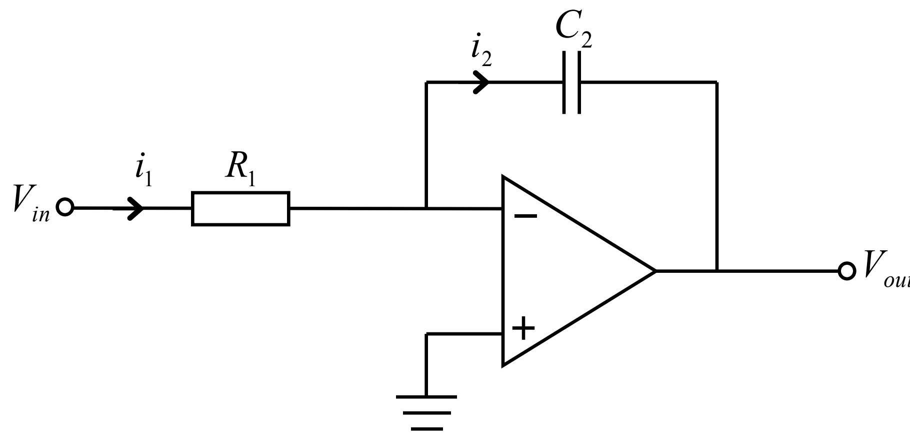
ELEC-E3530

Integrated Analog Systems L6 Continuous-time and current-mode filters

RC-Integrator



Current equations:

$$i_1 = \frac{V_{in}}{R_1}$$

$$i_2 = -C_2 \frac{dV_{out}}{dt}$$

$$i_1 = i_2$$

solve for V_{OUT}

$$\Rightarrow V_{OUT} = -\frac{1}{R_1 C_2}\int V_{in}(t)\,dt$$

Laplace-transformation

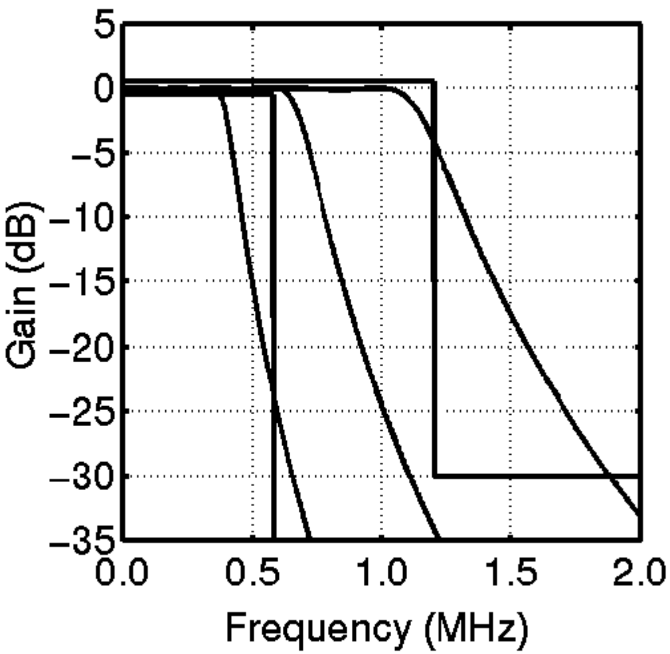
$$V_{out}(s) = \frac{-V_{in}(s)}{sR_1C_2}$$

$$\boxed{H(s) = \frac{V_{out}}{V_{in}} = -\frac{1}{sR_1C_2}}$$

Component Type	Range of Values	Relative Accuracy	Temperature Coefficient	Voltage Coefficient	Absolute Accuracy
Poly/poly capacitor	0.3 - 0.4 fF/μ^2	0.06%	25 ppm/$^\circ$C	- 50 ppm/V	20%
MOS capacitor	0.35 - 0.5 fF/μ^2	0.06%	25 ppm/$^\circ$C	- 20 ppm/V	10%
Diffused resistor	10 - 100 ohms/sq.	2% (5 μm width)	1500 ppm/$^\circ$C	220 ppm/V	35%
Poly resistor	30 - 200 ohms/sq.	2% (5 μm width)	1500 ppm/$^\circ$C	100 ppm/V	30%
Ion impl. resistor	0.5 - 2k ohms/sq.	1% (5 μm width)	400 ppm/$^\circ$C	800 ppm/V	5%
p-well resistor	1 - 10k ohms/sq.	2%	8000 ppm/$^\circ$C	10k ppm/V	40%
pinch resistor	5 - 20k ohms/sq.	10%	10k ppm/$^\circ$C	20k ppm/V	50%

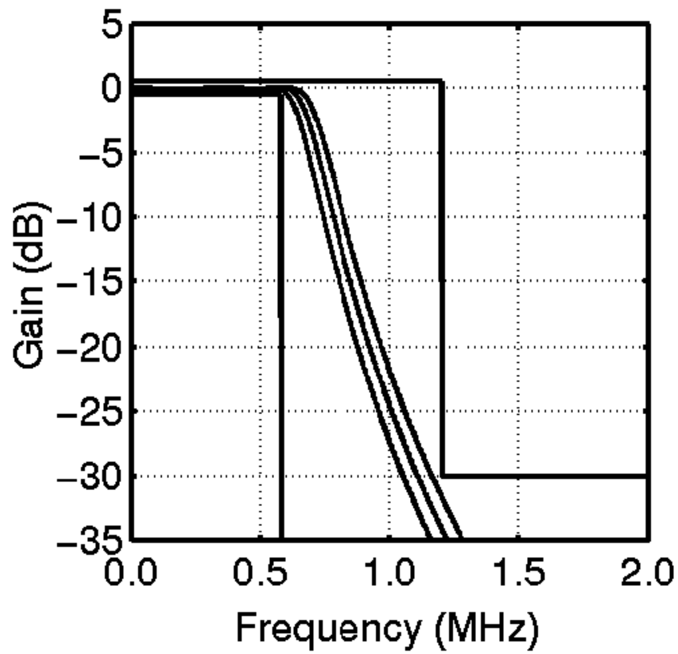
Variation of Filter Time Constants

untuned:



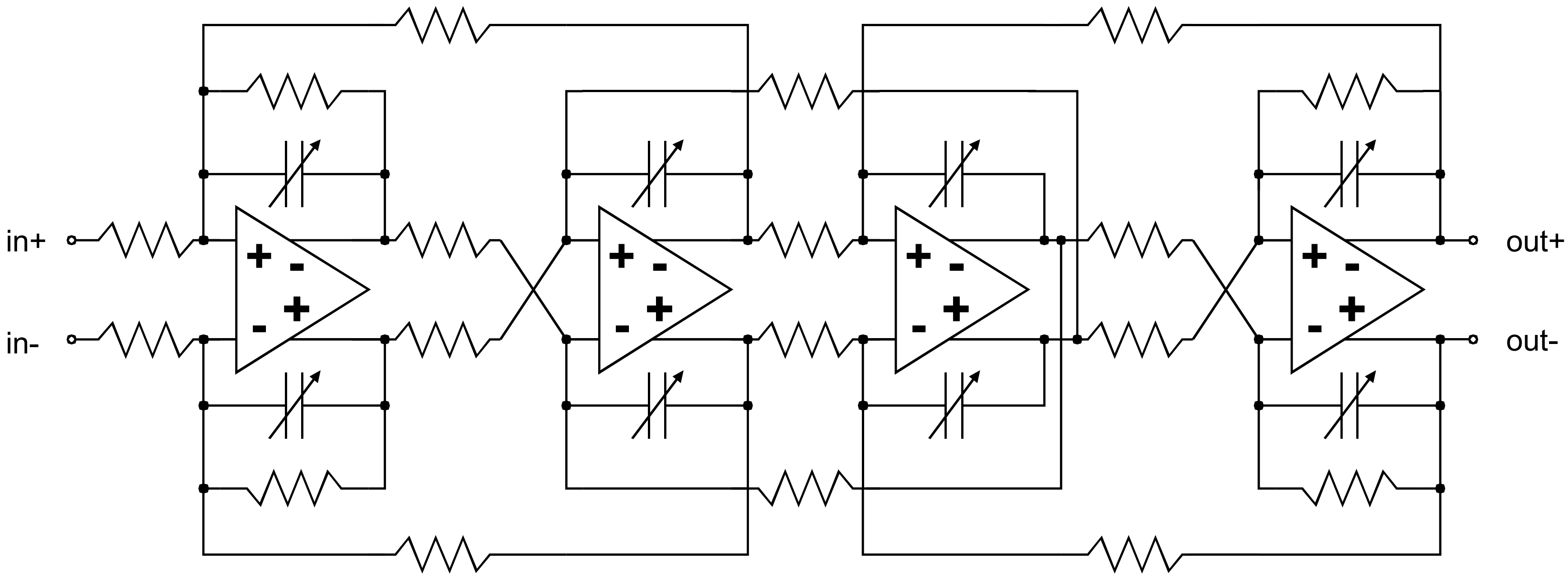
$$\frac{1}{1,7} \le \frac{RC}{RC_{NOM}} \le 1,7$$

tuned:



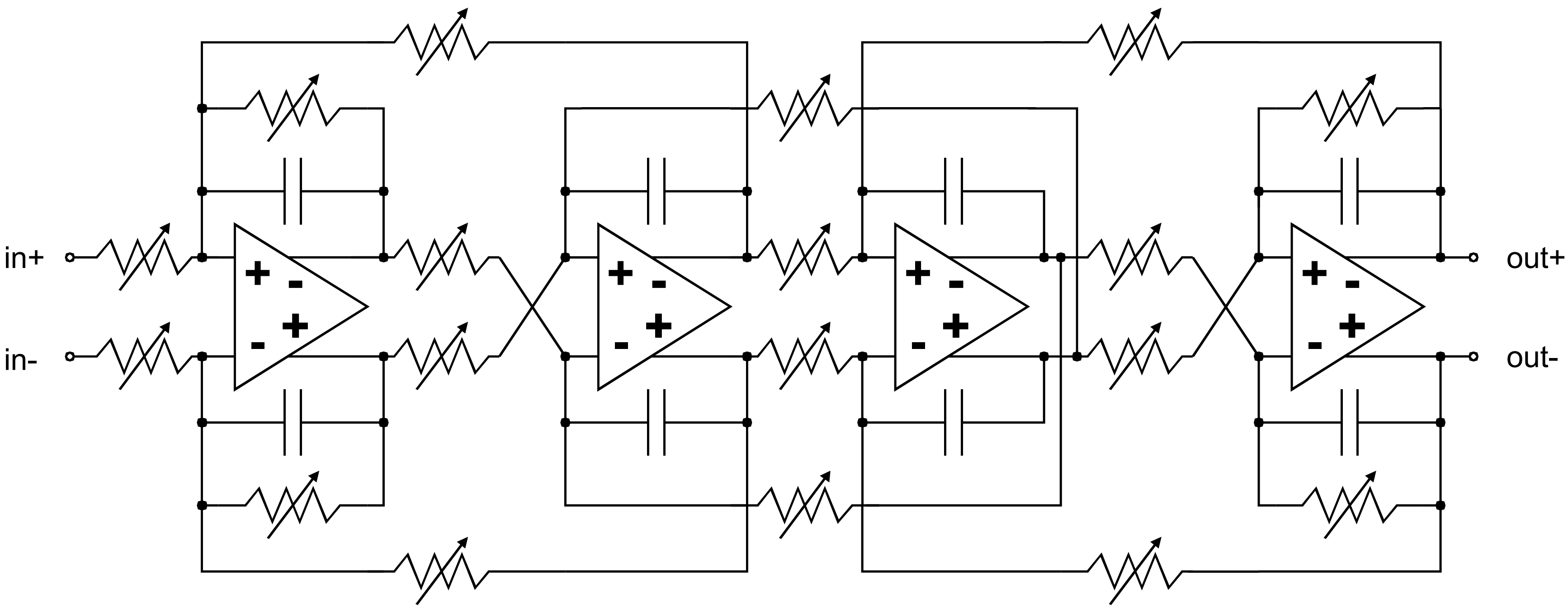
Tuning accuracy ± 5%

Fourth-order leapfrog opamp-RC filter



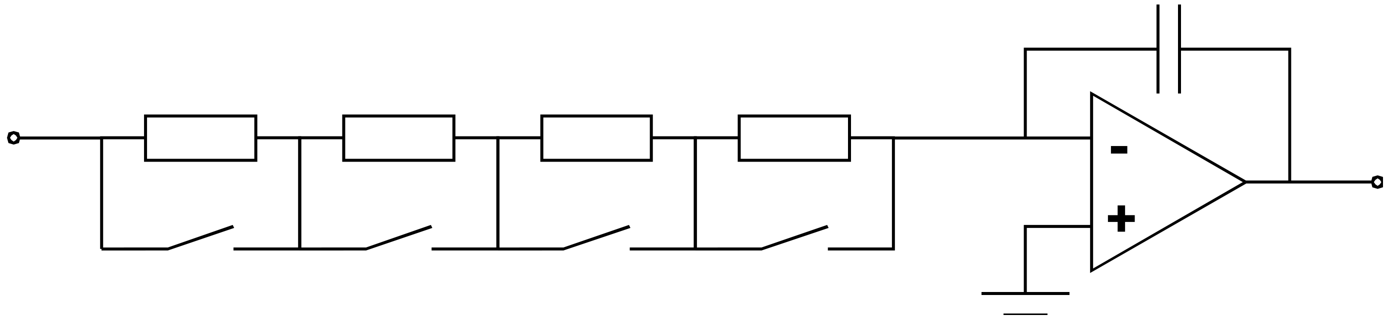
- tuning with switched capacitor matrix
- large area
- extra parasitic capacitors
- switch on-resistance does not affect directly the performance

Resistor tuning

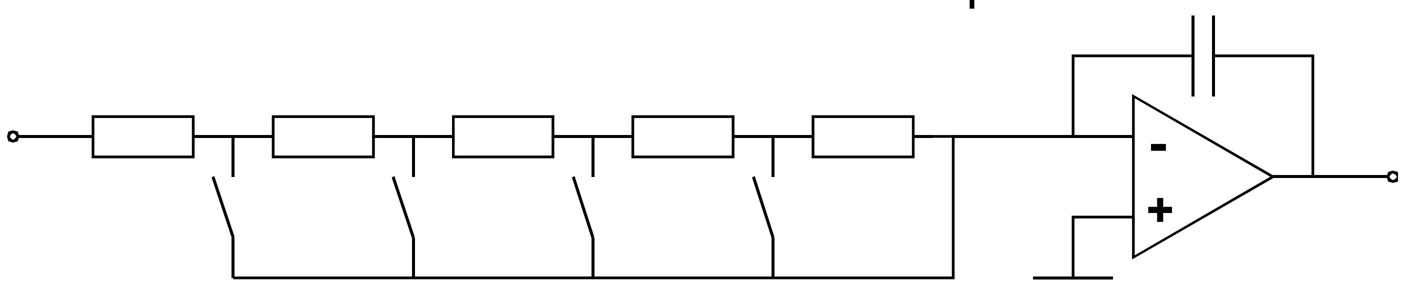


- series or parallel connection of resistors
- switch on-resistor affects the perfomance

Resistor tuning

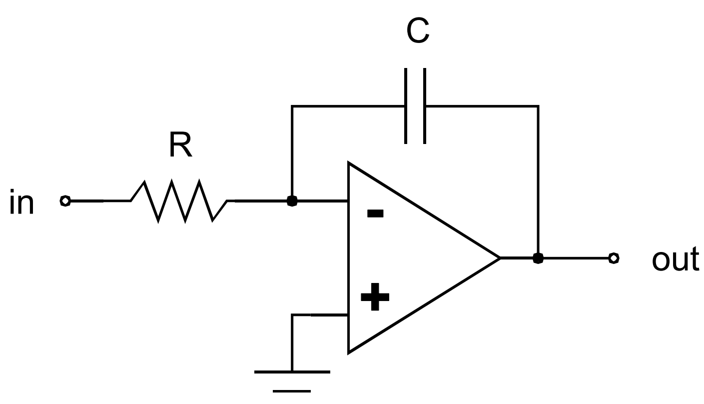


Switch on-resistance signal dependent:
- multiple series on-resistors
- nonlinearity large
- temperature dependence
- process variations



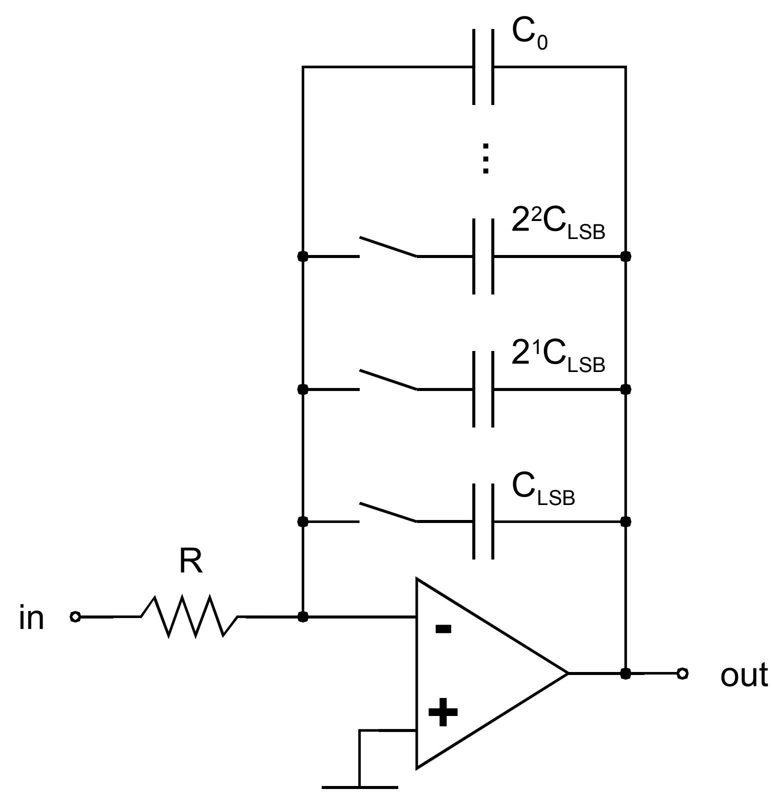
Switch on-resistance not signal dependent:
- single on-resistor
- nonlinearity small
- temperature dependence
- process variations

