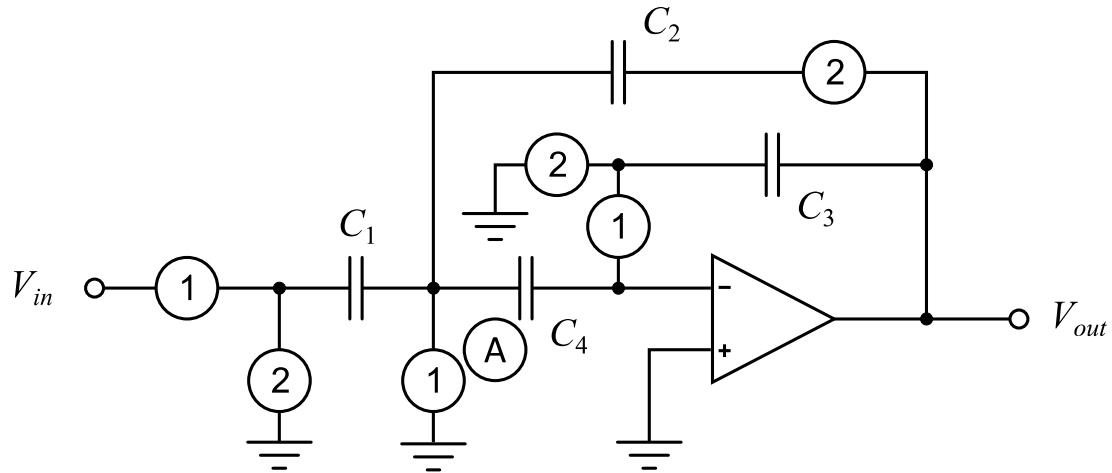
Phase ϕ_1 :



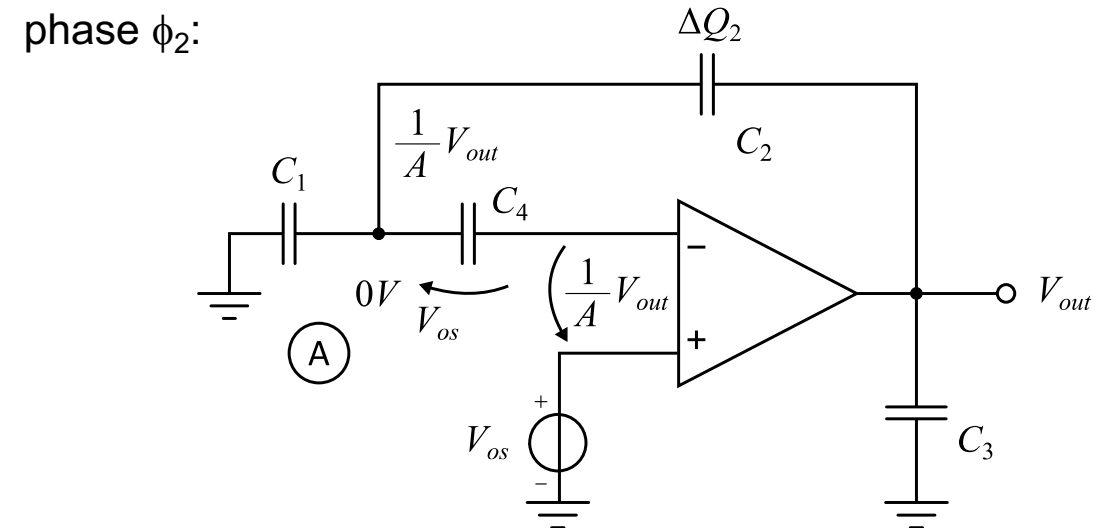
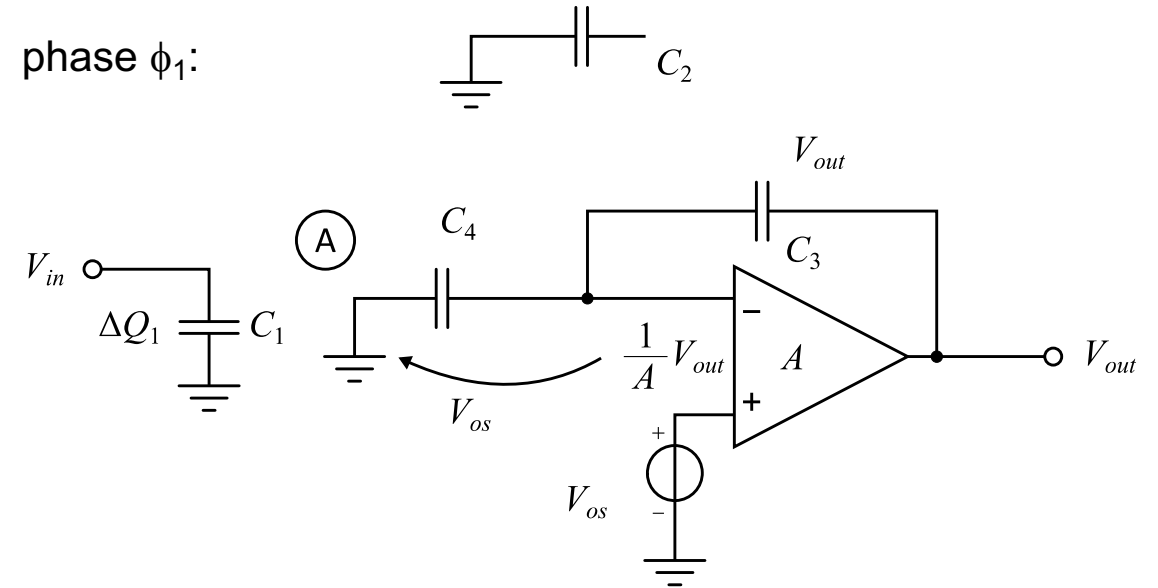
Phase ϕ_2 :



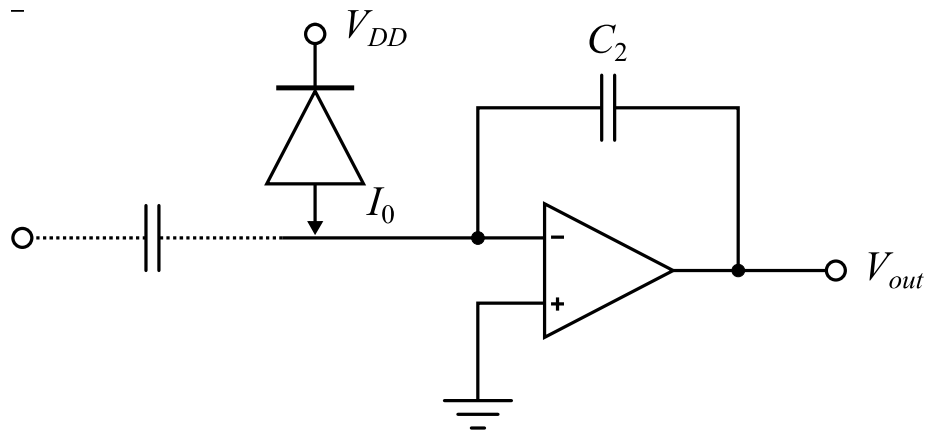
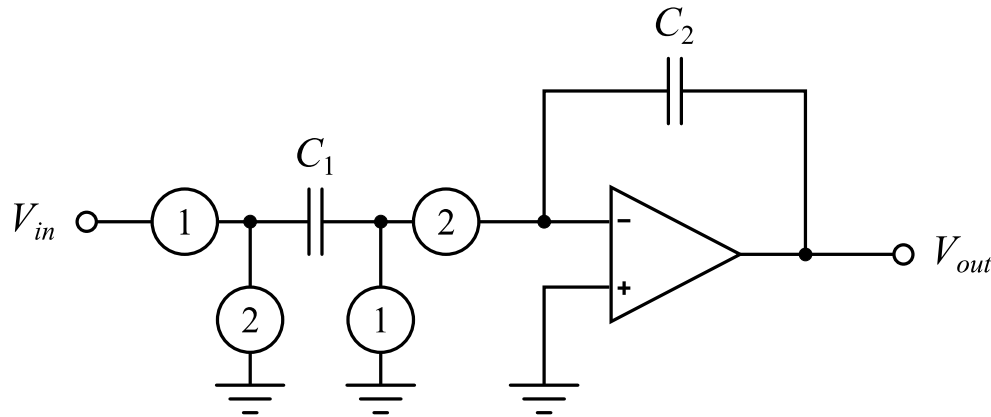
Offset free SC integrator



Node (A) is offset compensated virtual ground.
 C_1 samples against node (A) $\Rightarrow$ offset eliminated
 $\Delta Q_2 \equiv \Delta Q_1 = C_1 V_{in}$ exactly!
 No charge transfer error!
 Also gain error is eliminated!



Leakage currents of switches



Leakage current I_0 charges C_2
Change of charge in C_2 during one clock-cycle

$$\Delta Q = I_o \cdot T$$

Voltage droop in C_2

$$\Rightarrow \Delta V_{C_2} = \frac{\Delta Q}{C_2} = \frac{I_o \cdot T}{C_2} = \frac{I_o}{f_{CLK} C_2}$$

Leakage current causes offset!

Example

$$I_o = 1pA$$

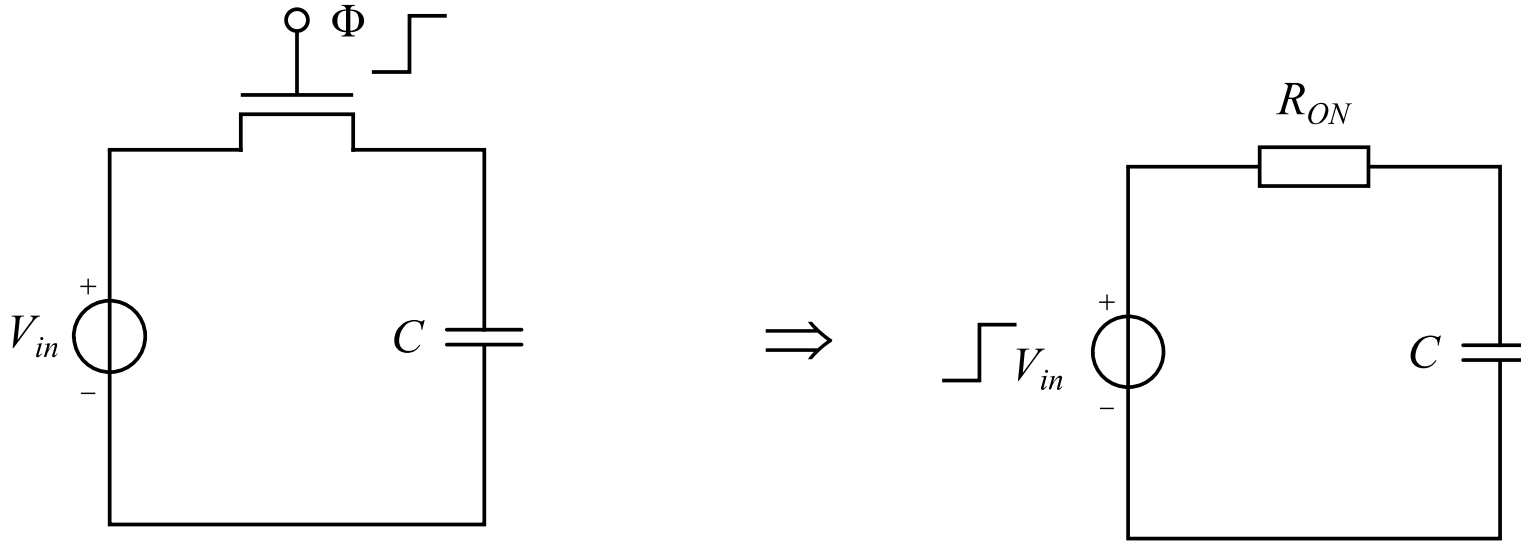
$$C_2 = 5pF \Rightarrow \underline{\underline{V_{OS} = 0,5mV}}$$

$$f_{CLK} = 1kHz$$

I_o doubles every $10^\circ C$ rise of temperature!

$$@100^\circ C \Rightarrow V_{OS} = 50mV!!$$

Error due to switch on-resistance



Settling during sampling the input

$$V_C = V_{in} (1 - e^{-t/R_{ON}C})$$

Sampling time $t = \frac{T}{2}$; $f_{CLK} = \frac{1}{T}$

$$\Rightarrow V_C = V_{in} (1 - e^{-T/2R_{ON}C})$$

The error in charge transfer is

$$\Delta V = V_{in} e^{-T/2R_{ON}C}$$

$$\varepsilon = \frac{\Delta V}{V_{in}} = e^{-T/2R_{ON}C}$$

Example:

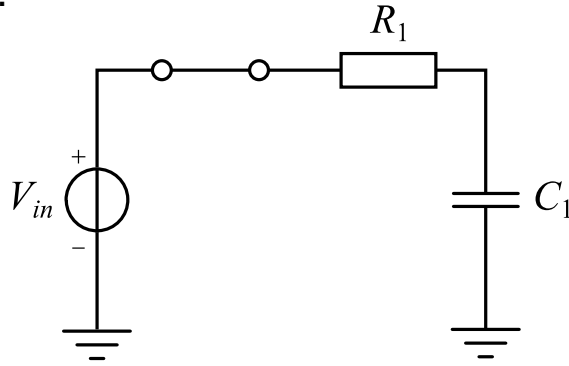
$$\varepsilon < 0,1\% \Rightarrow \frac{T}{2RC} < 5$$

$$\Rightarrow \frac{1}{2RC} > 5 \cdot f_{CLK}$$

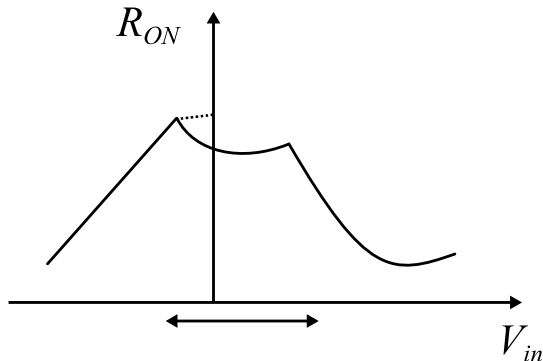
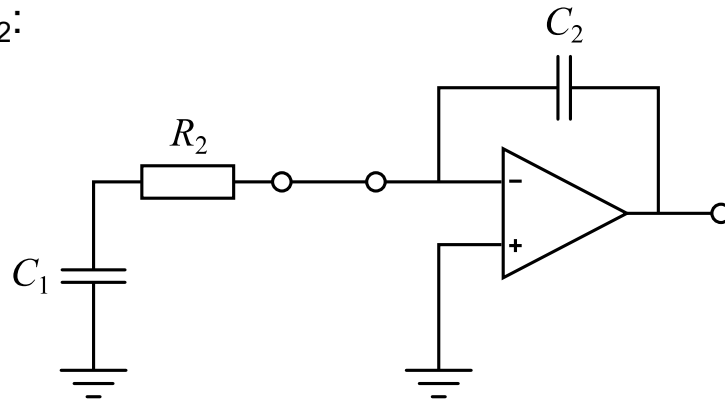
$$\left. \begin{array}{l} f_{CLK} = 1MHz \\ C = 5pF \end{array} \right\} \Rightarrow R_{ON} < 40k\Omega$$

