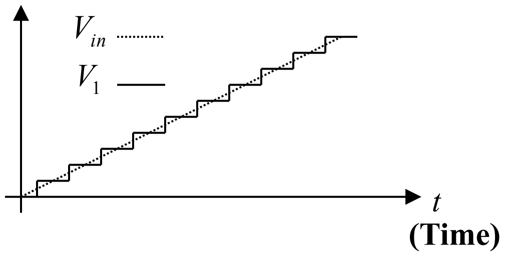
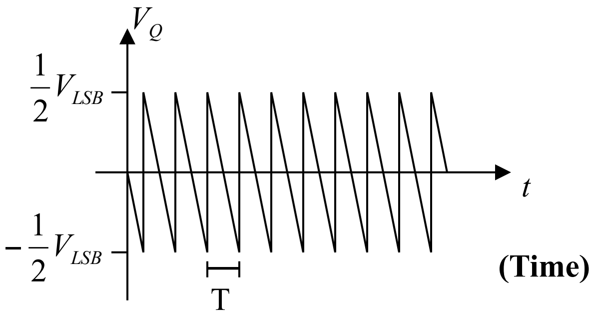
ELEC-E3530

Integrated Analog Systems L10 Oversampling Analog-to-Digital Converters

Quantization noise





$$V_{Q,ave} = 0$$

$$V_{Q(rms)} = \left[\frac{1}{T}\int_{-T/2}^{T/2} V_Q^2\, dt\right]^{1/2} = \left[\frac{1}{T}\int_{-T/2}^{T/2} V_{LSB}^2 \left(\frac{-t}{T}\right)^2 dt\right]^{1/2}$$

$$= \left[\frac{V_{LSB}^2}{T^3}\left(\frac{t^3}{3}\Big|_{-T/2}^{T/2}\right)\right]^{1/2}$$

$$V_{Q(rms)} = \frac{V_{LSB}}{\sqrt{12}} \qquad ;\ V_{LSB} = \frac{V_{REF}}{2^n}$$

Signal to noise ratio with sinusoidal signal $V_{in(rms)} = \dfrac{V_{REF}}{2\sqrt{2}}$

$$SNR = 20\log\left(\frac{V_{in(rms)}}{V_{Q(rms)}}\right)$$

$$= 20\log\left(\frac{V_{ref}\ /(2\sqrt{2})}{V_{LSB}\ /(\sqrt{12})}\right)$$

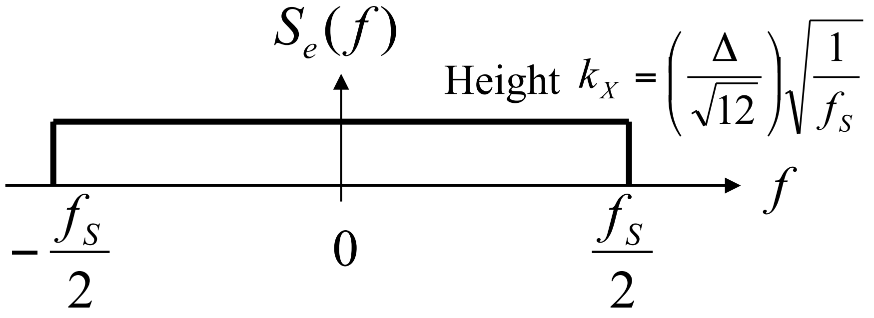
$$= 20\log\left(\sqrt{\frac{3}{2}}\, 2^N\right)$$

SNR improves 6dB by adding an extra bit

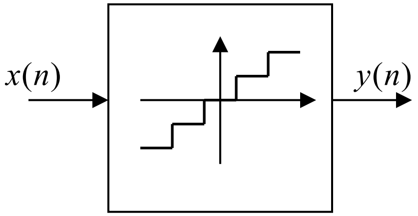
$$SNR = 6.02N + 1.76\, dB$$

Spectral density of quantization noise

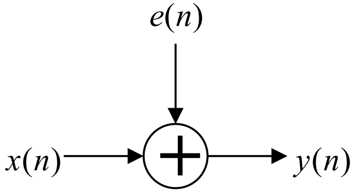
Quantization noise is evenly distributed with the noise density of k_x at Nyqvist-band



spectral density of quantization noise.



Quantizer



$$e(n) = y(n) - x(n)$$

Model

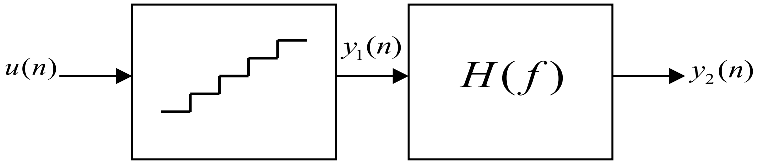
Quantization noise power

$$\int_{-f_S/2}^{f_S/2} S_e^2(f)df = \int_{-f_S/2}^{f_S/2} k_X^2 df = k_X^2 f_S = \frac{\Delta^2}{12} \quad ; \Delta = V_{LSB} = \frac{V_{REF}}{2^n}$$