Opamp-RC integrator time-constant tuning with a capacitor matrix



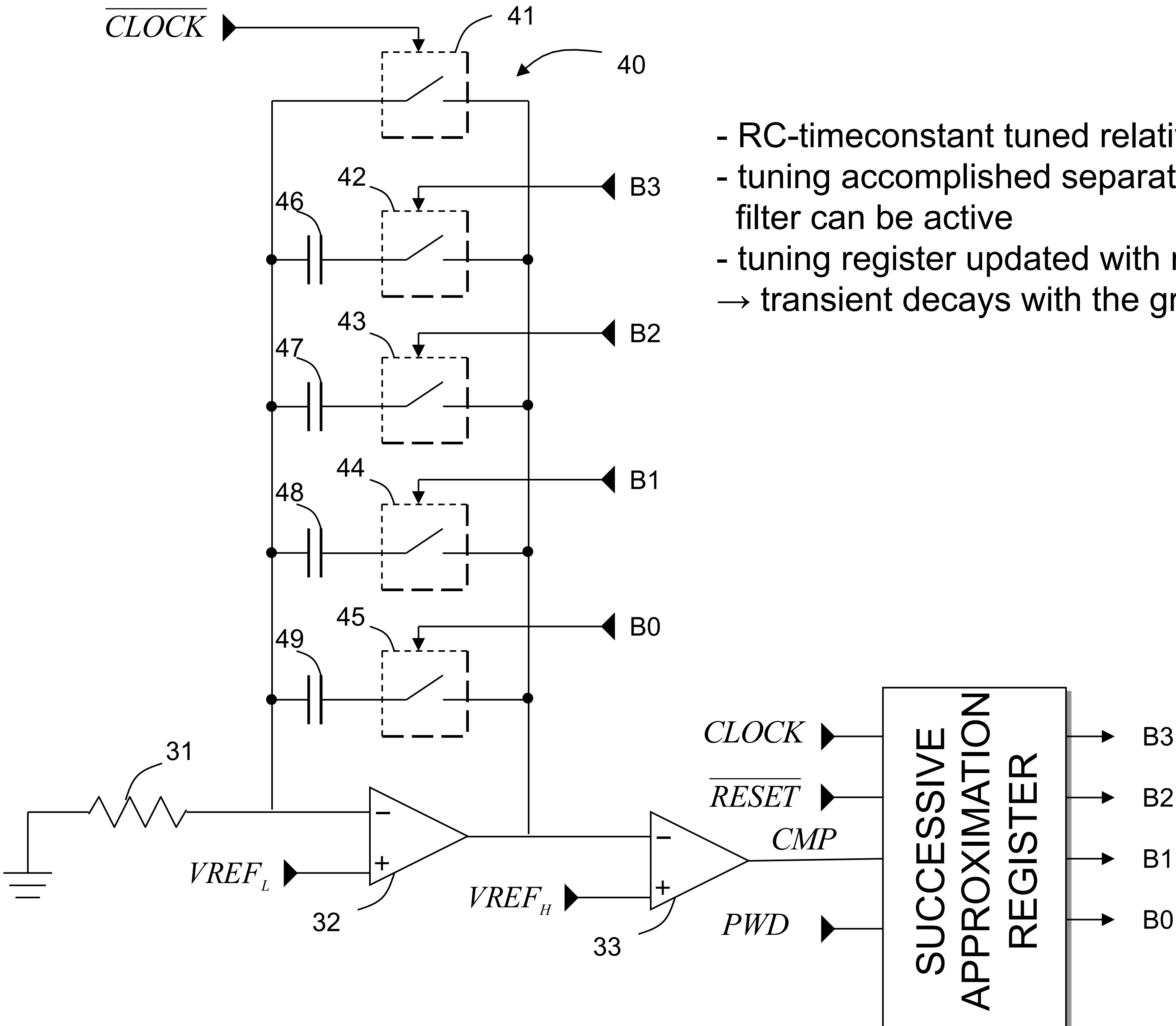
time-constant τ=RC
- R and C not matched
 (process and temperature)

→ τ varies with factor of 2
for accurate filtering
→ tuning of RC-time constant needed



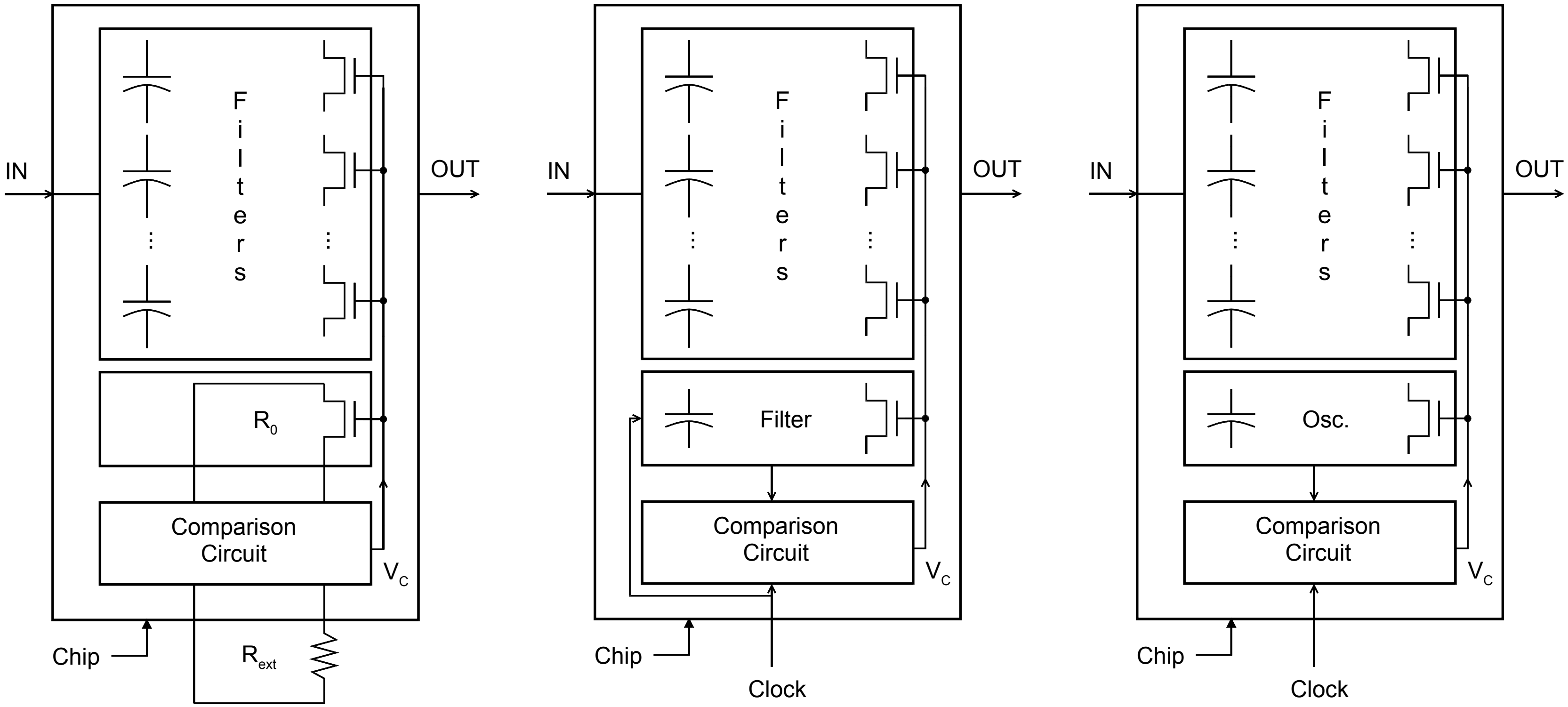
- tuning with programmable capacitor matrix
 or with tunable resistors

Tuning with separate RC-integrator



- RC-timeconstant tuned relative to an external clock
- tuning accomplished separately → filter can be active
- tuning register updated with new values → transient decays with the group delay of the filter

On-chip automatic indirect tuning schemes



(a) (b) (c)