Switch on-resistances in SC integrator

phase ϕ_1 :



phase ϕ_2 :



Phase ϕ_1 :

$$V_1(nT) = V_{in}(nT) \left(1 - e^{-T/2R_1C_1}\right) \quad T_{ov} = \frac{1}{f_{clk}}$$

Phase ϕ_2 :

$$\Delta q(nT + T/2) = C_1 v_1(nT) \left(1 - e^{-T/2R_2C_1}\right) \\ \Rightarrow \Delta v_{out} = \Delta q / C_2 = v_{out}(nT + T) - v_{out}(nT)$$

Z-domain transfer function:

$$H(z) = \frac{-\left(1 - e^{-T/2R_1C_1}\right)\left(1 - e^{-T/2R_2C_1}\right) \frac{C_1}{C_2}}{z - 1}$$

$$R_1 = R_2 = R$$

$$\Rightarrow \varepsilon = 1 - \left(1 - e^{-T/2RC_1}\right)^2 \approx 2e^{-T/2RC_1}$$

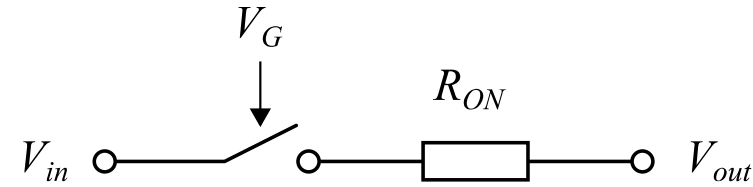
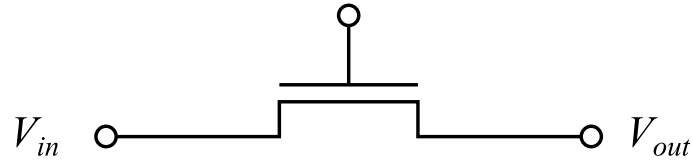
Example:

$$\varepsilon \leq 10^{-4} \dots 10^{-3} \Rightarrow \left(\frac{RC_1}{T}\right)^{-1} = \left(\frac{t_R}{T}\right)^{-1} \approx 7 \dots 10$$

$$C_1 = 5 \text{ pF} \quad f_c = 1 \text{ MHz}$$

$$R_{on} \approx 20 \text{ k}\Omega$$

MOS-switch



1. Switch open, $V_{GS} < V_T$

$$\Rightarrow R_{ON} = \infty$$

2. Switch closed, $V_{GS} > V_T$, V_{DS} small

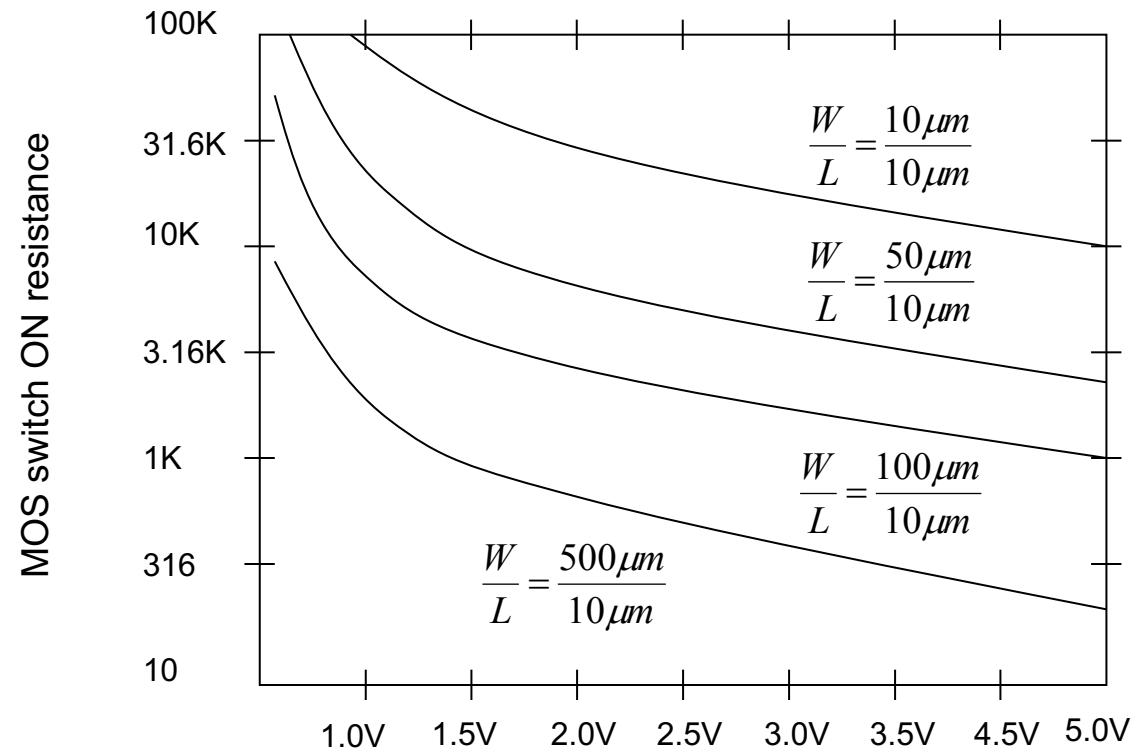
$\Rightarrow$ transistor in linear region

$$\frac{1}{R_{ON}} = \frac{\partial I_D}{\partial I_{DS}} = \mu C_{OX} \frac{w}{L} (V_{GS} - V_T)$$

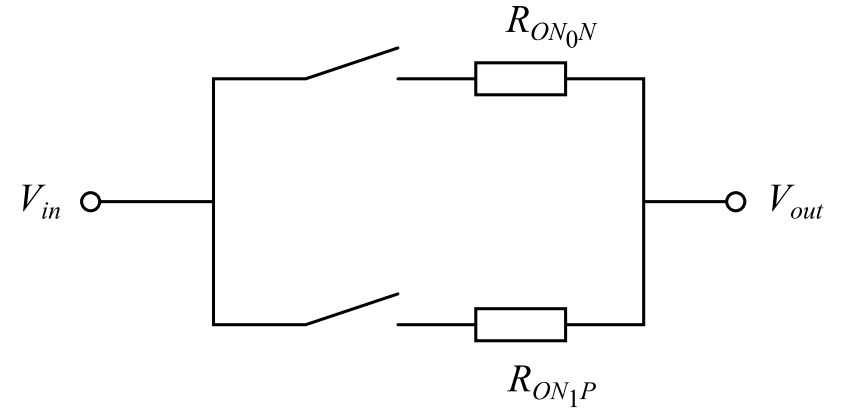
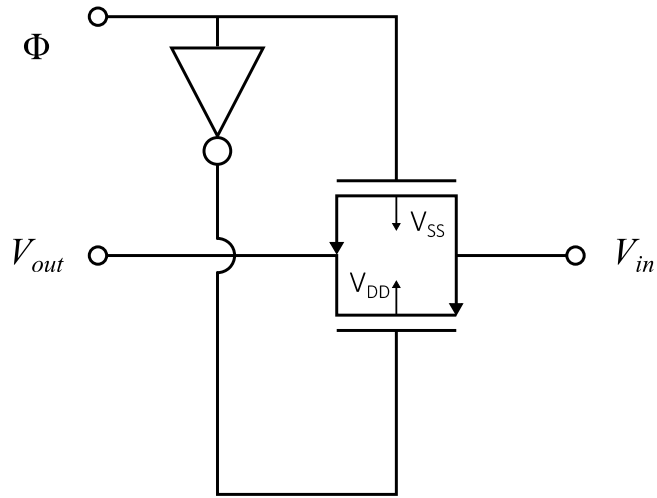
$$R_{ON} = \frac{1}{\mu C_{OX} \frac{w}{L} (V_{GS} - V_T)} \quad 200\Omega \dots 200k\Omega$$

- gate impedance infinite
- gate-capacitance causes clock-feedthrough
- C_{par} to substrate
- V_{in} , V_{out} limited
($V_{GS} = V_G - V_{in} > V_T$, $V_{DD} - V_{in} > V_T \Rightarrow V_{DD} - V_T > V_{in}$)

MOS-switch ON-resistance



CMOS-switch



Symmetrical design:

$$R_{ON_0N} \approx R_{ON_1P}$$

$$|V_{GSN} - V_T| = |V_{GSP} - V_T|$$

$$\left(\frac{w}{L}\right)_P \mu_P = \left(\frac{w}{L}\right)_N \mu_N$$

$$\Rightarrow \frac{\left(\frac{w}{L}\right)_P}{\left(\frac{w}{L}\right)_N} = \frac{\mu_N}{\mu_P} \quad (\sim 2,5)$$

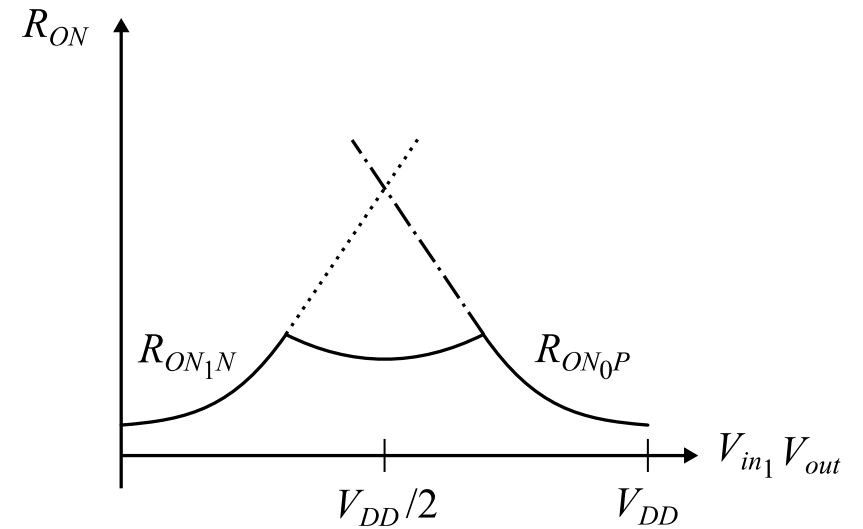
Not limited signal swings
(only by V_{DD} , V_{SS})

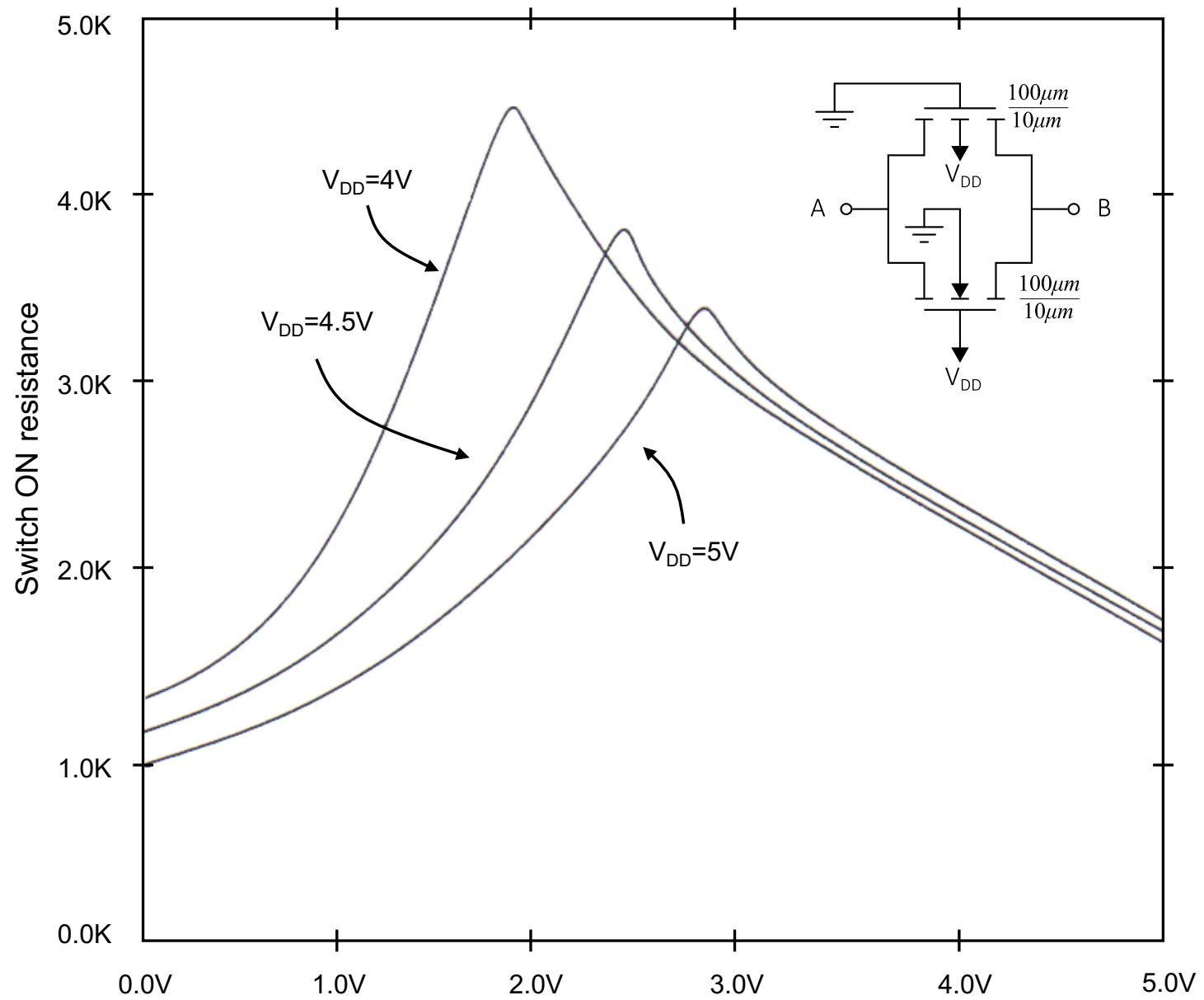
$$\frac{1}{R_{ON}} = \frac{1}{R_{ON_0N}} + \frac{1}{R_{ON_1P}}$$

$$R_{ON} = \frac{R_{ON_0N} \cdot R_{ON_1P}}{R_{ON_0N} + R_{ON_1P}}$$

$$V_{in,CM} = V_{out} \sim \frac{V_{DD}}{2}$$

half way between supplies





Effect of amplifier finite gain in SC integrator

Output voltage including finite amplifier gain

$$V_{out}(nT) = V_{C2}(nT) - \frac{1}{A_o} V_{out}(nT)$$

Charge transfer:

$$C_2[V_{C2}(nT) - V_{C2}(nT - T)] + C_1\left[V_{in}(nT) + \frac{1}{A_o} V_{out}(nT)\right] = 0$$