Solving this relation gives spectral noise density

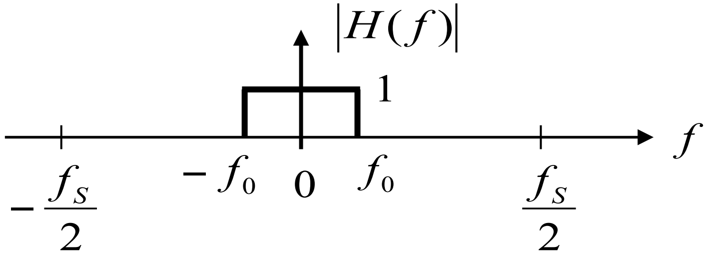
$$k_X = \left(\frac{\Delta}{\sqrt{12}}\right)\sqrt{\frac{1}{f_S}}$$

Over sampling A/D converter



N-bit quantizer

(a)



(b)

Maximum power of a sinuisoidal signal with a peak amplitude of

$$V_p = \frac{2^N \Delta}{2}$$

is

$$P_S = \left(\frac{\Delta 2^N}{2\sqrt{2}}\right)^2 = \frac{\Delta^2 2^{2N}}{8}$$

quantization noise power within frequency band of interest i.e. $f_0 << f_s/2$

$$P_e = \int_{-f_S/2}^{f_S/2} S_e^2(f)\left|H(f)\right|^2 df = \int_{-f_0}^{f_0} k_X^2 df = \frac{2 f_0 \Delta^2}{f_S 12} = \frac{\Delta^2}{12}\left(\frac{1}{OSR}\right)$$

in which the oversampling ratio is

$$OSR = \frac{f_S}{2 f_0}$$

Signal-to-noise ratio with sinusoidal input signal is

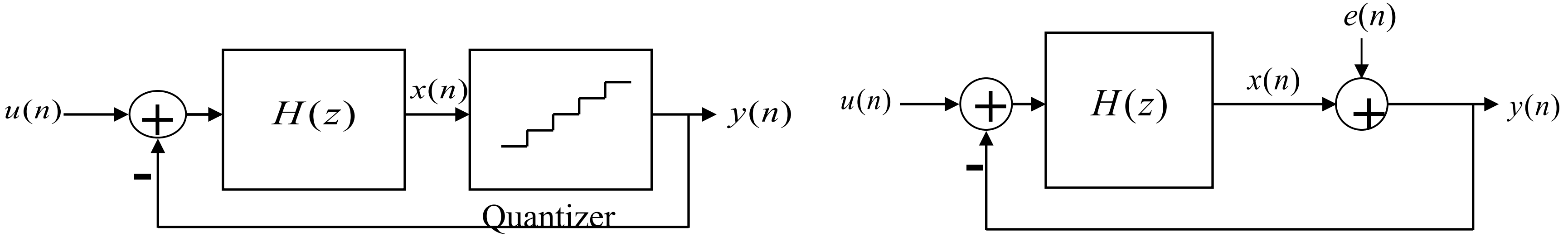
$$SNR_{\max} = 10\log\left(\frac{P_S}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log(OSR)$$

Or given in [dB]

$$SNR_{\max} = 6.02N + 1.76 + 10\log(OSR)$$

SNR improves 6dB by adding an extra bit and 3dB by doubling OSR

Sigma-delta modulator



(a) (b)

Output of the modulator with noise

$$Y(z) = S_{TF}(z)U(z) + N_{TF}(z)E(z)$$

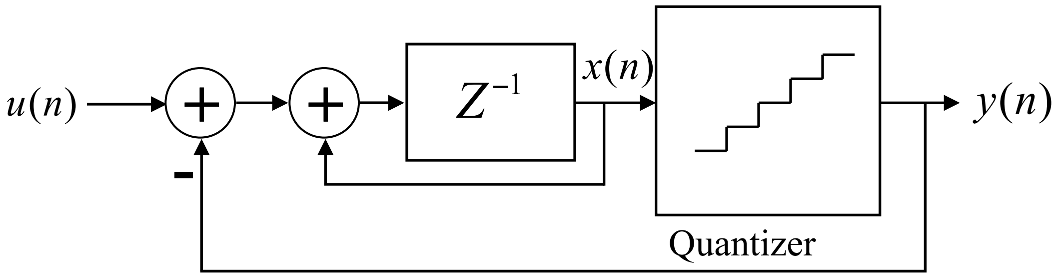
Signal transfer function

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1+H(z)}$$

Noise transfer function

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+H(z)}$$

First-Order Noise Shaping



Integrator (1st order) transfer function

$$H(z) = \frac{1}{z-1}$$

Signal transfer function

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1+1/(z-1)} = z^{-1} \qquad \text{; low-pass}$$

and the noise transfer function

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+1/(z-1)} = (1-z^{-1}) \text{ ; high-pass}$$

First-Order Noise Shaping

The signal transfer function

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1+1/(z-1)} = z^{-1}$$

The noise transfer function

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+1/(z-1)} = \left(1 - z^{-1}\right)$$