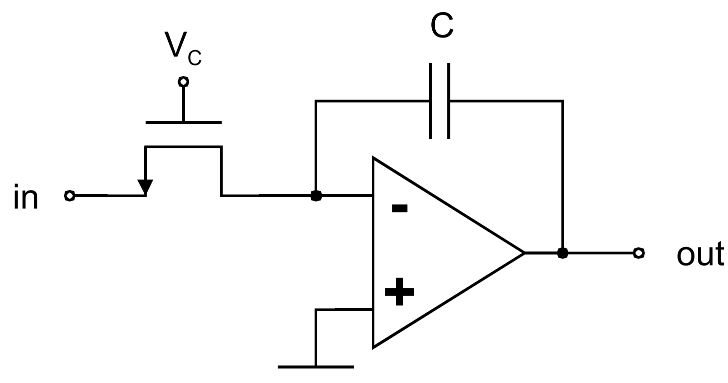
a) external resistor as reference

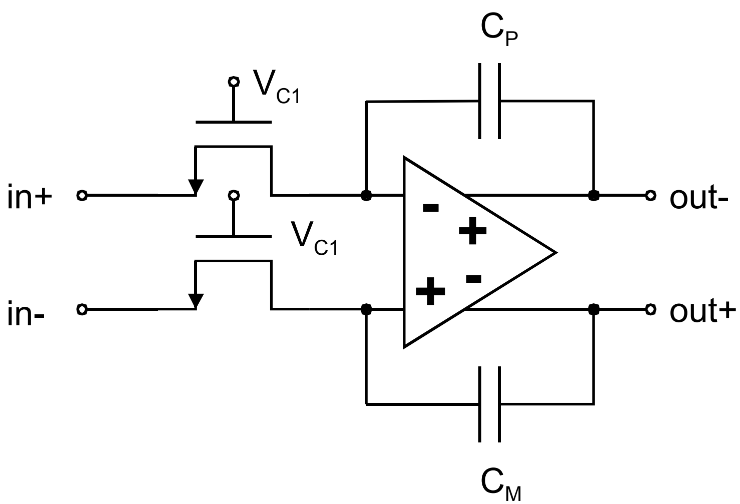
b) external clock and a tunable filter

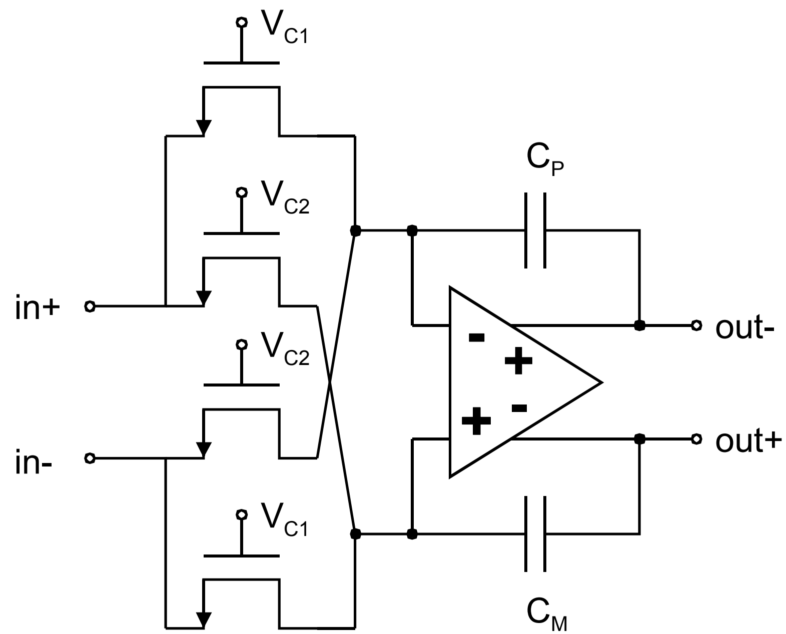
c) external clock and a voltage-controlled oscillator

MOSFET-C integrator

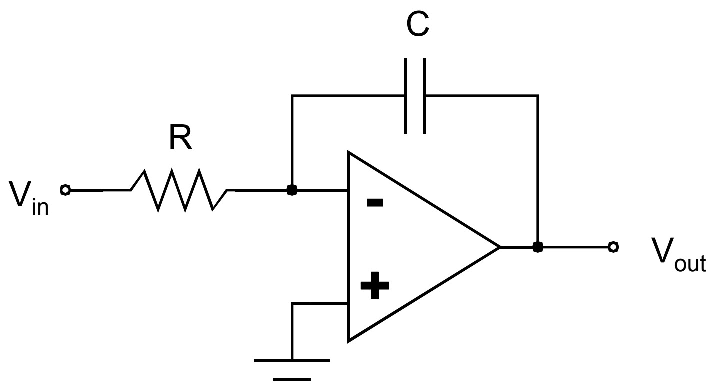
In a RC-integrator replace R with MOSFET biased into linear region:

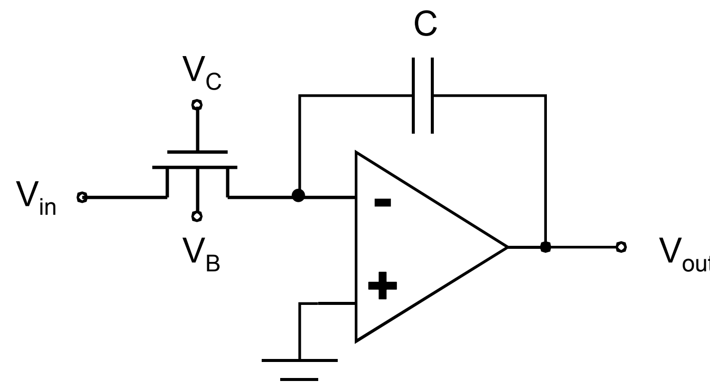


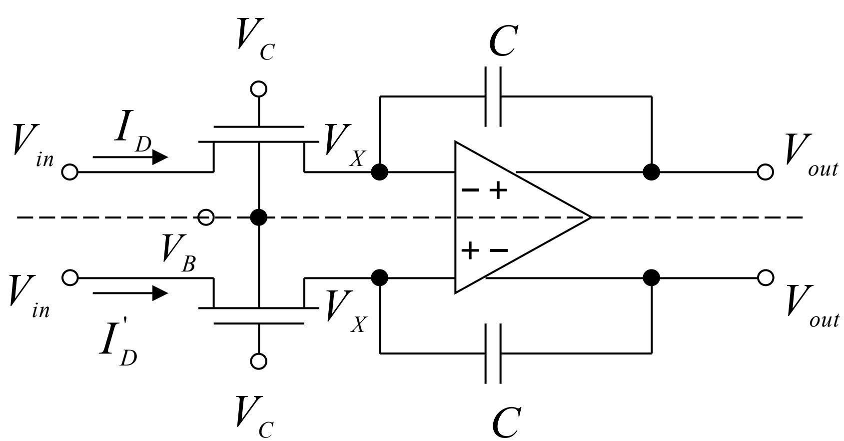




Analysing the cancellation of even harmonics







$$V_{out}(t) = -\frac{1}{C}\int_{-\infty}^{t} I_D dt' + V_x \quad (5a)$$

$$-V_{out}(t) = -\frac{1}{C}\int_{-\infty}^{t} I'_D dt' + V_x \quad (5b)$$

The solution for V_{out} is obtained by subtracting (5b) from (5a):

$$V_{out}(t) = -\frac{1}{2C}\int_{-\infty}^{t} \left(I_D - I'_D\right) dt' \quad (6)$$

The values of the currents I_D and I'_D are given according to (2):

$$I_D = K\left\{a_1\left[V_{in} - V_x\right] + a_2\left[V_{in}^2 - V_x^2\right] + a_3\left[V_{in}^3 - V_x^3\right] + \ldots\right\} \quad (7a)$$

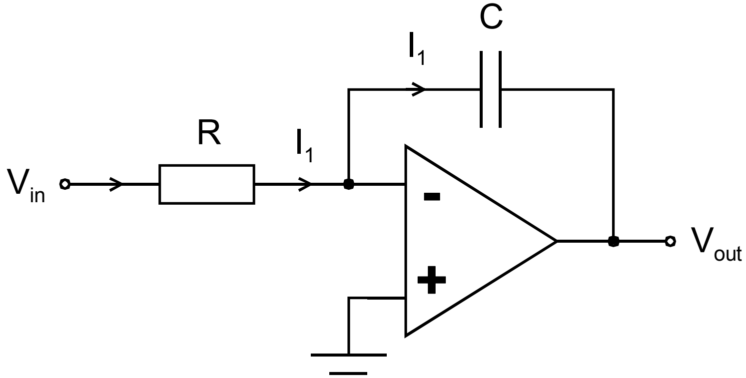
$$I'_D = K\left\{a_1\left[\left(-V_{in}\right) - V_x\right] + a_2\left[\left(-V_{in}\right)^2 - V_x^2\right] + a_3\left[\left(-V_{in}\right)^3 - V_x^3\right] + \ldots\right\} \quad (7b)$$

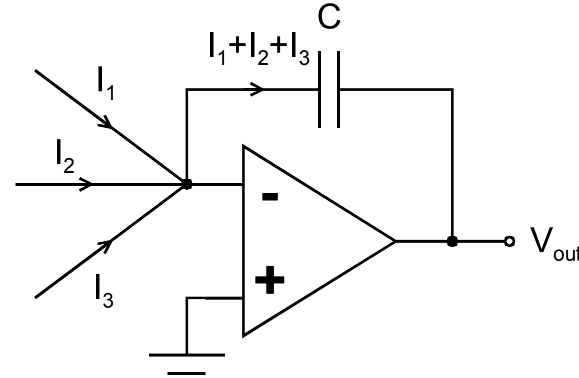
(7b) is subtracted from (7a), all the even order terms and all the terms in V_x cancel out:

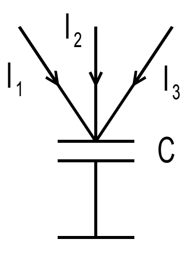
$$I_D - I'_D = 2K\left[a_1 V_{in} + a_3 V_{in}^3 + a_5 V_{in}^5 + \ldots\right] \quad (8)$$

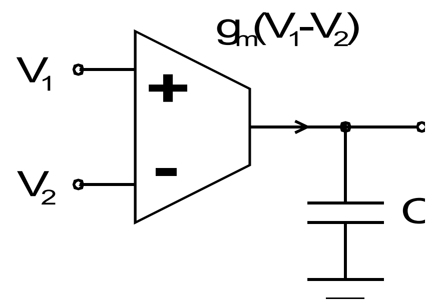
$$V_{out}(t) \cong -\frac{1}{RC}\int_{-\infty}^{t} V_{in} dt' \quad (9)$$

Current mode filtering



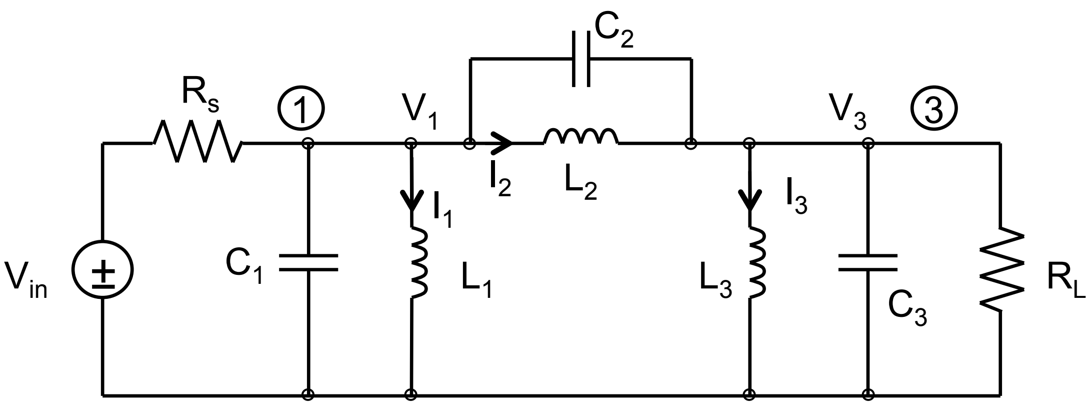




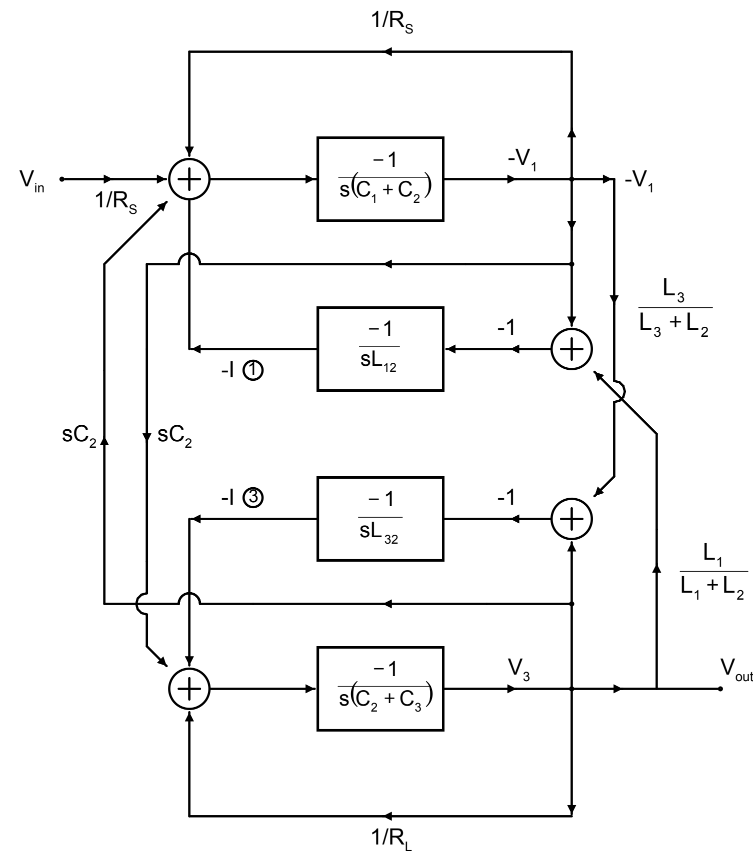


$$f_{GBW} \sim \frac{g_m}{C} \sim f_{3dB}$$

Band-pass filters



Signal flow-graph:



State equations:

$$-V_1 = \frac{-1}{s(C_1 + C_2)}\left[\frac{V_{in} - V_1}{R_S} + sC_2V_3 - I_{(1)}\right],$$

$$-I_{(1)} = -(I_1 + I_2) = \frac{-1}{sL_{12}}\left[V_1 - \frac{L_1V_3}{L_1 + L_2}\right],$$

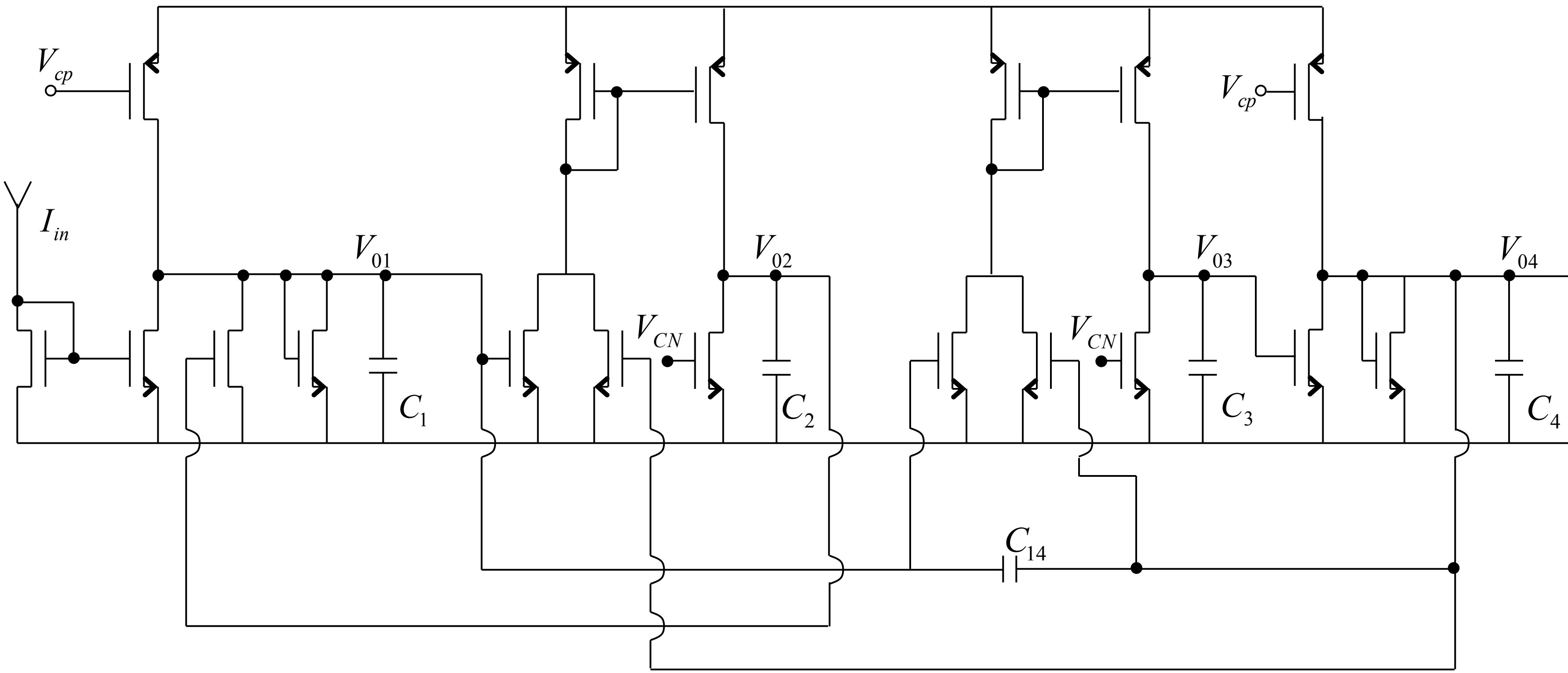
$$V_3 = \frac{-1}{s(C_2 + C_3)}\left[-sC_2V_1 - I_{(3)} + \frac{V_3}{R_L}\right],$$

$$-I_{(3)} = I_3 - I_2 = \frac{-1}{sL_{32}}\left[-V_3 + \frac{L_3V_1}{L_3 + L_2}\right],$$