Apply z-transformation:

$$V_{out}(z) = V_{C2}(z) - \frac{1}{A_o} V_{out}(z)$$

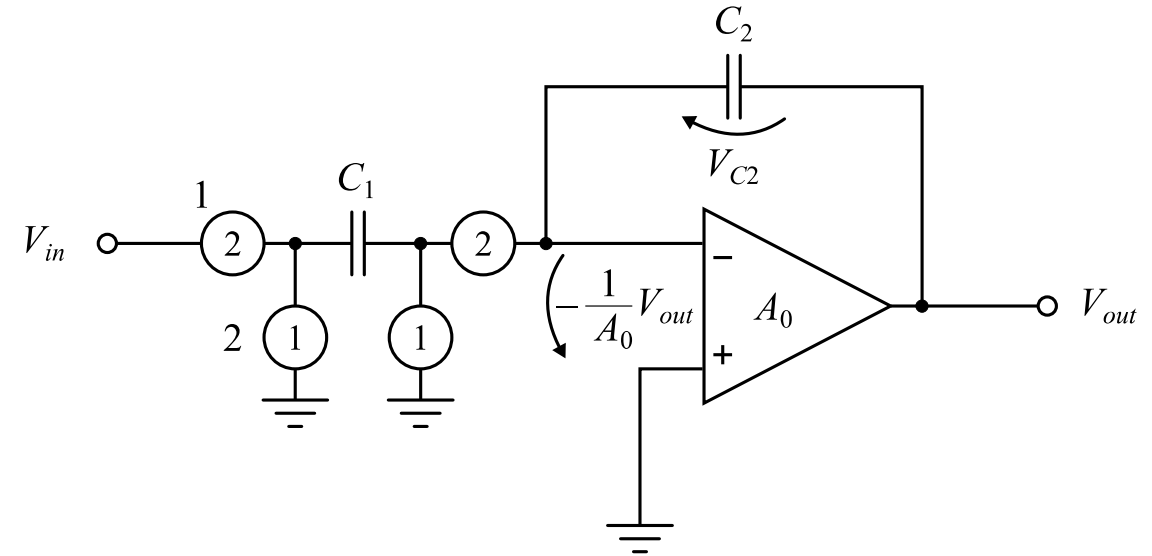
$$C_2[V_{C2}(z) - z^{-1}V_{C2}(z)] + C_1\left[V_{in}(z) + \frac{1}{A_o} V_{out}(z)\right] = 0$$

Transfer function in z-domain

$$H(z) = \frac{V_{out}(z)}{V_{in}(z)} = \frac{-(C_1/C_2)[1 + (1 + C_1/C_2)/A_o]^{-1}z}{z - (1 + 1/A_o)/[1 + (1 + C_1/C_2)/A_o]}$$

when $A_o \Rightarrow \infty \Rightarrow$ we obtain ideal integrator in z - domain i.e.

$$H_i(z) = \frac{-(C_1/C_2)z}{z - 1}$$



Finite amplifier gain A_0 causes gain and phase error

Insert $z = e^{j\omega T}$ into $H(z)$

$$H(e^{j\omega T}) = \underbrace{\frac{-(C_1/C_2)e^{j\omega T/2}}{j2\sin(\omega T/2)}}_{H_i(e^{j\omega T})} \cdot \frac{1}{1 + (1/A_0)(1 + C_1/C_2) - j(C_1/C_2)/2A_0 \tan(\omega T/2)}$$

ideal z-domain integrator

error-term

$$H(e^{j\omega T}) = H_i(e^{j\omega T}) \cdot \frac{1}{1 - m(\omega) - j\theta(\omega)}$$

$m(\omega)$ = magnitude error

$\theta(\omega)$ = phase error

$$m(\omega) = -\frac{1}{A_0} \left(1 + \frac{C_1}{C_2} \right)$$

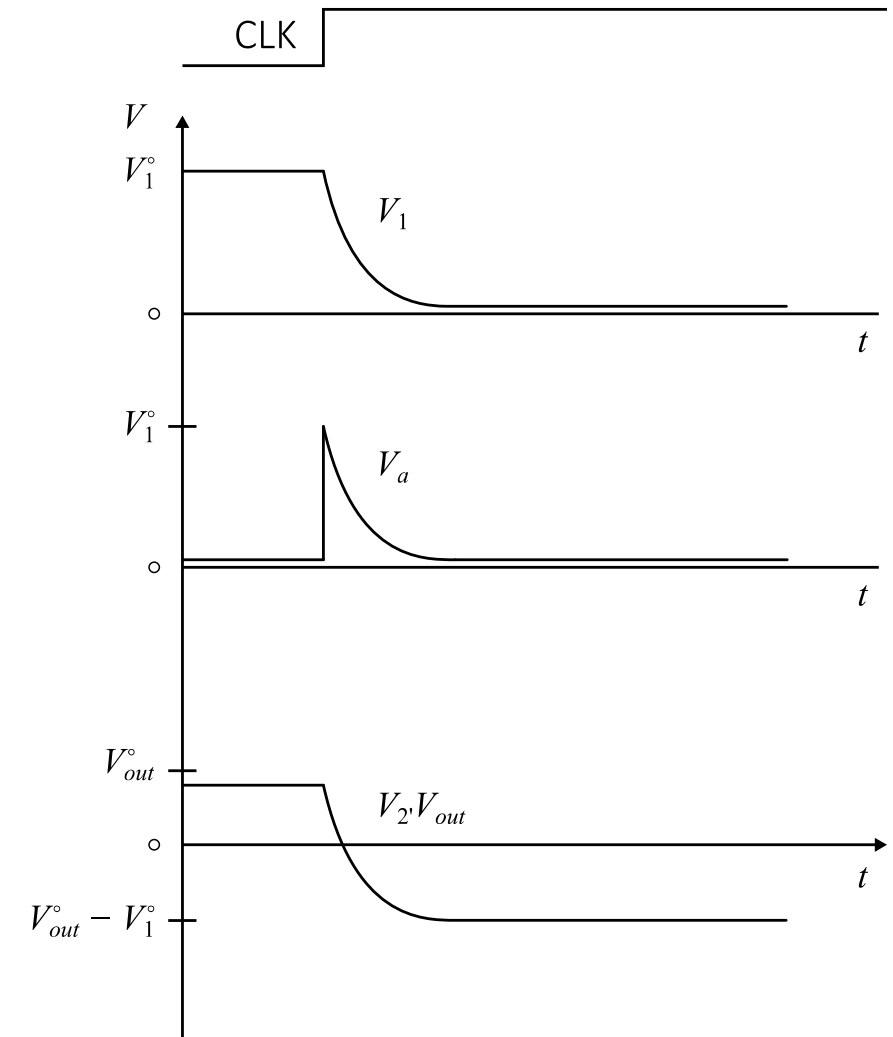
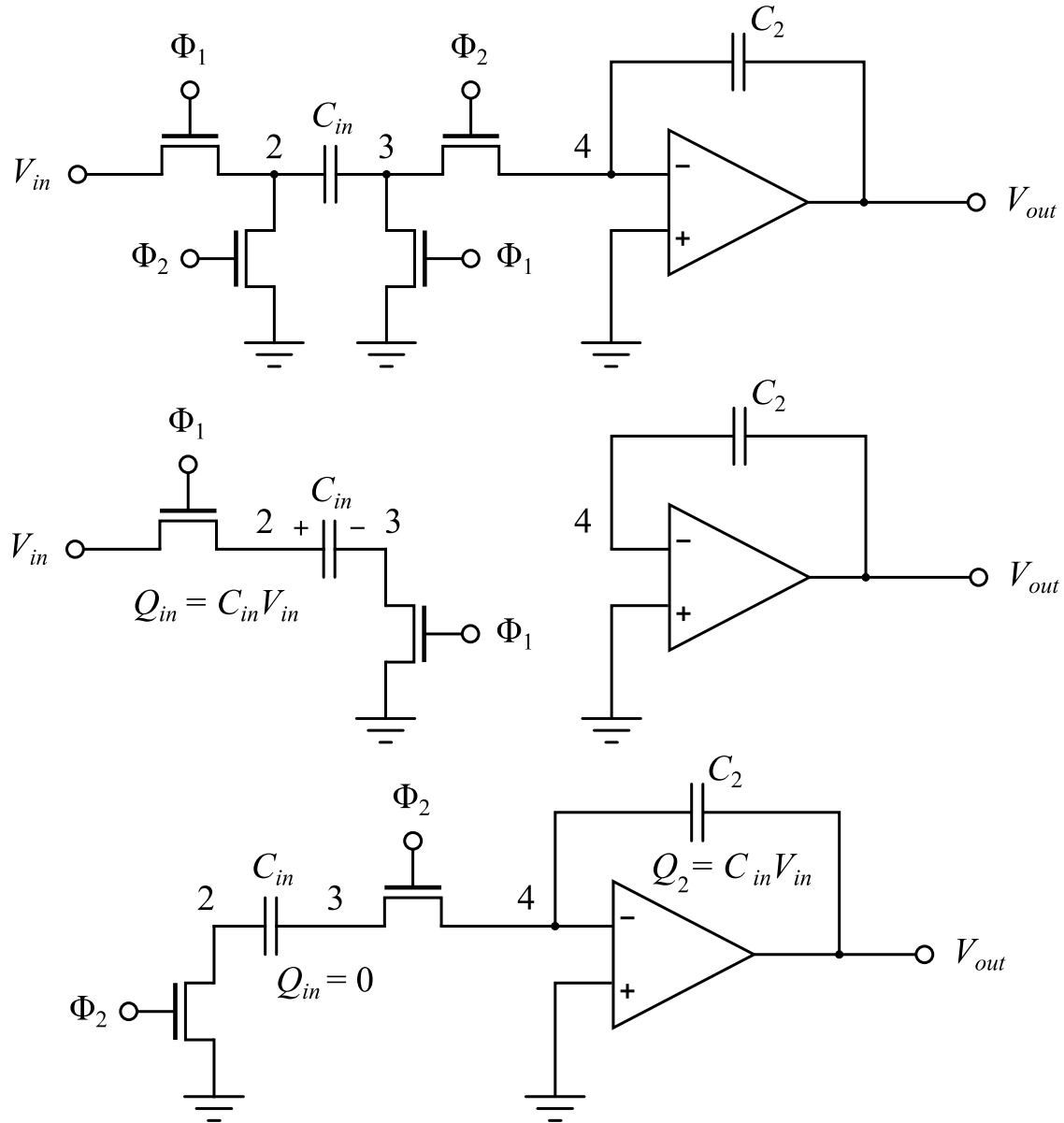
$$\theta(\omega) = \frac{C_1/C_2}{2A_0 \tan(\omega T/2)} \approx \frac{C_1/C_2}{A_0 \omega T}$$

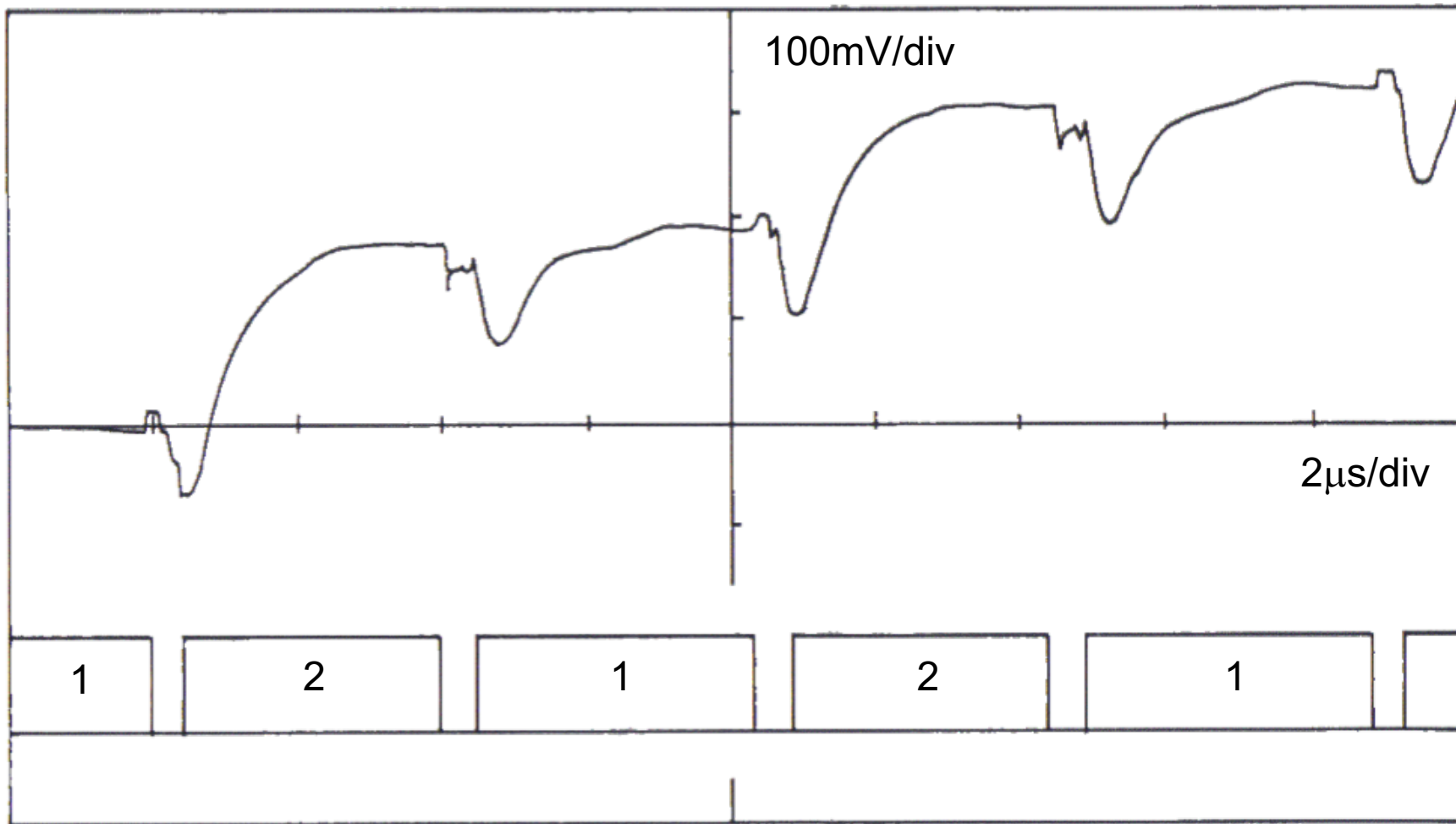
$$m(\omega), \theta(\omega) \propto \frac{1}{A_0}$$