Insert z = $e^{j\omega T} = e^{j2\pi f/fs}$

$$N_{TF}(z) = 1 - e^{-j2\pi f/f_s} = \frac{e^{j\pi f/f_s} - e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s}$$

$$= \sin\left(\frac{\pi f}{f_s}\right) \times 2j \times e^{-j\pi f/f_s}$$

The magnitude of noise transfer function is

$$\left|N_{FT}(f)\right| = 2\sin\left(\frac{\pi f}{f_s}\right)$$

Quatization noise power within band from 0 to f_0

$$P_e = \int_{-f_0}^{f_0} S_e^2(f)\left|N_{TF}(f)\right|^2 df$$

$$= \int_{-f_0}^{f_0} \left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}\left[2\sin\left(\frac{\pi f}{f_s}\right)\right]^2 df$$

assume $f_0 << f_s$ (i.e. OSR >> 1), then $\sin((\pi f)/f_s) = (\pi f)/f_s$, we have

$$P_e \cong \left(\frac{\Delta^2}{12}\right)\left(\frac{\pi^2}{3}\right)\left(\frac{2f_0}{f_s}\right)^3 = \frac{\Delta^2\pi^2}{36}\left(\frac{1}{OSR}\right)^3$$

With sinusoidal signal the maximum SNR is

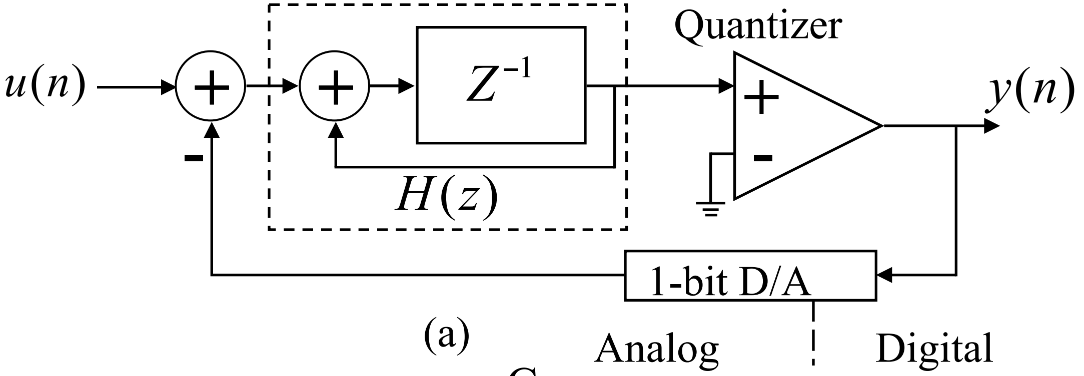
$$SNR_{\max} = 10\log\left(\frac{P_s}{P_e}\right)$$

$$= 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log\left[\frac{3}{\pi^2}(OSR)^3\right]$$

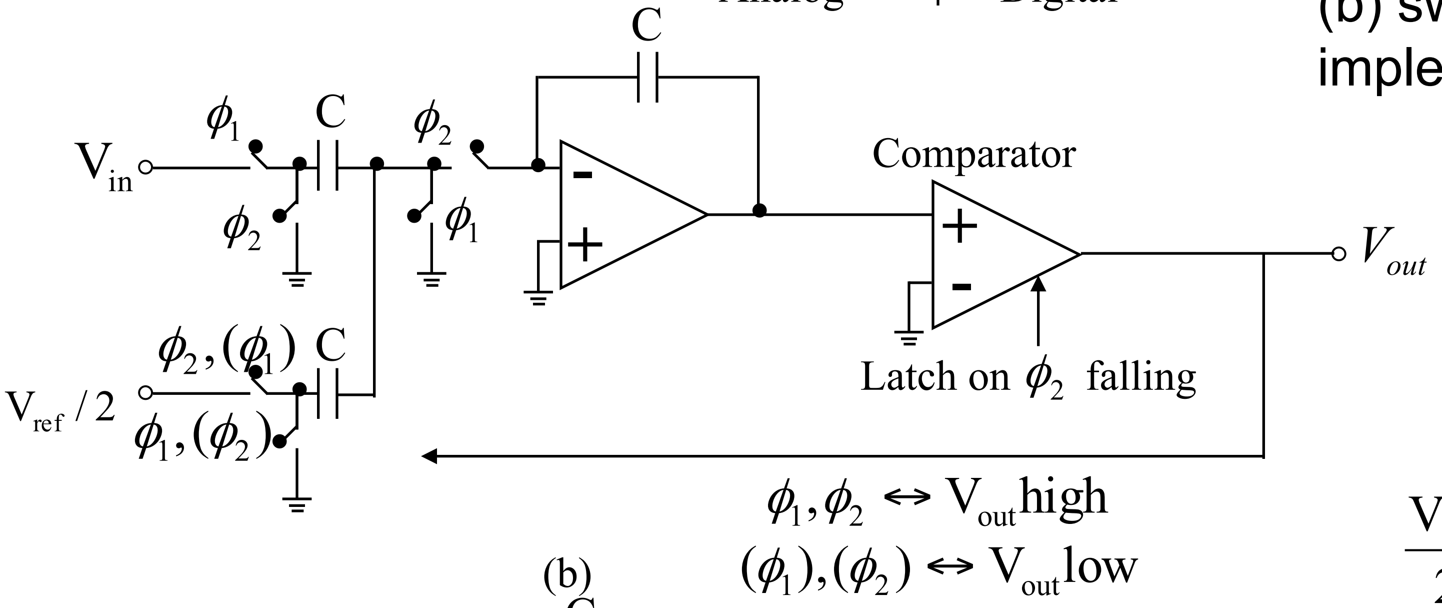
or in [dB]