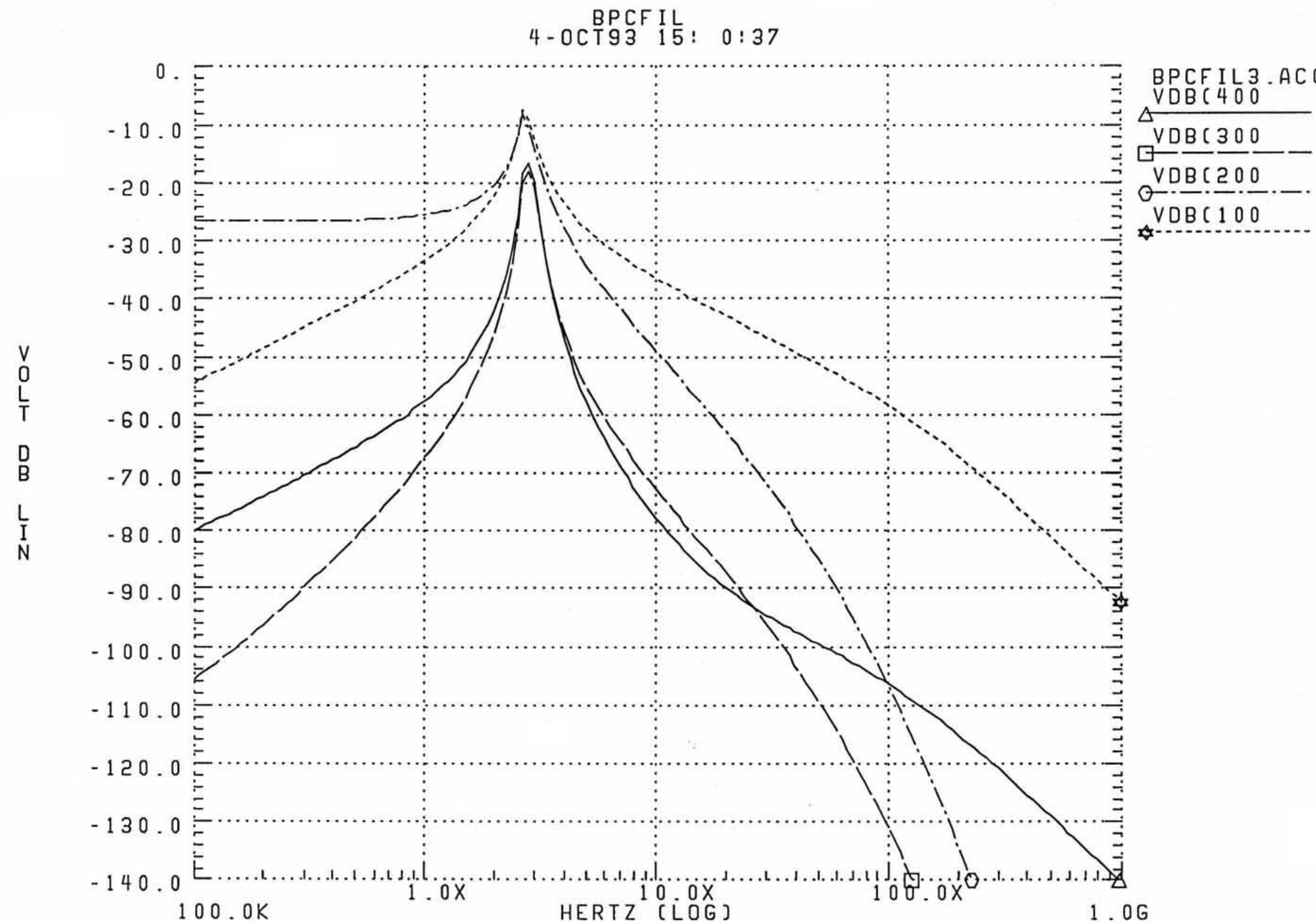
where $L_{12} = L_1 \,||\, L_2 = \dfrac{L_1L_2}{L_1 + L_2}.$

$L_{32} = L_3 \,||\, L_2$

Current mode implementation of bandpass filter



Frequency response of current mode BB-filter



High frequency filtering

1) SC:
requirement for OPAMP

$$f_{GBW} \geq 5 \cdot f_{CLK} \geq 10 \cdot f_{3dB}$$

$$\Rightarrow f_{GBW} \geq 50 \cdot f_{3dB}$$

2) RC:

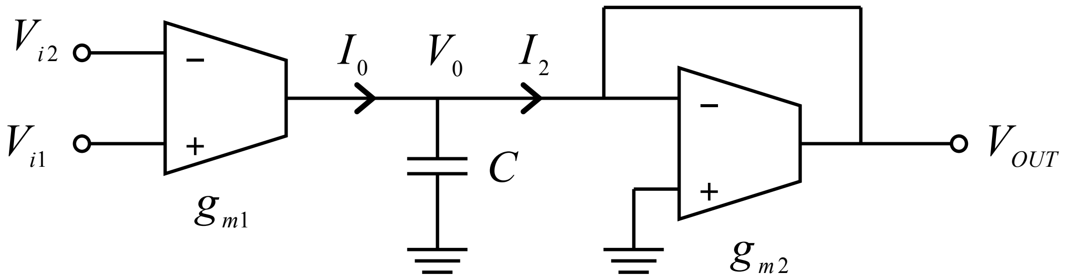
$$f_{GBW} \geq 10 \ldots 20 f_{3dB}$$

3) g_m-C:

$$f_{3dB} = f_{GBW}$$

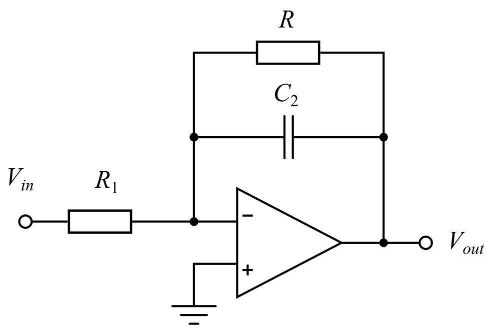
With same C and GBW
g_mC filter capable to 10...50 times higher frequencies than RC and SC filters

Synthesis of g_m-C filters



$$I_0 = g_{m1}(V_{i1} - V_{i2})$$
$$I_0 = I_C + I_2$$
$$I_2 = g_{m2} \cdot V_0$$
$$I_0 = sCV_0 + g_{m2} \cdot V_0 = g_{m1}(V_{i1} - V_{i2})$$
$$\Rightarrow \frac{V_0}{V_{i1} - V_{i2}} = \frac{g_{m1}}{g_{m2} + SC} = \frac{g_{m1}/g_{m2}}{1 + SC/g_{m2}}$$

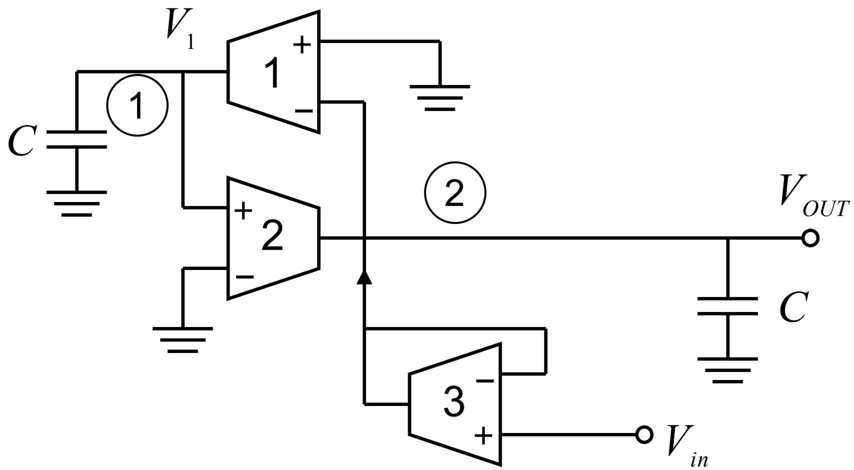
Lossy integrator

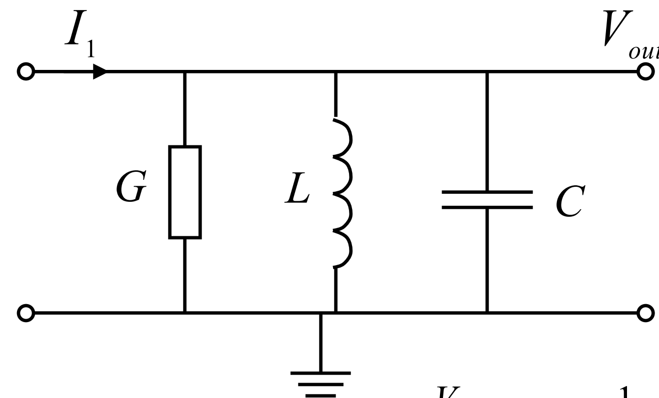


The transfer function of lossy RC integrator:

$$H(\omega) = -\frac{R}{R_1}\frac{1}{1 + j\omega C_2 R} = -\frac{1}{R_1 C_2}\frac{1}{j\omega - \frac{1}{RC_2}}$$

g_m-C realization of RLC-resonance circuit





$$\frac{V_8}{I} = \frac{1}{SC + 1/SL + G} = \frac{SL}{S^2LC + SLG + 1}$$

OTA currents:

$$I_1 = -g_{m1}V_{out}$$
$$I_2 = g_{m2}V_1$$
$$I_3 = g_{m3}\left(V_{in} - V_{out}\right)$$