$$\left(\frac{C_1}{C_2} \right) \rightarrow (1 + m(\omega)) \frac{C_1}{C_2} \propto \left(1 + \frac{1}{A_0} \right) \frac{C_1}{C_2}$$

$$\Rightarrow \frac{\Delta \frac{C_1}{C_2}}{C_1/C_2} = \frac{1}{A_0}$$

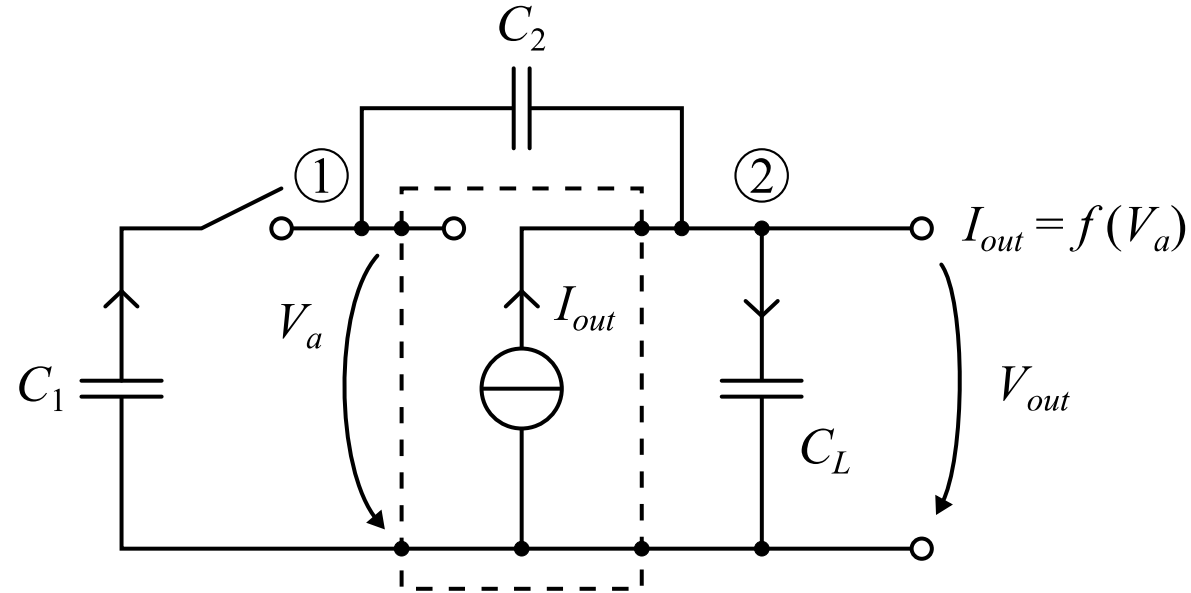
Charge transfer in SC integrator





Charge transfer transient of the 4th order elliptical low-pass filter.

Charge transfer transient



The current equations for nodes ① and ② $I(t) = C \frac{dV}{dt}$

$$(1) \quad C_1 \frac{dV_a}{dt} - C_2 \frac{d(V_{OUT} - V_a)}{dt} = 0$$

$$(2) \quad I_{OUT}(t) - C_L \frac{dV_{OUT}}{dt} - C_2 \frac{d(V_{OUT} - V_a)}{dt} = 0$$

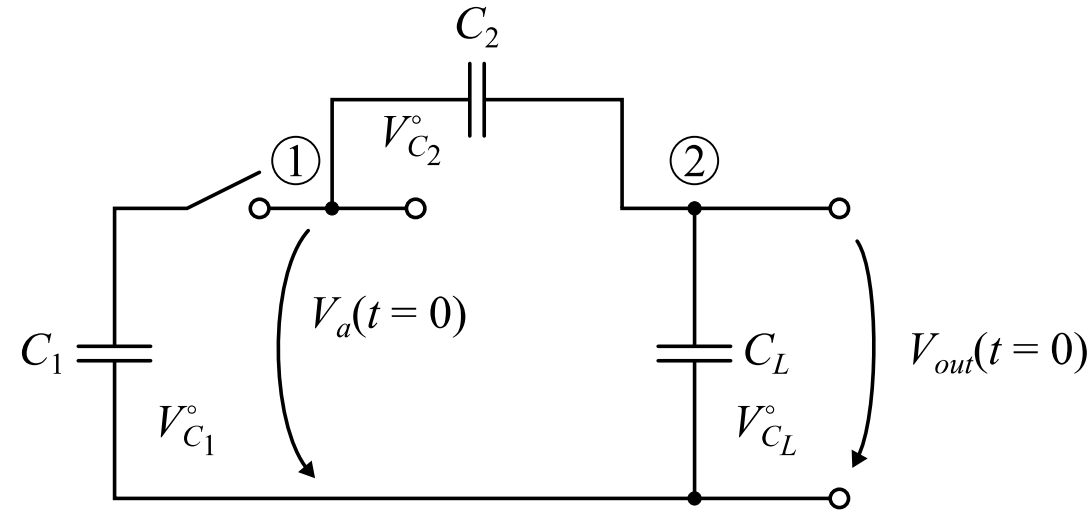
eliminating $\frac{dV_{OUT}}{dt} \Rightarrow$ we obtain charge transfer equation

$$\frac{C_1 C_2 + C_L C_1 + C_L C_2}{C_2} \cdot \frac{dV_a}{dt} + I_{OUT}(t) = 0$$

$V_{OUT}(t)$ is solved from eq. (1)

$$V_{OUT}(t) = V_{OUT}(t=0) + \frac{C_1 + C_2}{C_2} [V_a(t) - V_a(t=0)]$$

Calculation of initial voltages $V_a(t=0)$ and $V_{OUT}(t=0)$



Before switching the capacitor voltages are

$$V_{C1}^{\circ}, V_{C2}^{\circ} \text{ and } V_{CL}^{\circ}$$

After closing the switch the equilibrium is obtained between the charges of the capacitors

$$V_{C2} + V_a(t=0) = V_{OUT}(t=0)$$

Charge balance at nodes ① and ②

$$\Delta Q_{C1} = -\Delta Q_{C2}$$

$$-\Delta Q_{C2} = \Delta Q_{CL}$$

To obtain this a charge transfer between the capacitors occur

$$(1) \quad C_1(V_{C1}^{\circ} - V_a(t=0)) = C_2V_{C2}^{\circ} + C_2(V_a(t=0) - V_{OUT}(t=0))$$

$$(2) \quad C_L(V_{CL}^{\circ} - V_{OUT}(T=0)) = C_2V_{C2}^{\circ} + C_2(V_a(t=0) - V_{OUT}(t=0))$$

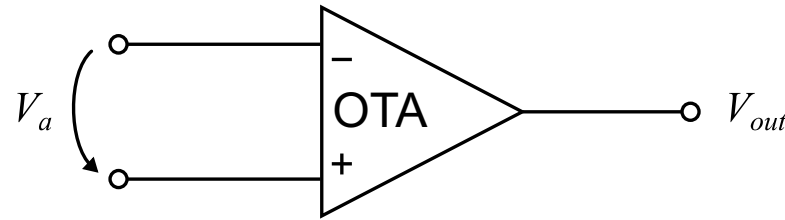
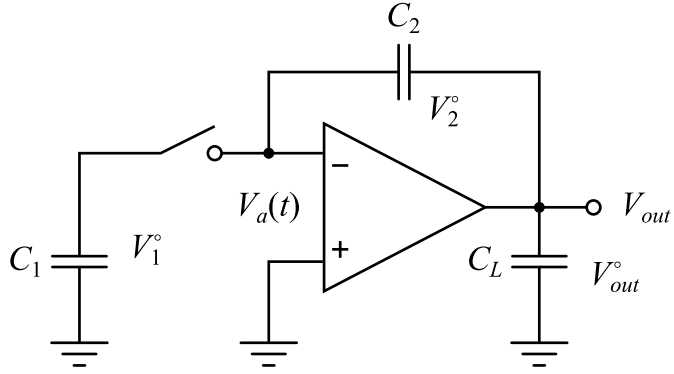
From these we solve the initial conditions

$$V_a(t=0) \text{ and } V_{OUT}(t=0)$$

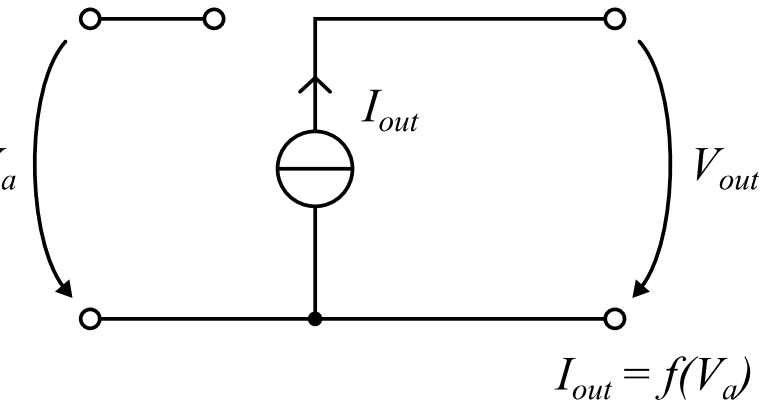
$$V_a(t=0) = -\frac{C_1C_2 + C_1C_L}{C_1C_L + C_1C_2 + C_2C_L}v_{C1}^{\circ} + \frac{C_2C_L}{C_1C_2 + C_1C_L + C_2C_L}(v_{CL}^{\circ} - v_{C1}^{\circ})$$

$$V_{out}(t=0) = \frac{C_1C_L + C_2C_L}{C_1C_L + C_1C_2 + C_2C_L}v_{CL}^{\circ} + \frac{C_1C_2}{C_1C_2 + C_1C_L + C_2C_L}(v_{C2}^{\circ} - v_{C1}^{\circ})$$

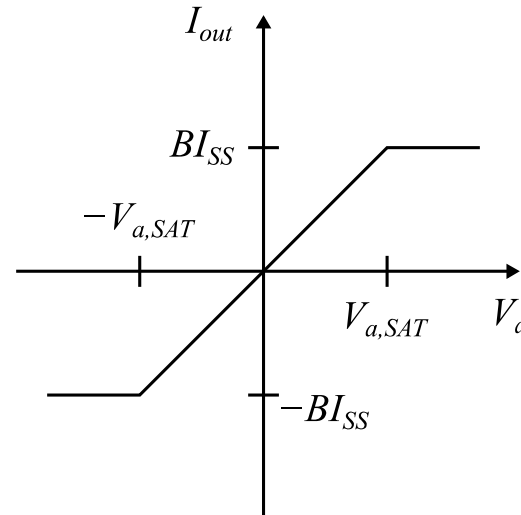
Model of OTA-amplifier (voltage controlled current-source)



Voltage controlled current-source



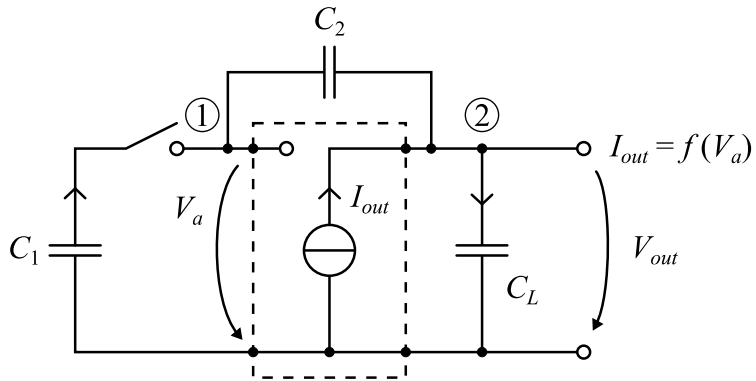
OTA transfercurve

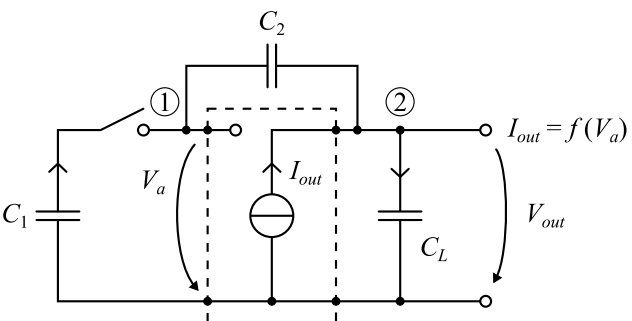


OTA output current:

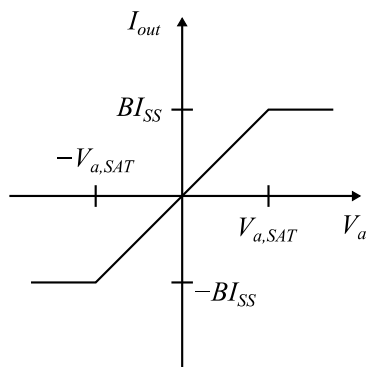
$$I_{out} = \begin{cases} -BI_{SS}; & V_a < -V_{a,SAT} \\ g_m V_a; & -V_{a,SAT} < V_a < V_{a,SAT} \\ BI_{SS}; & V_a > V_{a,SAT} \end{cases}$$

(Depending on V_a regions of operation:
linear and slewing)





OTA transfer curve:



OTA output current:

$$I_{out} = \begin{cases} -BI_{SS}; & V_a < -V_{a,SAT} \\ g_m V_a; & -V_{a,SAT} < V_a < V_{a,SAT} \\ BI_{SS}; & V_a > V_{a,SAT} \end{cases}$$

I Small-signal settling (linear):

assume $|V_a(t_0)| < V_{a,SAT} \Rightarrow I_{OUT}(t) = g_m V_a(t)$

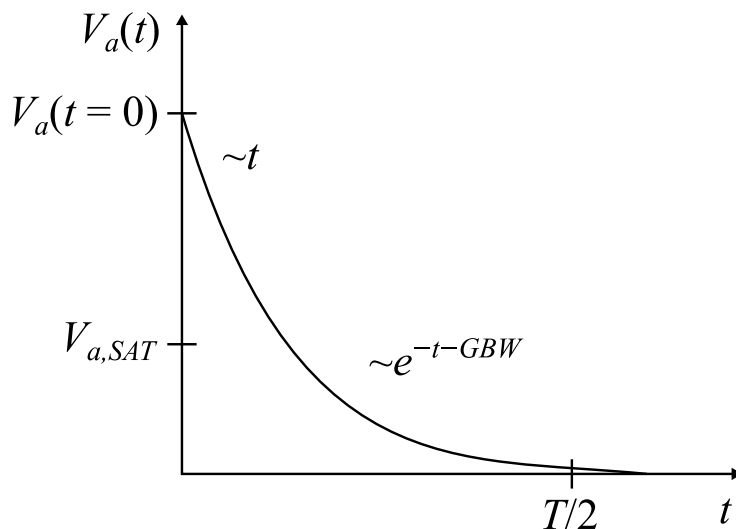
$$\frac{C_1 C_2 + C_L C_1 + C_L C_2}{C_2} \cdot \frac{dV_a(t)}{dt} + g_m V_a(t) = 0$$

solving $V_a(t)$:

$$V_a(t) = V_a(t=0) \exp\left(-\frac{g_m}{C_{L,eff}} \cdot t\right) \quad ; C_{L,eff} = \frac{C_1 C_2 + C_L C_1 + C_L C_2}{C_2}$$

$$\frac{g_m}{C_{L,eff}} \alpha GBW$$

$$|V_a(t)| = V_a(0) e^{-GBW \cdot t}$$



II Settling with slewing:

assume $|V_a(t_0)| > V_{a,SAT} \Rightarrow I_{OUT}(t) = (\pm) BI_{SS}$

$$\frac{C_1 C_2 + C_L C_1 + C_L C_2}{C_2} \cdot \frac{dV_a(t)}{dt} (\pm) BI_{SS} = 0$$

solving $V_a(t)$:

$$V_a(t) = V_a(t=0) (\mp) \frac{B \cdot I_{SS}}{C_{L,eff}} \cdot t \quad ; SR = \frac{B \cdot I_{SS}}{C_{L,eff}}$$

slew - period :

$$|V_a(T_{SR})| = V_{a,SAT} = |V_a(t=0)| - \frac{B \cdot I_{SS}}{C_{L,eff}} \cdot T_{SR}$$

$$\Rightarrow T_{SR} = \frac{|V_a(t=0)| - V_{a,SAT}}{B \cdot I_{SS}} \cdot C_{L,eff}$$

linear settling after slewing

$$|V_a(t)| < V_{a,SAT} \quad \text{i.e. } t > T_{SR}$$

$$V_a(t) = V_{a,SAT} \exp\left(-\frac{g_m}{C_{L,eff}} \cdot (t - T_{SR})\right)$$

Charge transfer error

Relative charge transfer error:

$$\varepsilon = \frac{\Delta Q(t \rightarrow \infty) - \Delta Q(T)}{\Delta Q(t \rightarrow \infty)}$$

$$\Delta Q(t \rightarrow \infty) = C_{in} V_{in}$$

I assume $|V_a(t=0)| < V_{a,SAT}$

$$\Delta Q(t \rightarrow \infty) - \Delta Q(T) = C_{in} \cdot V_a(T) = C_{in} V_a(t=0) e^{\frac{-g_m \cdot T}{C_{L,e}}}$$

Error with small signal settling

$$\Rightarrow \left[\varepsilon = \frac{C_{in} V_{in}}{C_{in} V_{in}} \cdot e^{\frac{-g_m \cdot T}{C_{L,e}}} = e^{\frac{-g_m \cdot T}{C_{L,e}}} \right] \quad (\phi V_{in}) \quad T = \frac{1}{2f_{CLK}}$$

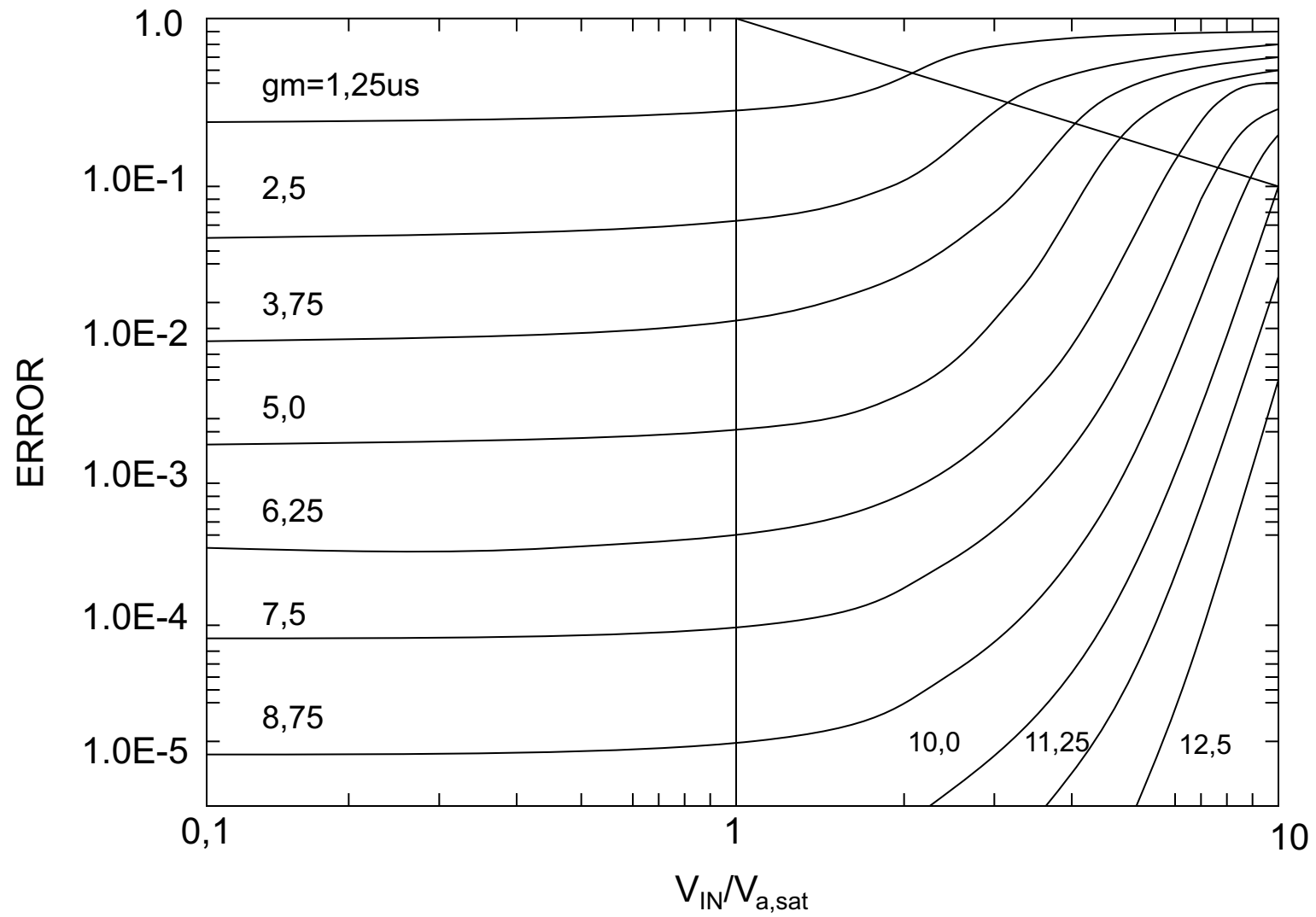
$\Rightarrow$ Only gain error i.e. linear settling-mode!

II error with slewing

assume $|V_a(t=0)| > V_{a,SAT}$

$$\Rightarrow \varepsilon = \frac{C_{in} V_{a,SAT}}{C_{in} V_{in}} \cdot e^{\frac{-g_m \cdot (T - T_{SR})}{C_{L,e}}}$$

$T_{SR} = f(V_{in}) \Rightarrow$ Slewing causes non-linearity!



Charge transfer error with the conventional OTA.

SC integrator transfer function including linear charge transfer error

$$\Delta Q_{C_2} = C_1 v_{in}(nT) \left(1 - e^{-\frac{g_m \cdot T/2}{C_{L,e}}} \right) \quad ; f_{CLK} = \frac{1}{T}$$

$$\Rightarrow \Delta V_{OUT} = \frac{\Delta Q_{C_2}}{C_2} = v_{OUT}(nT + T) - v_{OUT}(nT)$$

$$\frac{C_1}{C_2} v_{in}(nT) \left(1 - e^{-\frac{g_m \cdot T/2}{C_{L,e}}} \right) = v_{OUT}(nT + T) - v_{OUT}(nT)$$

Perform z-transformation

$$\frac{C_1}{C_2} v_{in}(z) \left(1 - e^{-\frac{g_m \cdot T/2}{C_{L,e}}} \right) = v_{OUT}(z) \cdot z - v_{OUT}(z)$$

$$\Rightarrow H(z) = \frac{v_{OUT}(z)}{v_{in}(z)} = \frac{C_1}{C_2} \frac{\left(1 - e^{-\frac{g_m \cdot T/2}{C_{L,e}}} \right)}{z - 1}$$

Linear error causes only gain error!

Slewing causes also phase error!!

Non-ideal amplifiers in biquad

Magnitude error $m(\omega)$ causes only capacitor ratio error in the integrators.

Ideal integrator:

$$H(s) = \frac{-1/c}{s}$$

Non-ideal integrator:

$$H(s) = \frac{-(1+m)/c}{s + \delta i}$$

In the SC integrator:

$$H_{SC}(s) = \frac{-(1+m) \frac{C_1}{TC_2}}{s + \delta i} = \frac{-\frac{1}{T} \cdot \left(\frac{C_1}{C_2} \right)'}{s + \delta i}$$

Thus magnitude error causes the shift of the pole and cut-off frequency, which can be compensated by predistorting the capacitor ratio

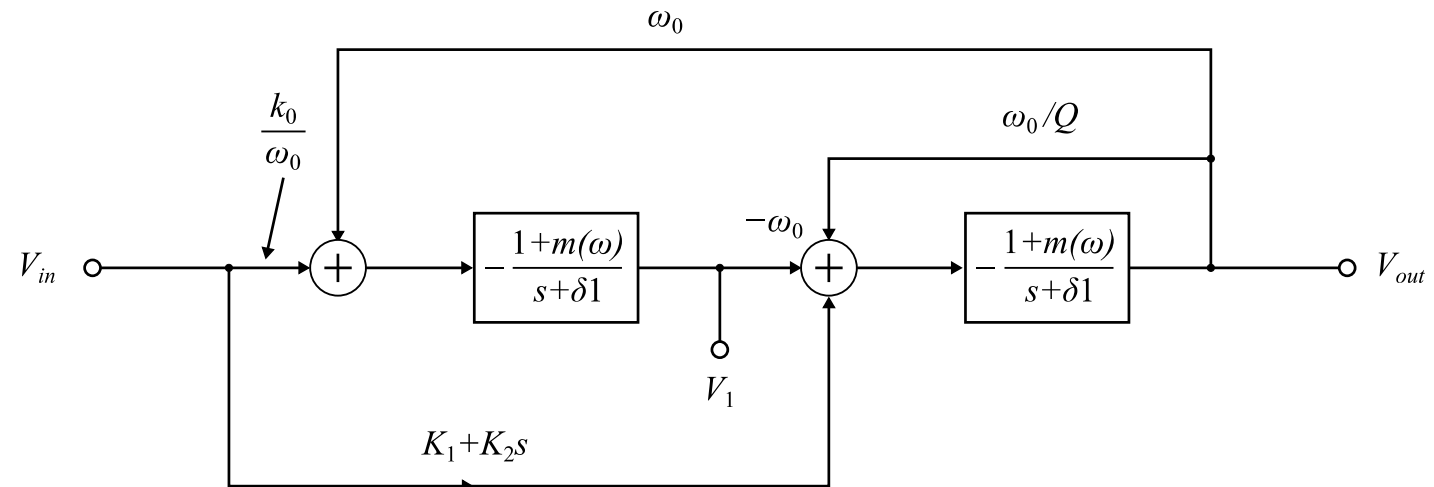
$$\frac{C_1}{C_2} \rightarrow (1+m) \frac{C_1}{C_2}$$