$$SNR_{\max} = 6.02N + 1.76 - 5.17 + 30\log(OSR)$$

Doubling OSR improves SNR by 9dB



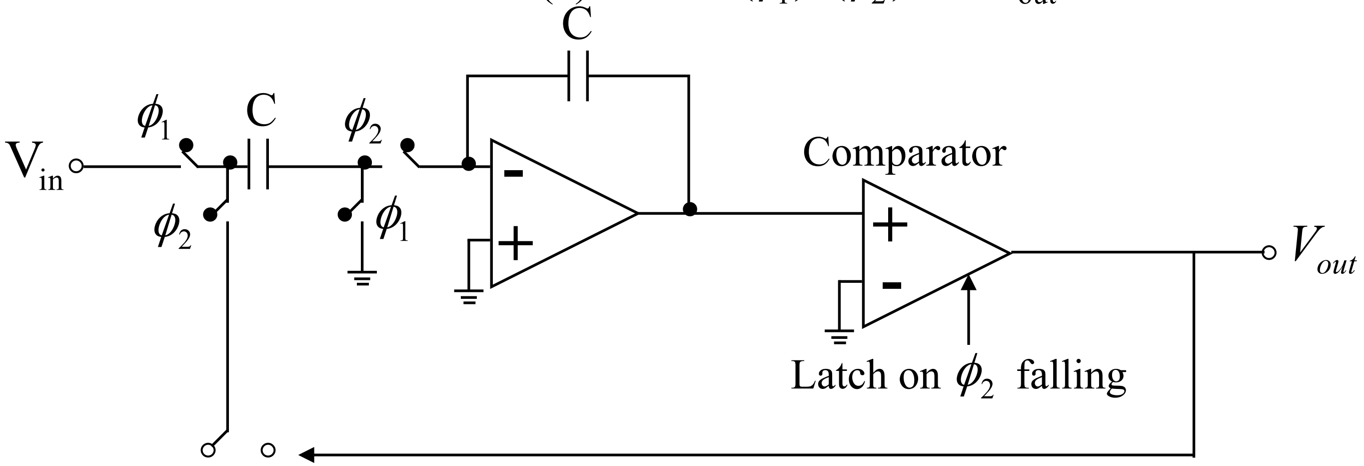
(a)



(b)

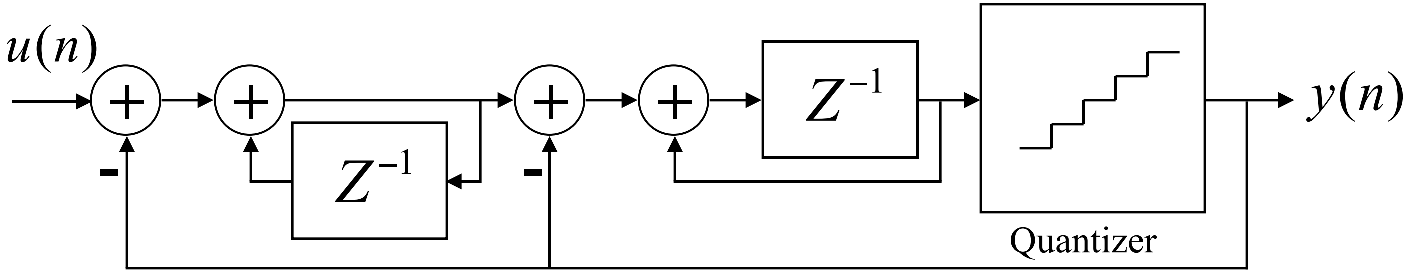
First-order A/D modulator:
(a) block diagram;
(b) switched capacitor implementation.

$$\frac{V_{ref}}{2} \quad \frac{-V_{ref}}{2}$$



First-order A/D modulator using only one input capacitance to the discrete-time integrator.