Current equations at nodes (1) and (2)

(1) $I_1 = SCV_1$

(2) $I_3 + I_2 = SCV_{out}$

Inserting currents:

(1) $-g_{m1}V_{out} = SCV_1$

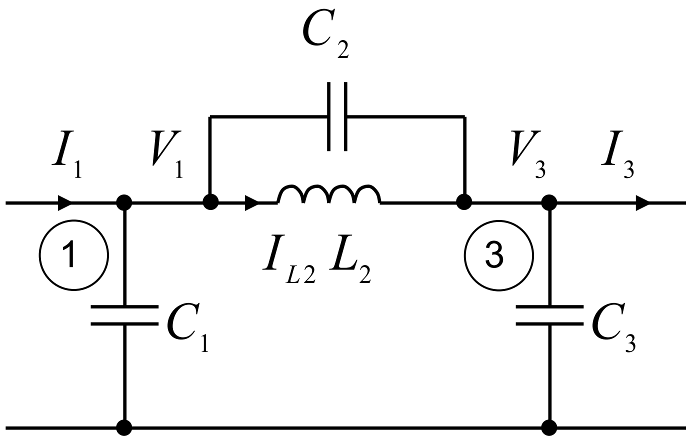
(2) $g_{m2}V_1 + g_{m3}\left(V_{in} - V_{out}\right) = SCV_{out}$

Eliminating V_1

$$\frac{V_{out}}{V_{in}} = H_{BP} = \frac{Sg_{m3}C}{S^2C^2 + Sg_{m3}C + g_{m1}g_{m2}}$$

$$I = g_{m3}V_{in}$$
$$G = g_{m3}$$
$$L = \frac{C}{g_{m1} \cdot g_{m2}}$$
$$C \equiv C$$

Design of g_mC ladder filters



Current equations at nodes (1) and (3)

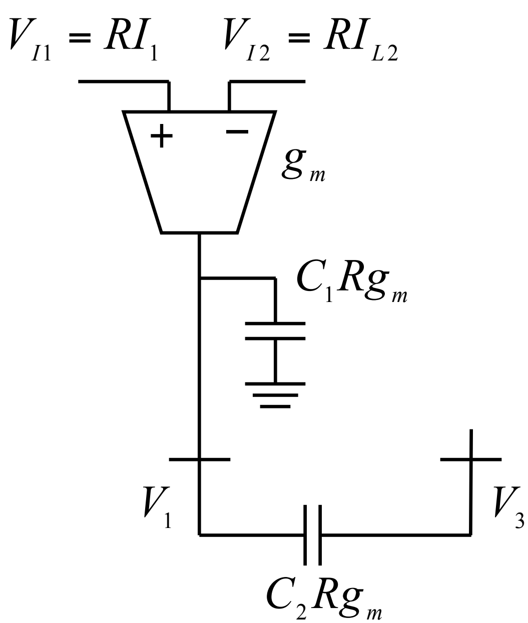
$$
\left.\begin{aligned}
&(1)\quad I_1 = SC_1V_1 + I_{L2} + SC_2\left(V_1 - V_3\right) \\
&(3)\quad I_3 = I_{L2} + SC_2\left(V_1 - V_3\right) - SC_3V_3
\end{aligned}\right| \cdot g_mR
$$

$\Rightarrow$

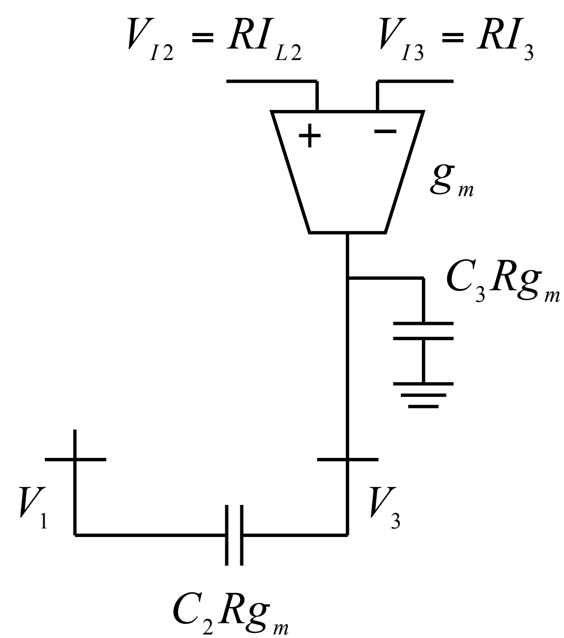
(1) $$g_m\left(RI_1 - RI_{L2}\right) = SC_1Rg_mV_1 + SC_2Rg_m\left(V_1 - V_3\right)$$

(3) $$g_m\left(RI_{L2} - RI_3\right) = SC_3Rg_mV_3 + SC_2Rg_m\left(V_3 - V_1\right)$$

Current eq. (1)



Current eq. (3)

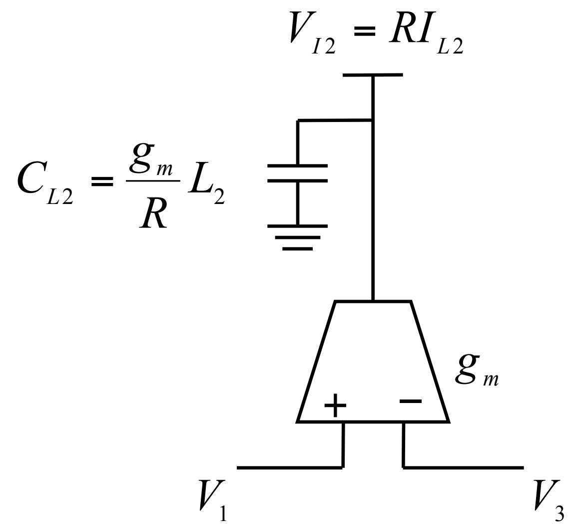


Current of inductor L_2

$$I_{L2} = \frac{1}{SL_2}(V_1 - V_3) \quad \Big| \cdot R$$

$$\Rightarrow RI_{L2} = \frac{R}{SL_2}(V_1 - V_3)$$

$$V_{I2} = RI_{L2} = \frac{R}{SL_2 g_m} \cdot g_m (V_1 - V_3)$$

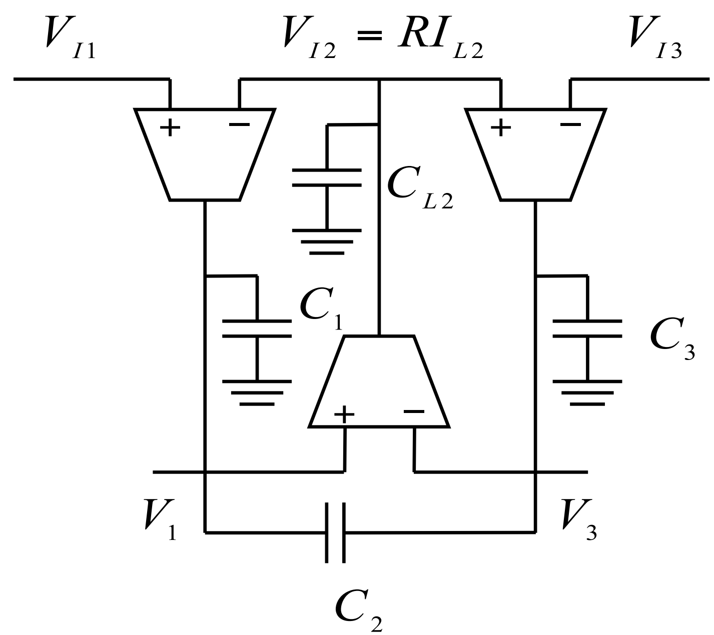


select:

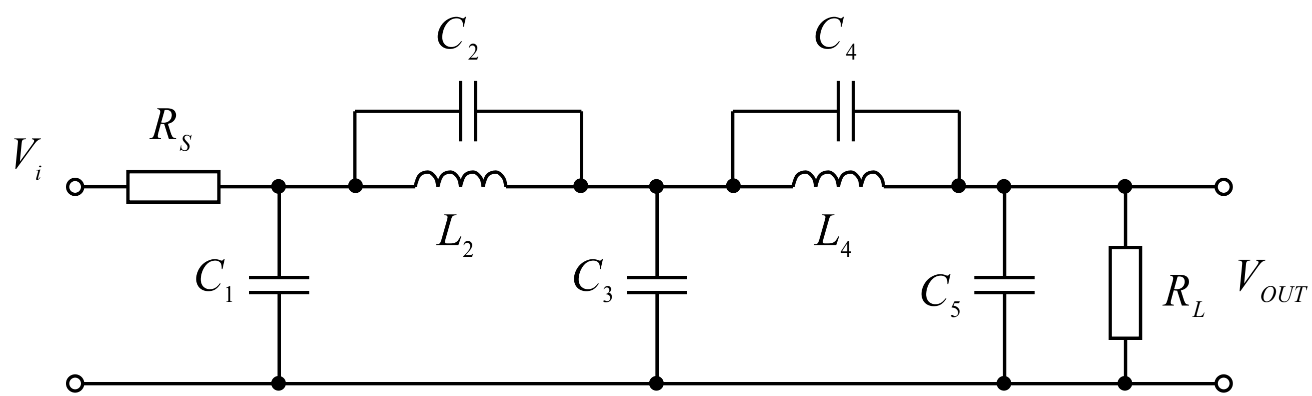
$$R = \frac{1}{g_m}$$

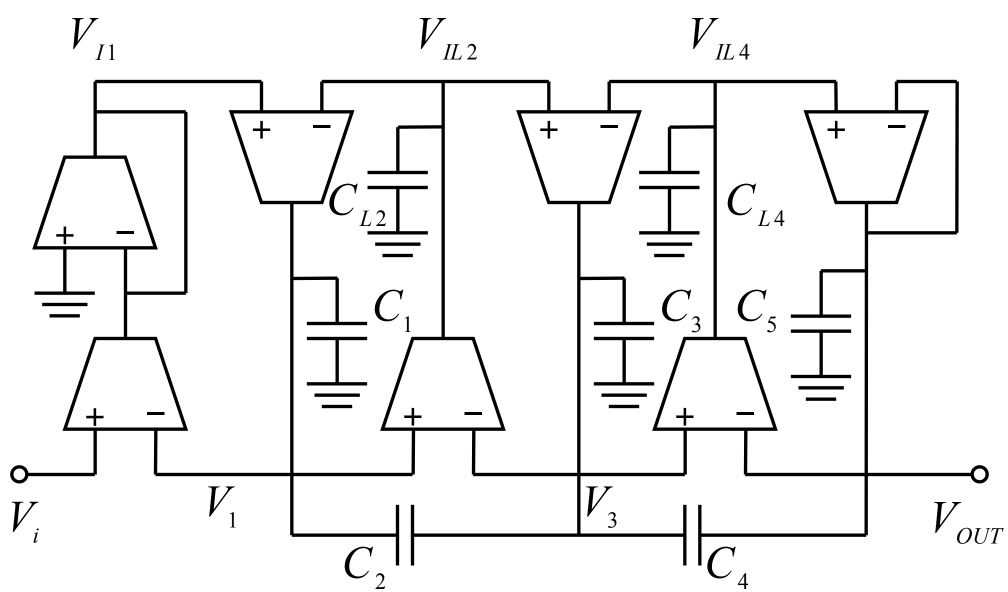
$$\Rightarrow C_{L2} = g_m^2 L_2$$

C_1, C_2, C_3 unaltered



gmC ladder filter implementation





select:

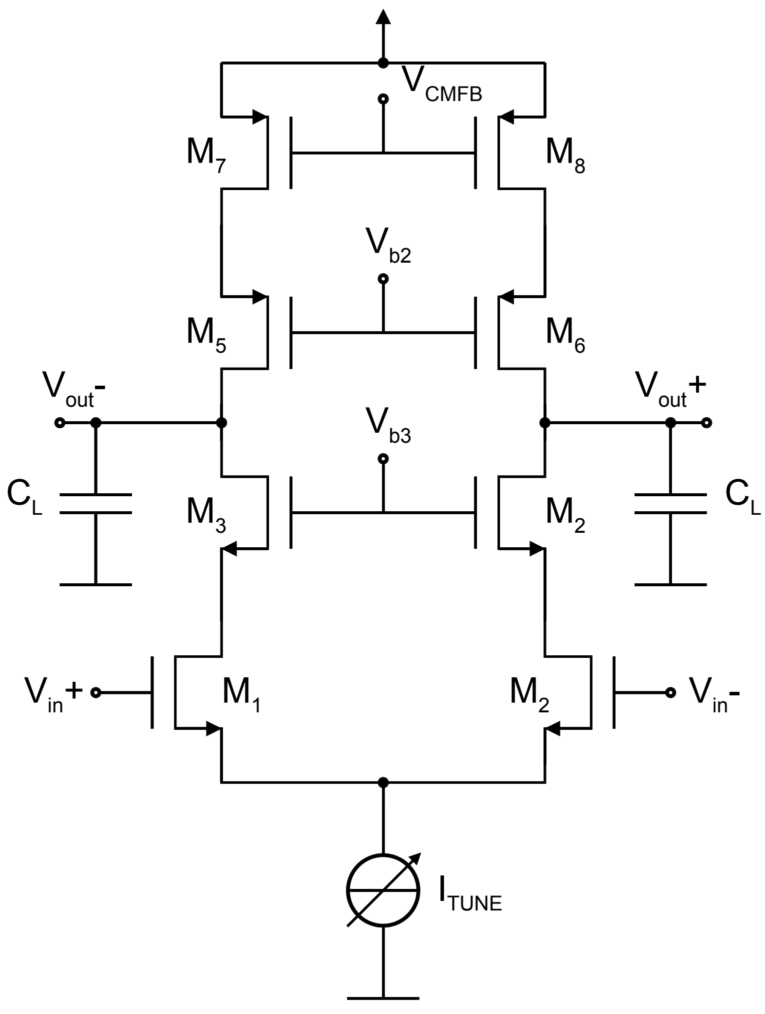
$$R_S = R_L = R = \frac{1}{g_m}$$

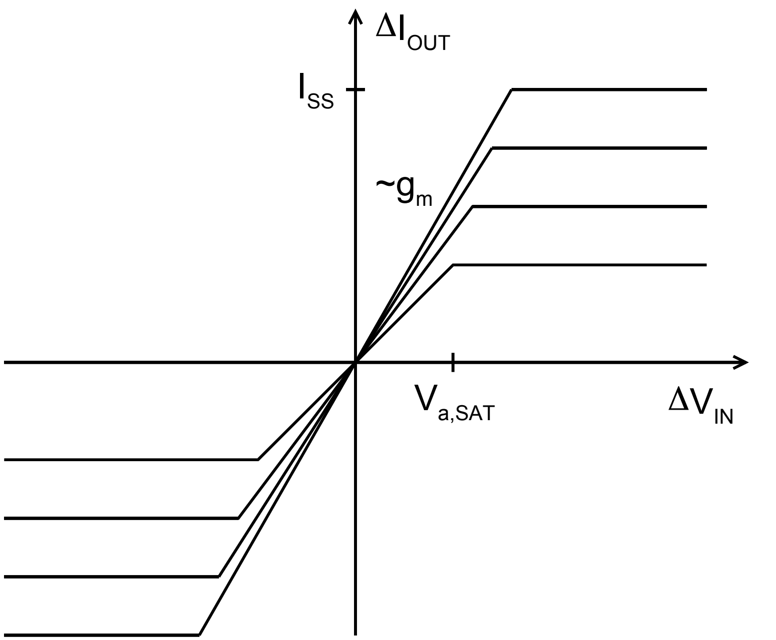
$$\Rightarrow C_{L2} = g_m^2 L_2$$

$$C_{L4} = g_m^2 L_4$$

$C_{1...5}$ remain unaltered

Tunable OTA



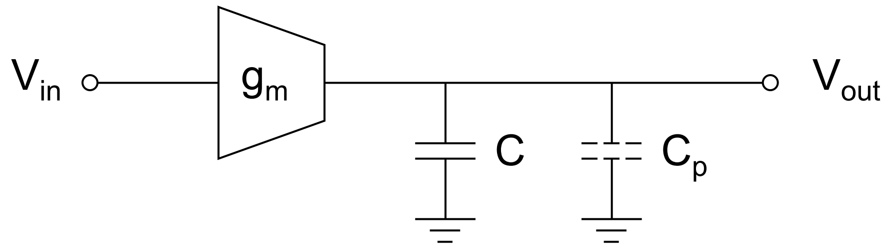


$$I_{SS} \sim I_{tune}$$

$$V_{a,SAT} \sim \sqrt{I_{tune}}$$

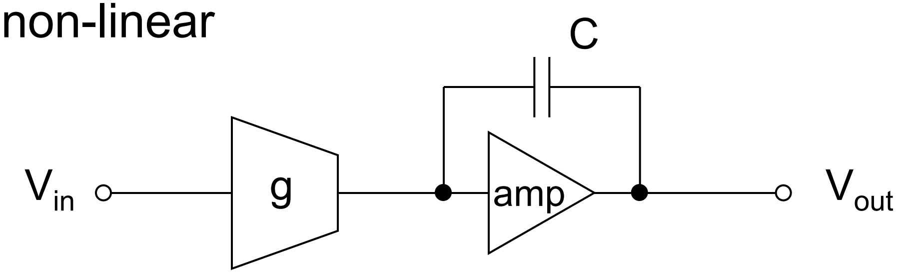
$$\sim g_m \sim \sqrt{I_{tune}}$$

• Parasitic capacitors



$$C' = C + C_p$$

• C_p is non-linear



No parasitic affects.
-limited speed due to amplifier
-increased power consumption

• If no floating capacitors (i.e. No elliptical transfer function)

$\Rightarrow$ transistor only processing
(fine linewidth digital CMOS)

• Non-linearity of g_m-realisation

$\Rightarrow$ THD > 0,1%