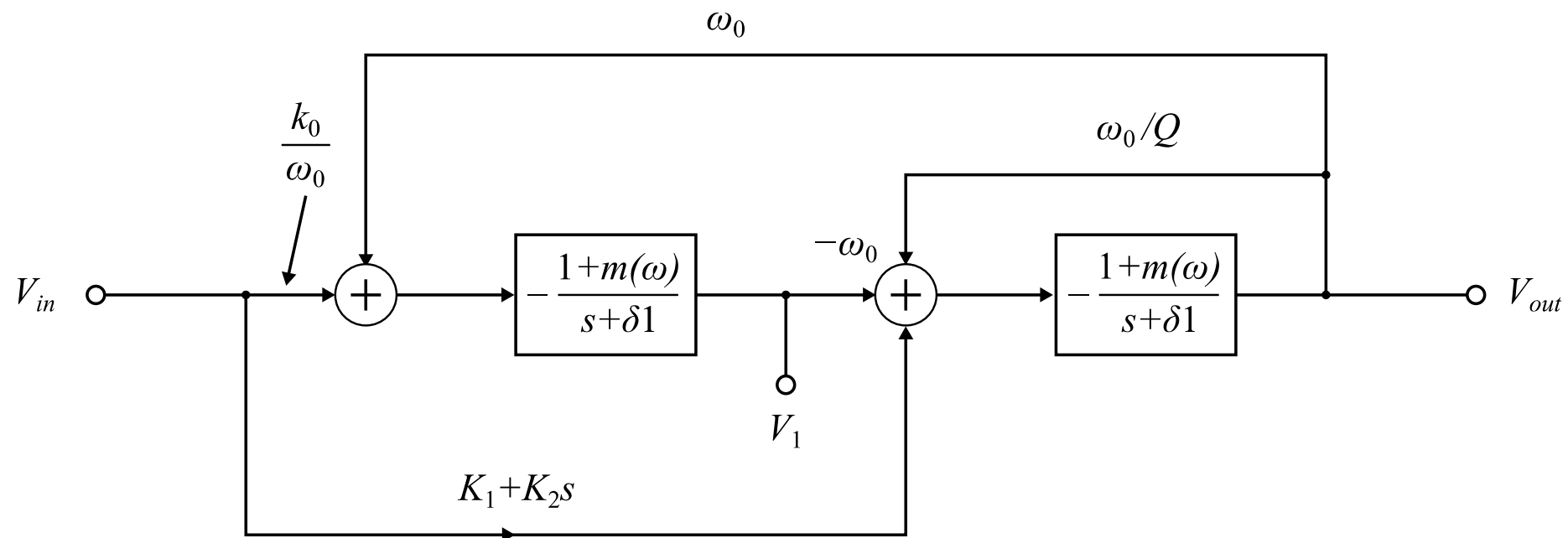
For amplifier finite gain:

$$m \sim \frac{1}{A_0} \Rightarrow \omega_p' \simeq \left(1 - \frac{1}{A_0} \right) \omega_p$$

$$A = 60dB = 1000$$

$$\Rightarrow \frac{\Delta \omega_p}{\omega_p} \approx 0,1\%$$





Phase error due to amplifier finite gain (from integrator analysis):

$$\text{phase error } \theta(\omega): \quad H(S) \cong -\frac{\frac{C_1}{C_2 T}}{S + \frac{C_1}{C_2 A_o T}}$$

integrator is lossy and has a pole

$$\delta i = \frac{C_1/C_2}{A_o T}$$

To analyse the biquad's performance we insert the lossy integrator into the SFG

$$\frac{1}{S} \rightarrow \frac{1}{S + \delta_i}$$

Now the state equations are

$$(1) \quad V_1(S) = -\frac{1}{S + \delta_{i1}} \left[\omega_o V_{OUT}(S) + \frac{K_o}{\omega_o} V_{in} \right]$$

$$(2) \quad V_{OUT}(S) = -\frac{1}{S + \delta_{i2}} \left[(K_1 + K_2 S) V_{in} + \frac{\omega_o}{Q} V_{OUT}(S) - \omega_o V_1(S) \right]$$

From these we may solve the transfer function $\frac{V_{OUT}}{V_{in}}$ with the lossy integrations

The poles are given by the denominators, which is

$$S_p^2 + \left(\frac{\omega_o}{Q} + \delta_{i1} + \delta_{i2} \right) S_p + \left(\omega_o^2 + \frac{\omega_o \delta_{i1}}{Q} + \delta_{i1} \delta_{i2} \right) = 0$$

with the ideal amplifiers :

$$S^2 + \frac{\omega_o}{Q} S + \omega_o^2 = 0$$

Thus the phase error distorts the pole Q

$$\frac{\omega_o}{Q'} \sim \frac{\omega_o}{Q} + \delta_{i1} + \delta_{i2}$$

with ideal integrator $\omega_o = \frac{C_1}{TC_2}$

$$\Rightarrow \frac{\delta_{i1}}{\omega_o} = \frac{1}{A_o}$$

dividing with

$$\frac{1}{Q'} \sim \frac{1}{Q} + \frac{\delta_{i1}}{\omega_o} + \frac{\delta_{i2}}{\omega_o}$$

$$\boxed{\frac{1}{Q'} \sim \frac{1}{Q} + \frac{2}{A_o}}$$

$$\left. \begin{array}{l} \text{Example : } Q = 15 \\ A = 1000 \end{array} \right\} \Rightarrow Q' = 14.56$$

Effect on the pass - band gain at center frequency

$$S \cong j\omega_o \quad S^2 \simeq \omega_o^2$$

the denominator is given by

$$S^2 + \left(\frac{\omega_o}{Q}\right)S + \omega_o^2 \approx \frac{\omega_o}{Q} \cdot S \quad H(S \approx j\omega_o) \propto \frac{1}{\frac{\omega_o \cdot S}{Q}}$$

At center frequency the gain depends on the Q - factor

$$H(S \sim j\omega_o) \propto \frac{1}{Q}$$

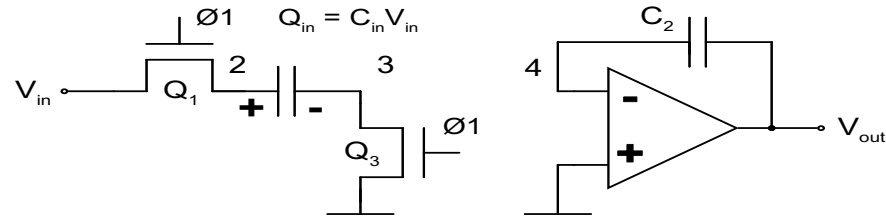
The error in Q - factor causes errors in the gain

$$\Rightarrow \varepsilon = \frac{|H'(S = j\omega_o)|}{|H(S = j\omega_o)|} \approx \frac{Q}{Q'}$$

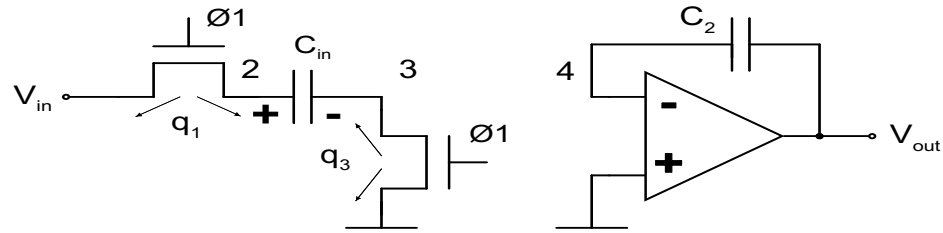
$$\Rightarrow \Delta\alpha[dB] = 20 \log\left(\frac{Q}{Q'}\right) = 20 \log\left(1 + 2 \frac{Q}{A_o}\right)$$

$$\text{Example : } \left. \begin{array}{l} Q = 15 \\ A = 1000 \end{array} \right\} \Rightarrow \Delta\alpha = 0,26dB$$

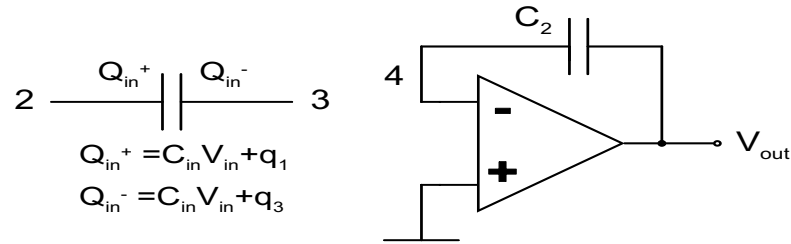
Release of switch transistor channel charge in SC integrator:



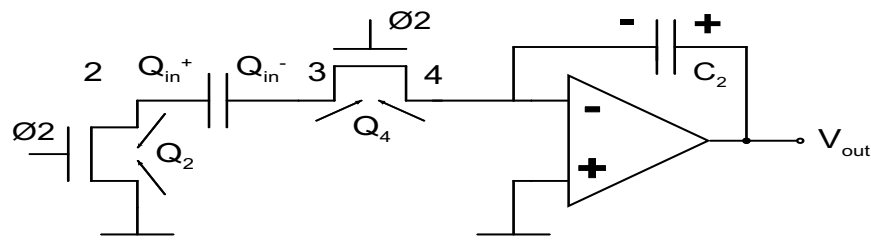
a) holding phase



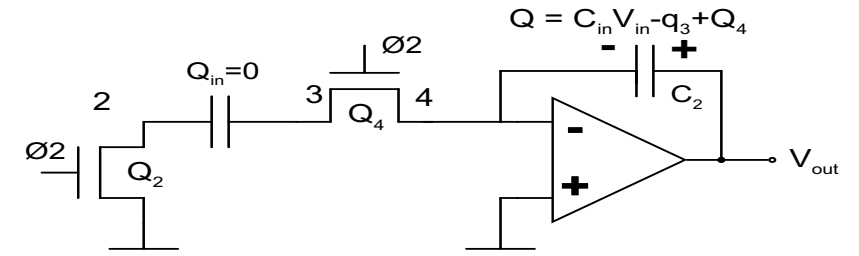
b) closer of SW1 and SW3



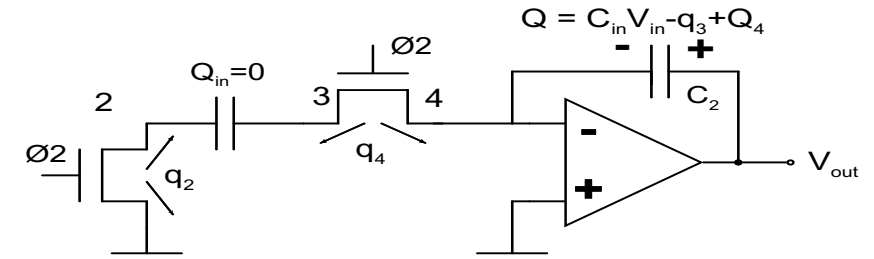
c) non-overlap period between holding and integrating phases



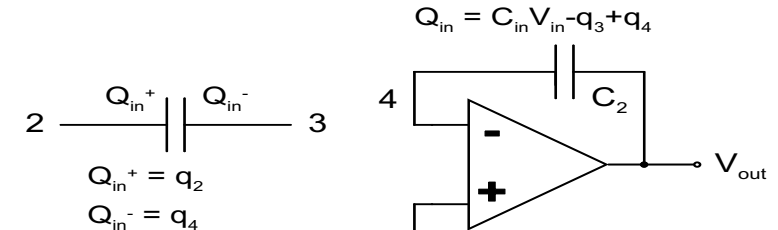
d) opening of SW2 and SW4



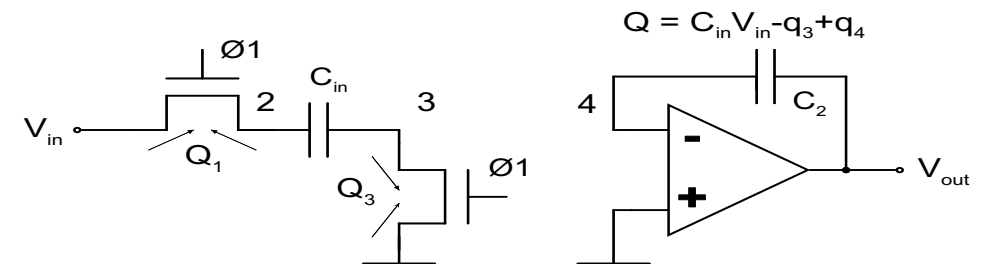
e) integrating phase



f) closer of SW2 and SW4

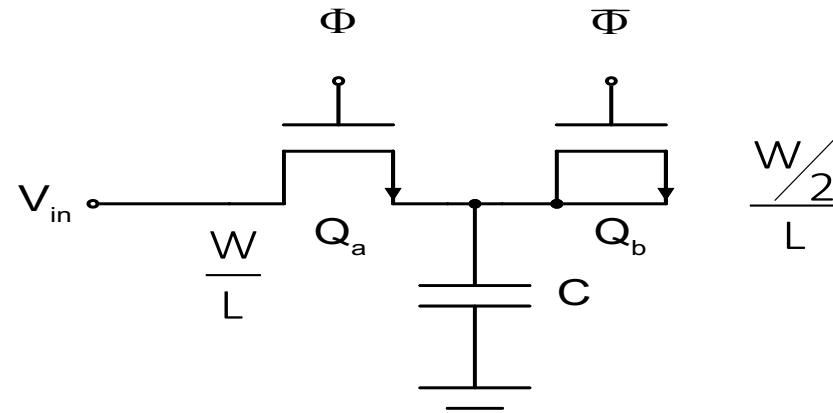


g) non-overlap period between integrating and holding phases

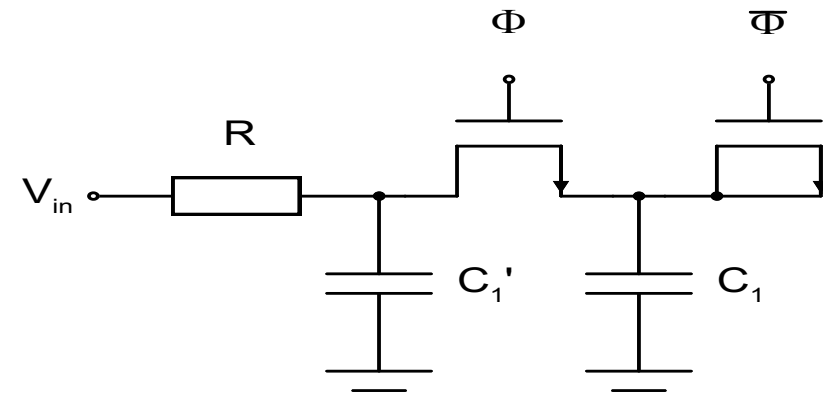
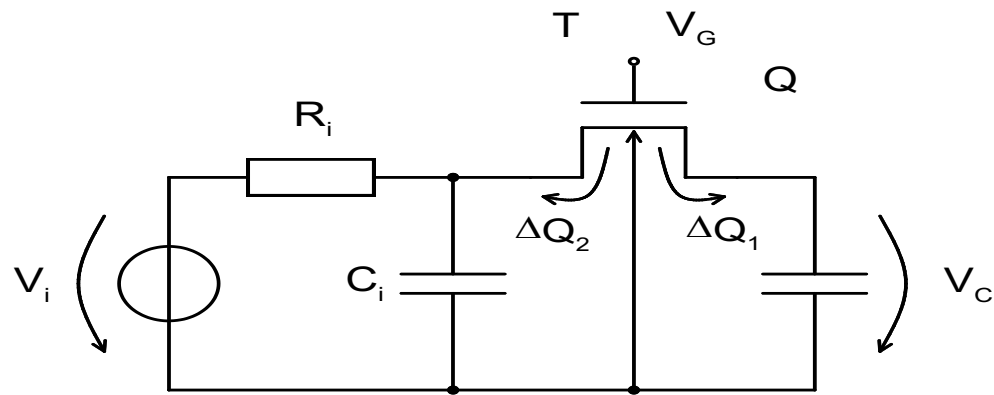