Second-Order Noise Shaping

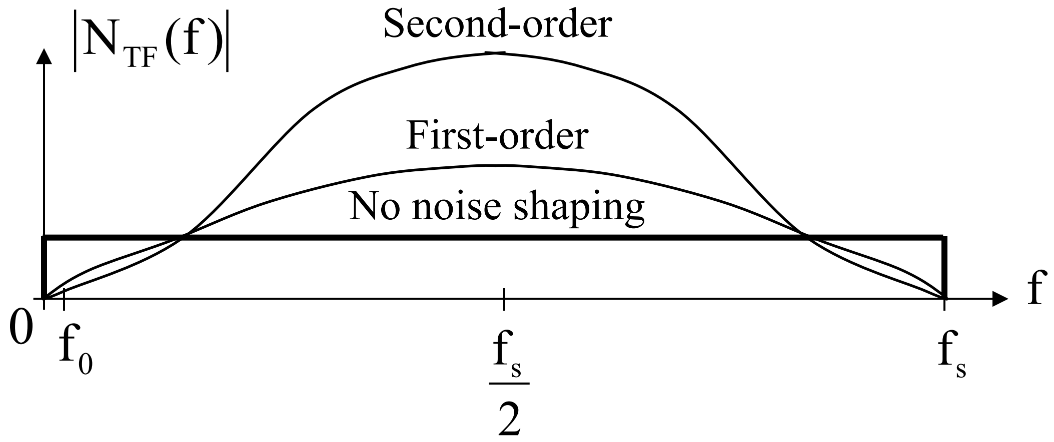


Signal transfer function

$$S_{TF}(f) = Z^{-1}$$; low-pass

Noise transfer function

$$N_{TF}(f) = (1 - Z^{-1})^2$$; high-pass



The magnitude of the noise transfer function

$$\left|N_{TF}(f)\right| = \left[2\sin\left(\frac{\pi f}{f_s}\right)\right]^2$$

The quantization noise power

$$P_e = \frac{\Delta^2 \pi^4}{60}\left(\frac{1}{OSR}\right)^5$$

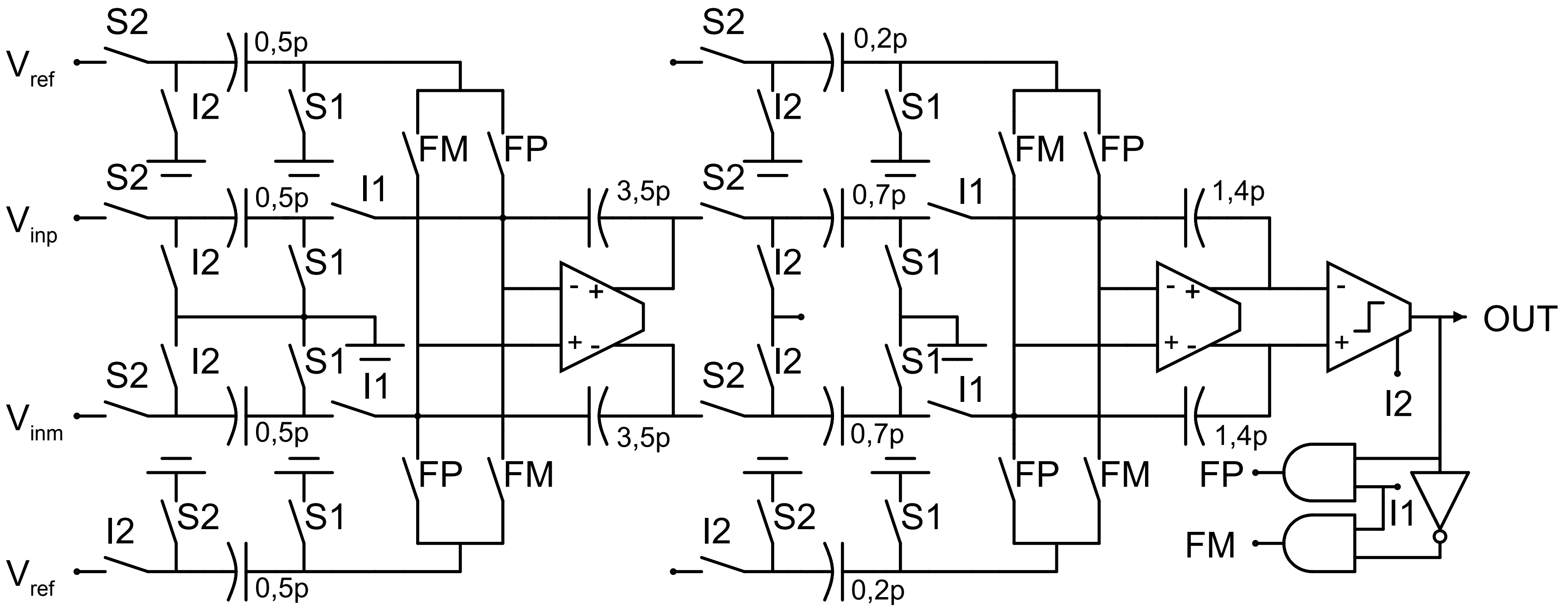
The maximum SNR

$$SNR_{max} = 10\log\left(\frac{P_s}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log\left[\frac{5}{\pi^4}(OSR)^5\right]$$