d) opening of SW3 and SW3

Cancellation of switch channel charge with a half sized transistor

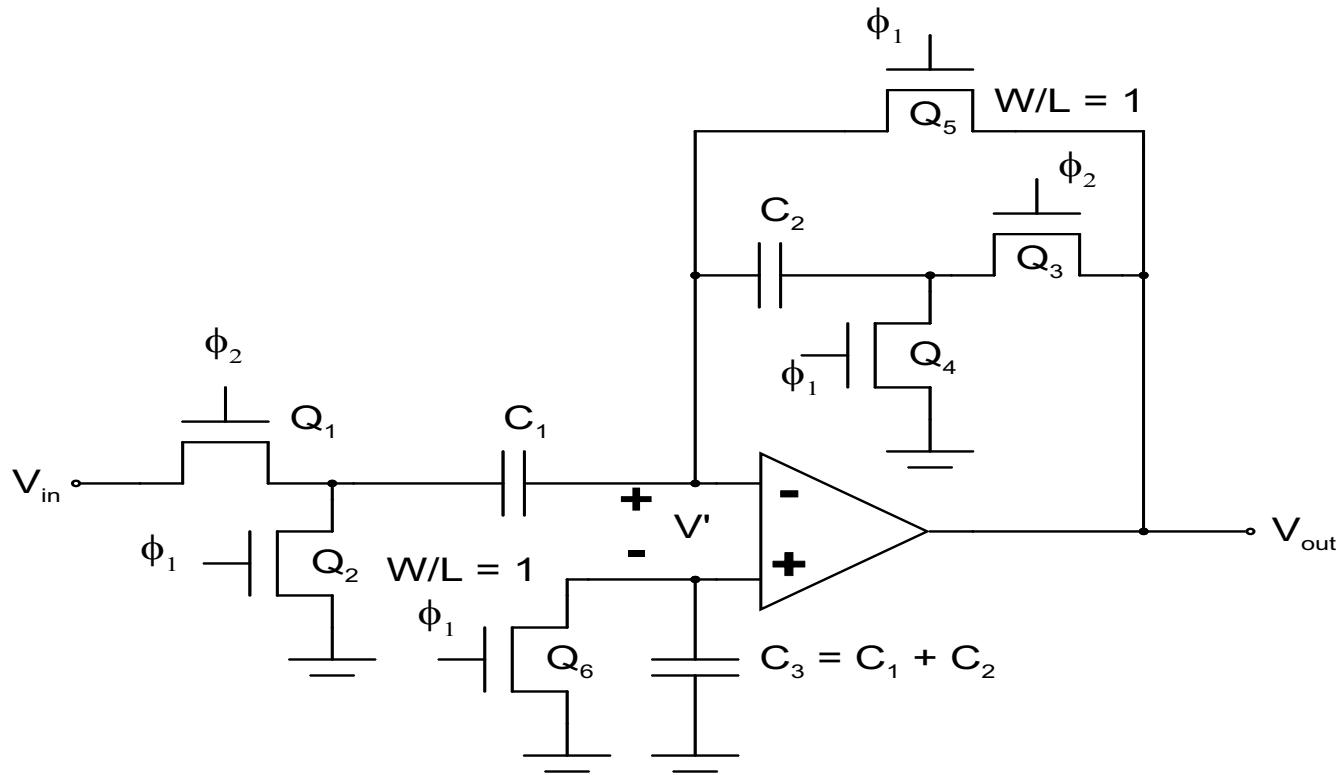


Equalization of termination impedances of the switch transistor

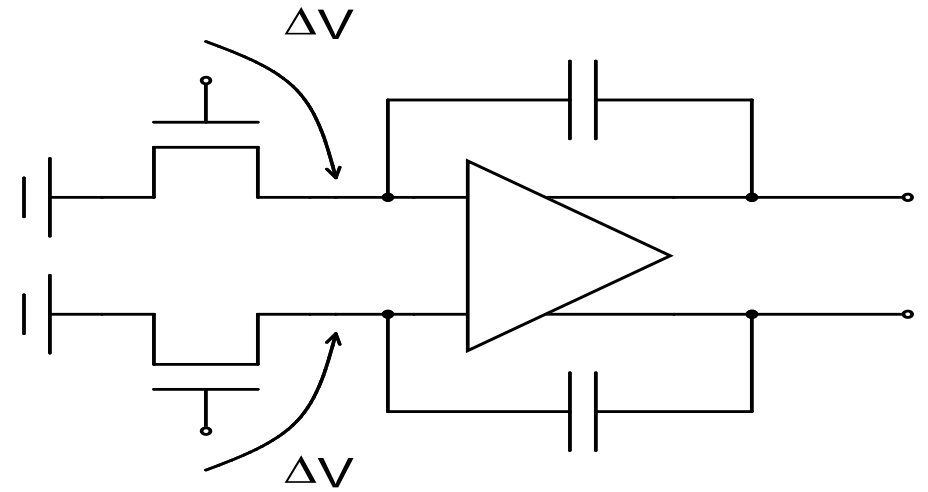


Cancellation of switch charge with fully-differential circuit topology

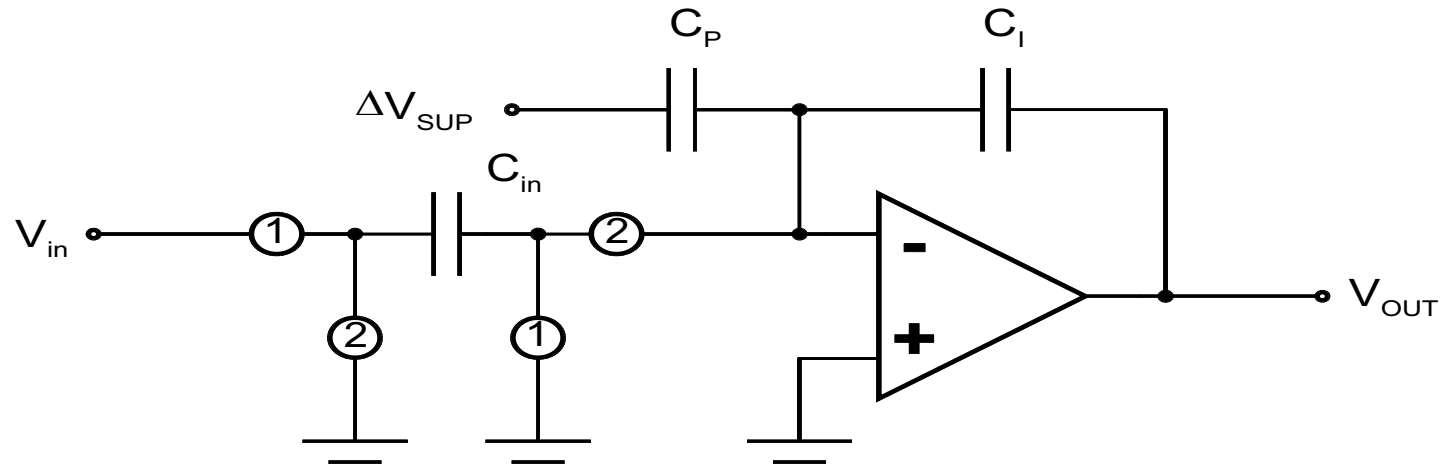
Semi-differential circuit using transistor Q_6 to cancel Q_5 channel charge:



Fully-differential circuit where channel charge of equal sized switch transistors appears as a common-mode disturbance:



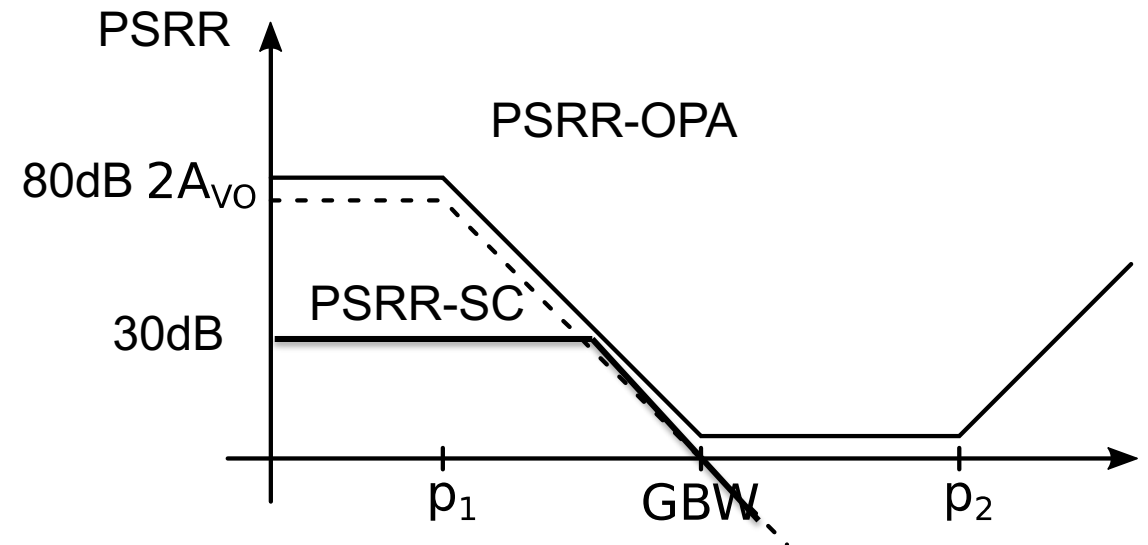
The SC-integrator model for the system PSRR_analysis



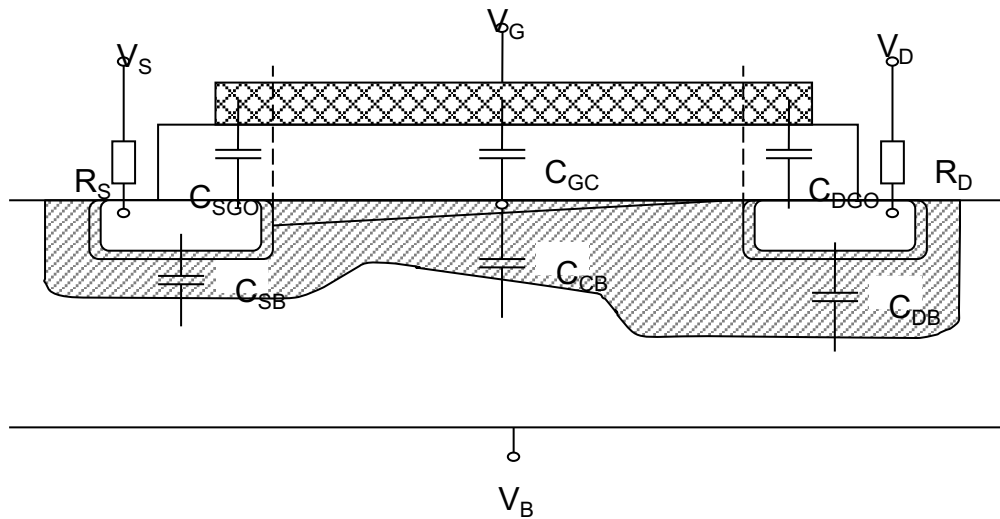
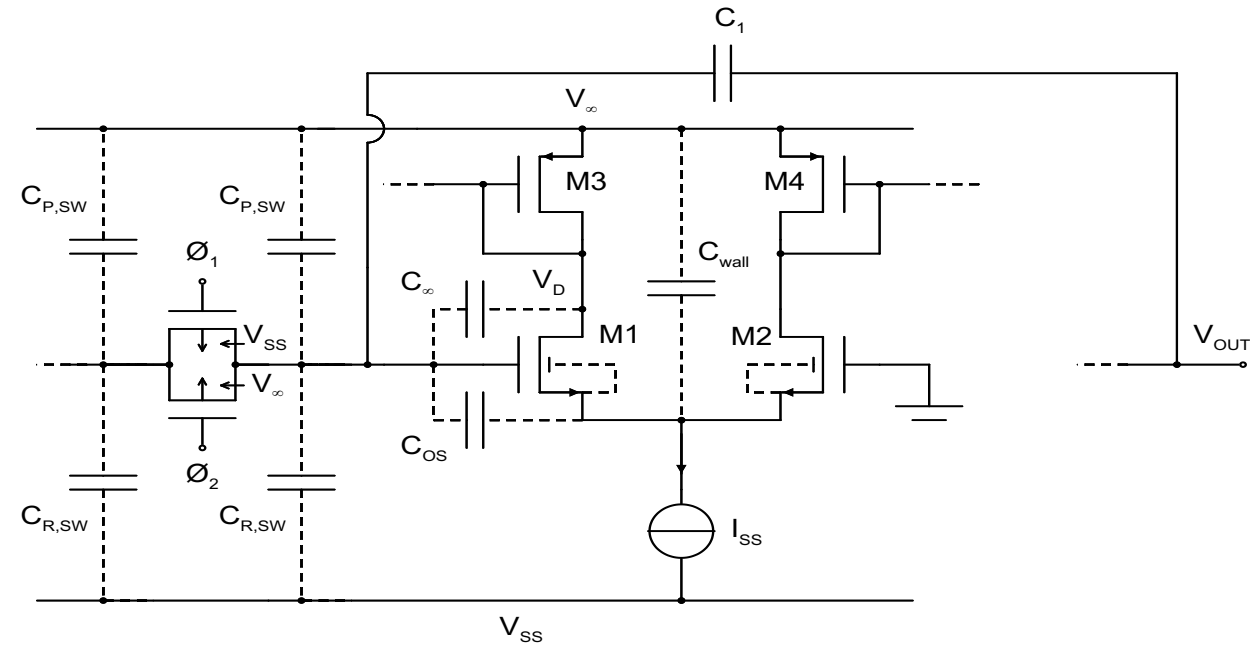
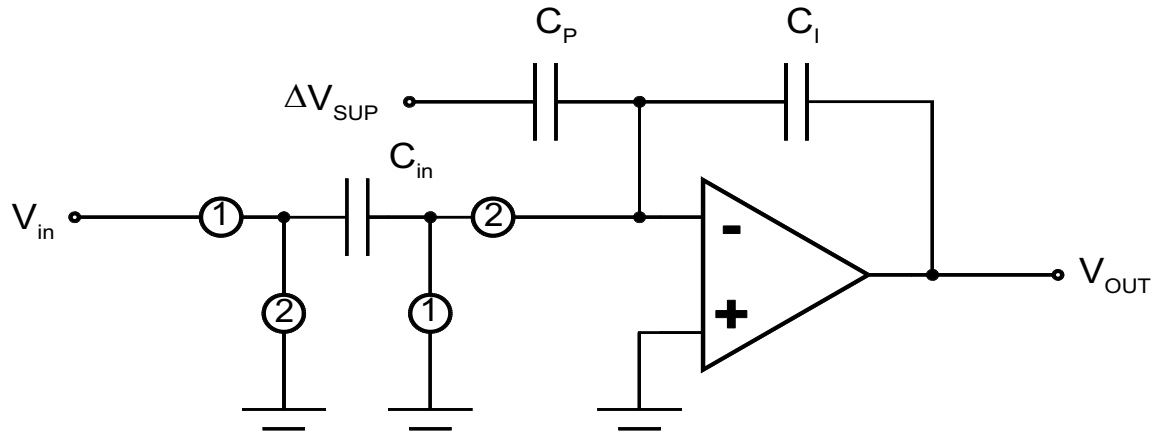
$$V_{OUT}(z) = \frac{z^{-1}}{1 - z^{-1}} \frac{C_{in}}{C_I} V_{in} + \frac{C_P}{C_I} \Delta V_{SUP}$$