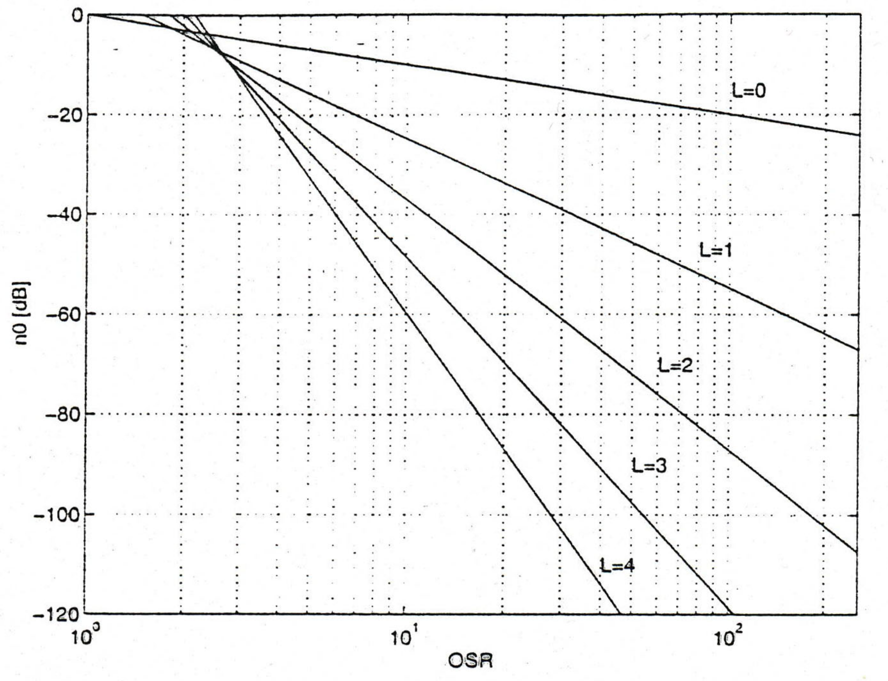
$$SNR_{max} = 6.02N + 1.76 - 12.9 + 50\log(OSR)$$

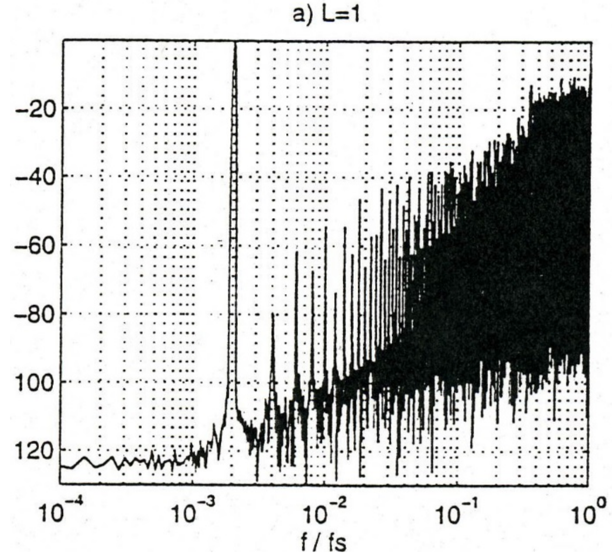
Doubling the OSR improves the SNR of a second-order modulator by 15 dB

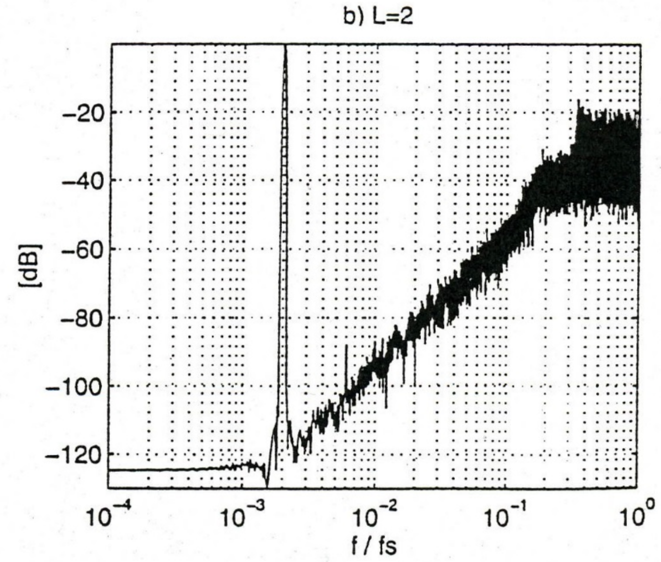
Second-order sigma-delta modulator



Quantization noise with different modulator order

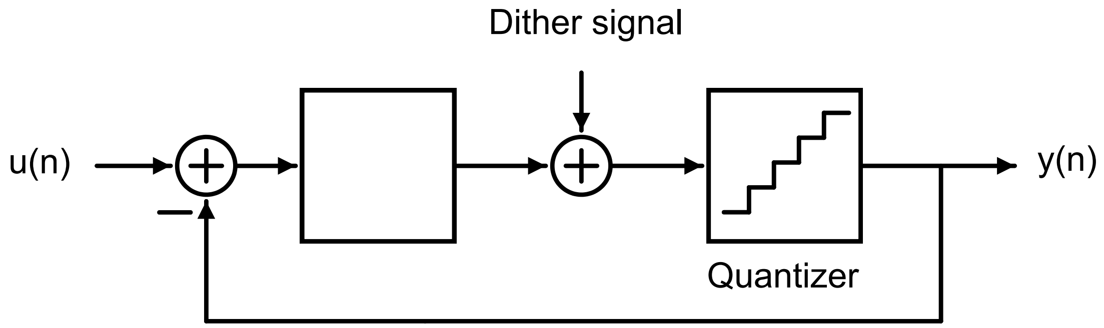






Higher order modulators are not inherently stable, to avoid instability cascaded (MASH) second order modulators are used for high order noise shaping.