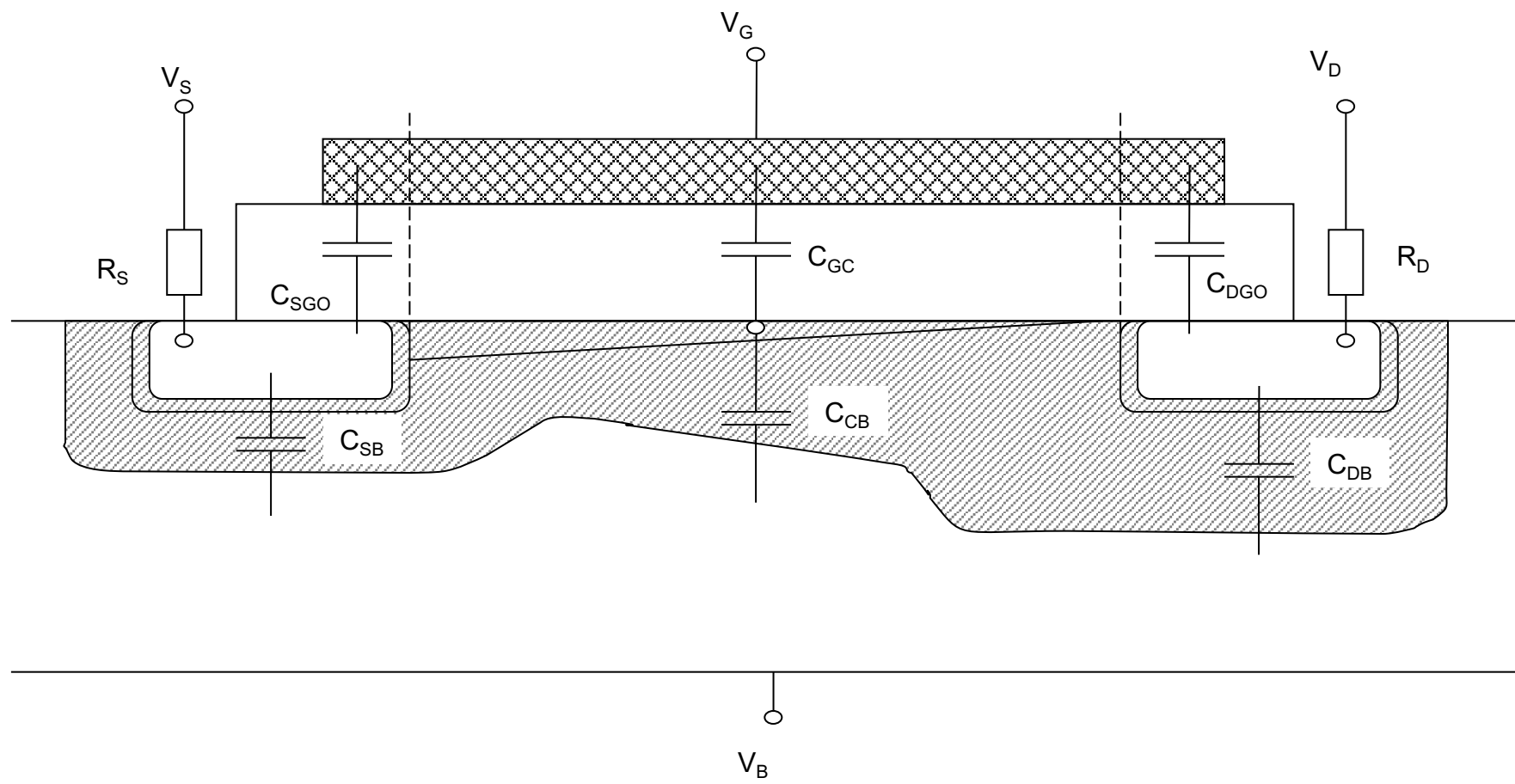
Parasitic capacitor C_P is order of tens of femtofarads
 Integrator capacitor C_I is order of picofarads

=>PSRR is reduced into order of 30-40 dB!
 (Amplifier PSRR is better than 80dB)



The model for the power supply coupling in the SC-integrator





Noise in SC-circuits

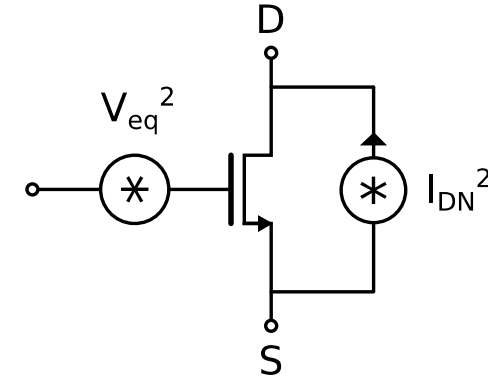
- 1) Amplifier have a) thermal noise (broad-band) and b) 1/f-noise (low-frequency)

Resistivity of conductive channel causes thermal noise

$$\frac{\partial I_{DN}^2}{\partial f} = \frac{8}{3} kT g_m$$

Charge traps in oxide-silicon surface are slow causing 1/f-noise

$$\frac{\partial V_{GN}^2}{\partial f} = \frac{B}{fW \cdot L}$$



- 2) Switches has noise due to R_{ON} whose the noise density is

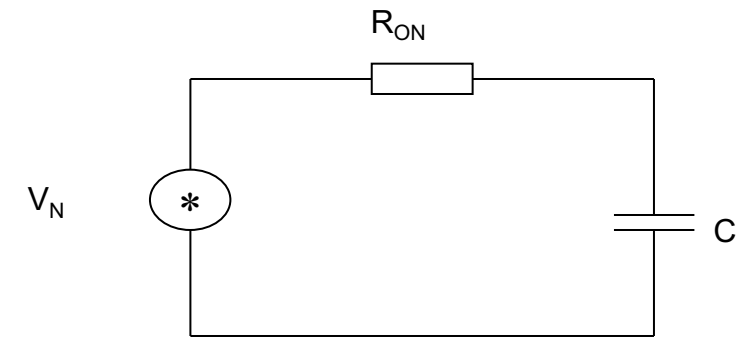
$$S_R(\omega) = 4kT R_{ON}$$

The noise power is obtained by integrating over the noise bandwidth.

$$N_R = \int_{-\infty}^{\infty} S_R(\omega) H_{RC}^2(\omega) d\omega = \int_{-\infty}^{\infty} S_R(\omega) \frac{1}{1 + (\omega/\omega_{3dB})^2} d\omega$$

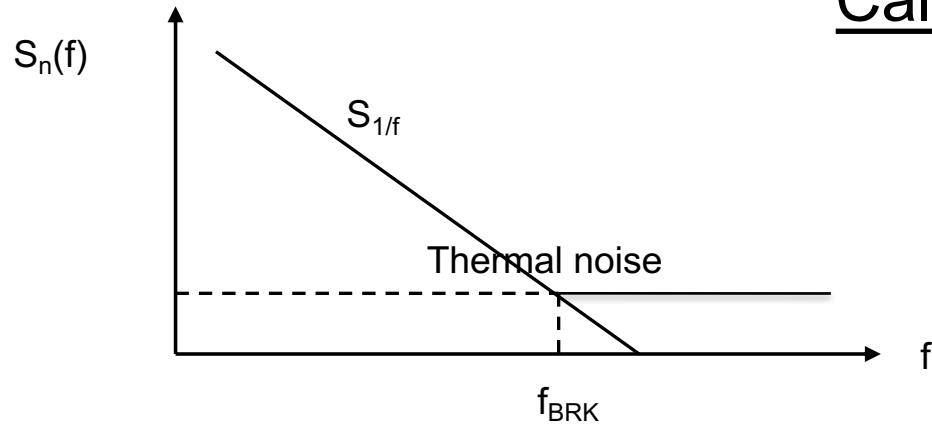
$$= \frac{kT}{C}$$

Also the off-switch is noisy but the bandwidth is so narrow that it is meaningless.

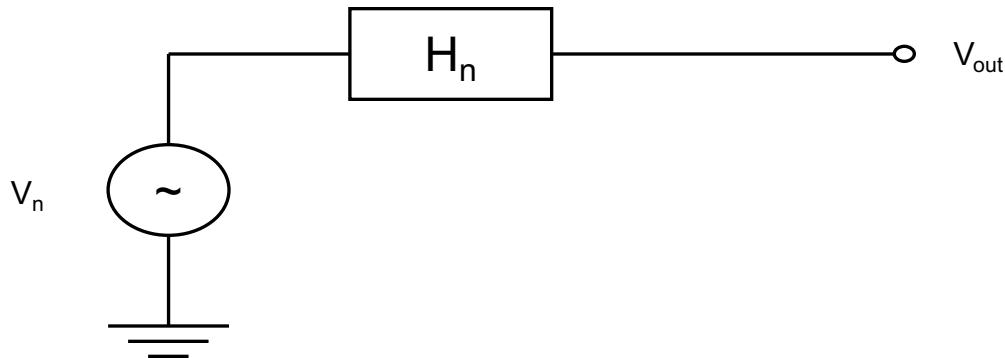
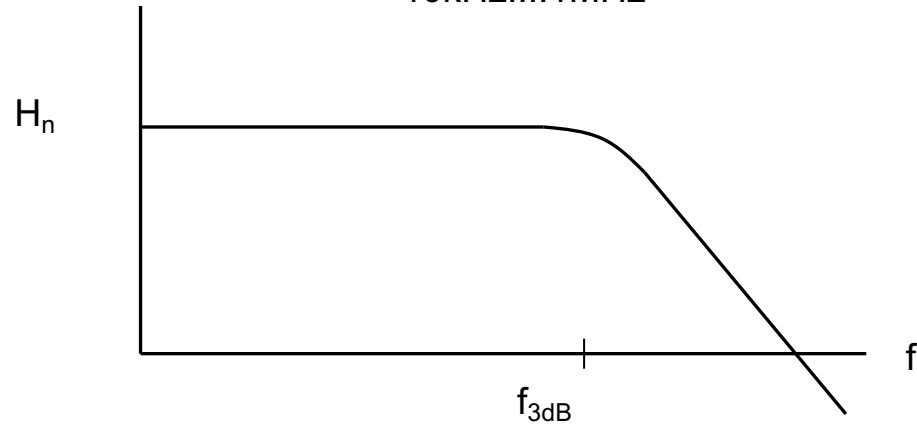


$$\Rightarrow \omega_{3dB} = \frac{1}{R_{ON} C}$$

Calculation of noise power



10kHz....1MHz



Amplifier noise density:
$$S_{eq} = \left[\frac{8kT}{3g_m} + \frac{KF}{2fC_{ox}WLK'} \right]$$

↑ Thermal noise ↙ 1/f-noise

Noise power is obtained by integrating the noise density over the BW:

$$V_n^2 = \int_0^\infty S_\omega(f) |H_n(f)|^2 df + \int_0^\infty S_{1/f}(f) |H_n(f)|^2 df$$

$$\approx \int_0^{f_{3dB}} S_\omega(f) |H_n(f)|^2 df + \int_1^{f_{3dB}} S_{1/f}(f) |H_n(f)|^2 df$$

$$S_{1/f} = \frac{B}{WLf} \quad , \quad S_\omega \approx \frac{8kT}{3g_m} \quad , \quad f_{3dB} = GBW = \frac{g_m}{2\pi C_L}$$

Integrated noise power over noise BW:

$$v_n^2 \approx \frac{8kT}{3g_m} \cdot GBW + \frac{B}{WL} \cdot \ln GBW$$

$$\approx \frac{8kT}{3C_L} + \frac{B}{WL} \cdot \ln \frac{g_m}{C_L}$$

$$H_y = \begin{cases} 1 & f < f_{3dB} \\ 0 & f > f_{3dB} \end{cases}$$

Noise folding in the sampled system

Amplifier bandwidths (GBW) are typically 5-7 times larger than the clock frequency

=> Noise folding occurs from several side-bands increasing the noise level at the signal band