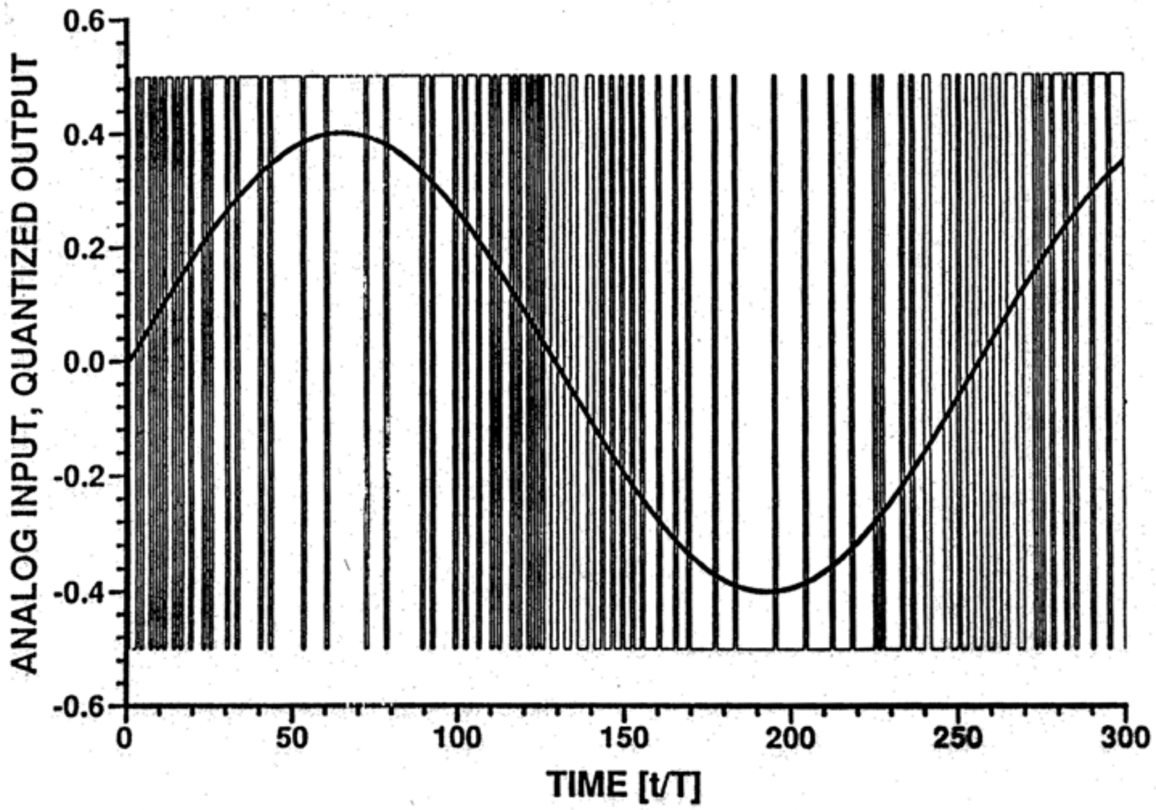
Stabilization of first-order modulator



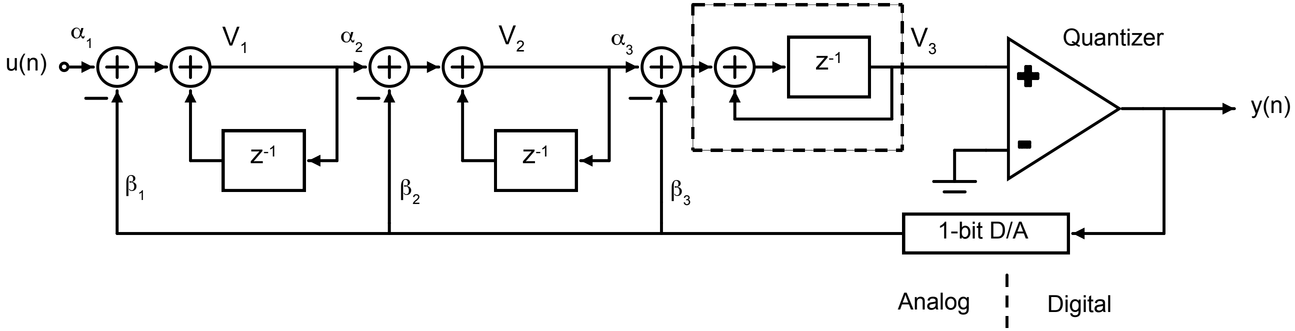
Properties of dither signal

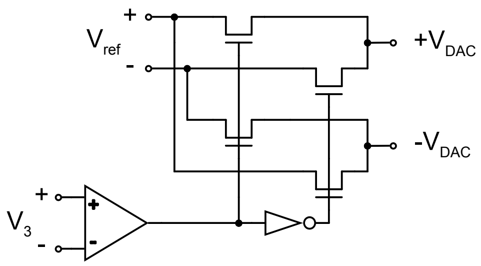
- random → white noise
- becomes noise shaped by the loop

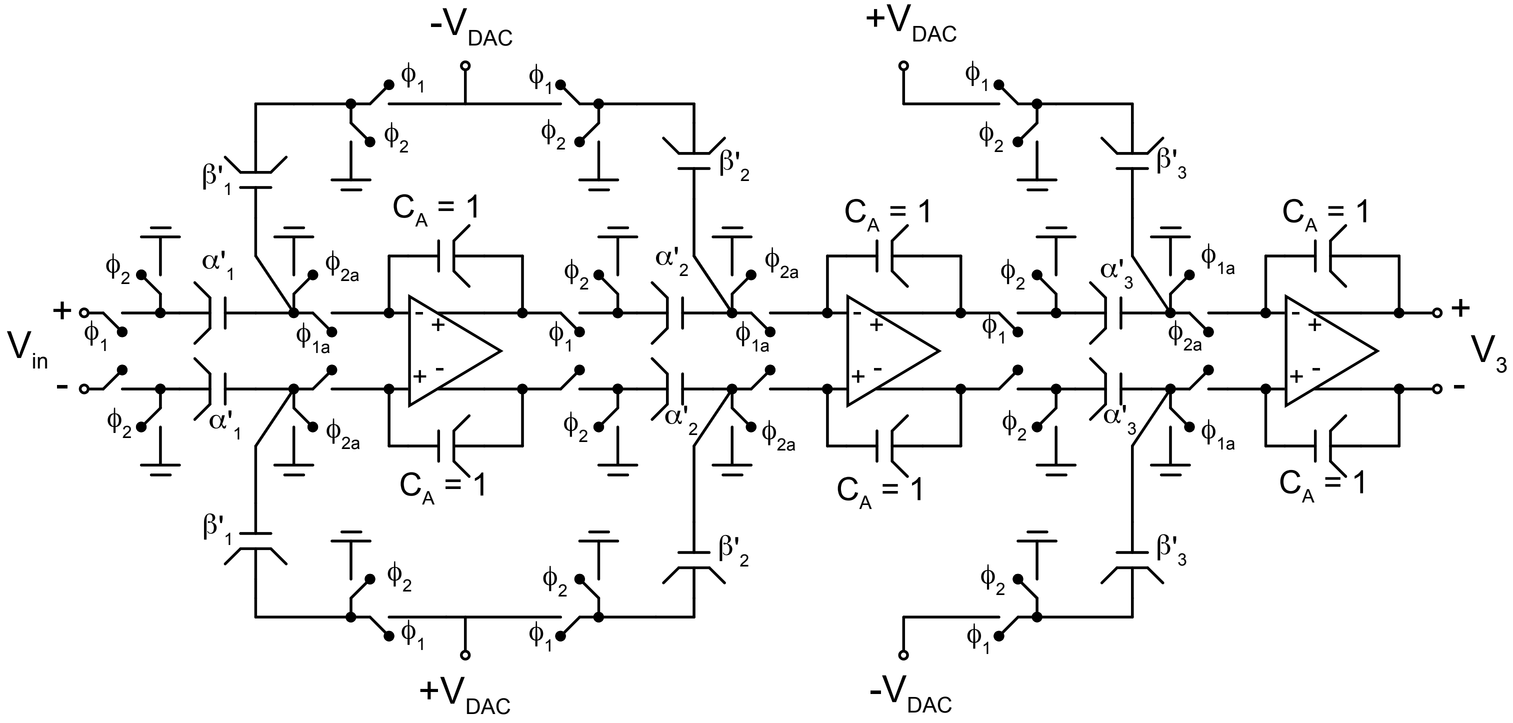
ΣΔ modulator output



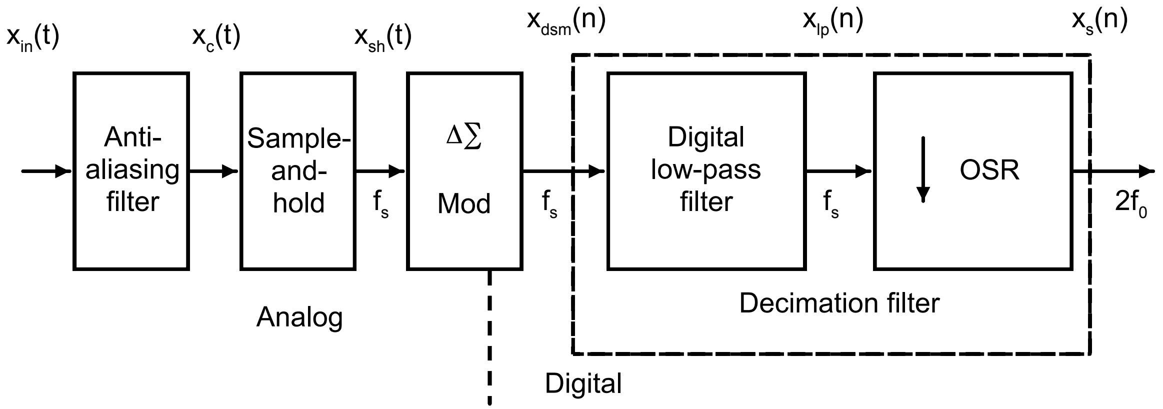
Third-order modulator realized with cascade of integrators







Functional ΣΔ analog-to-digital converter



ΣΔ modulator:

- converting analog signal into digital bit line and transferring quantization noise outside the signal band

Decimation stage:

- Decreasing the high sampling frequency
- Increasing resolution of modulator’s 1-bit data

Digital low pass filter:

- Filtering components external to signal band

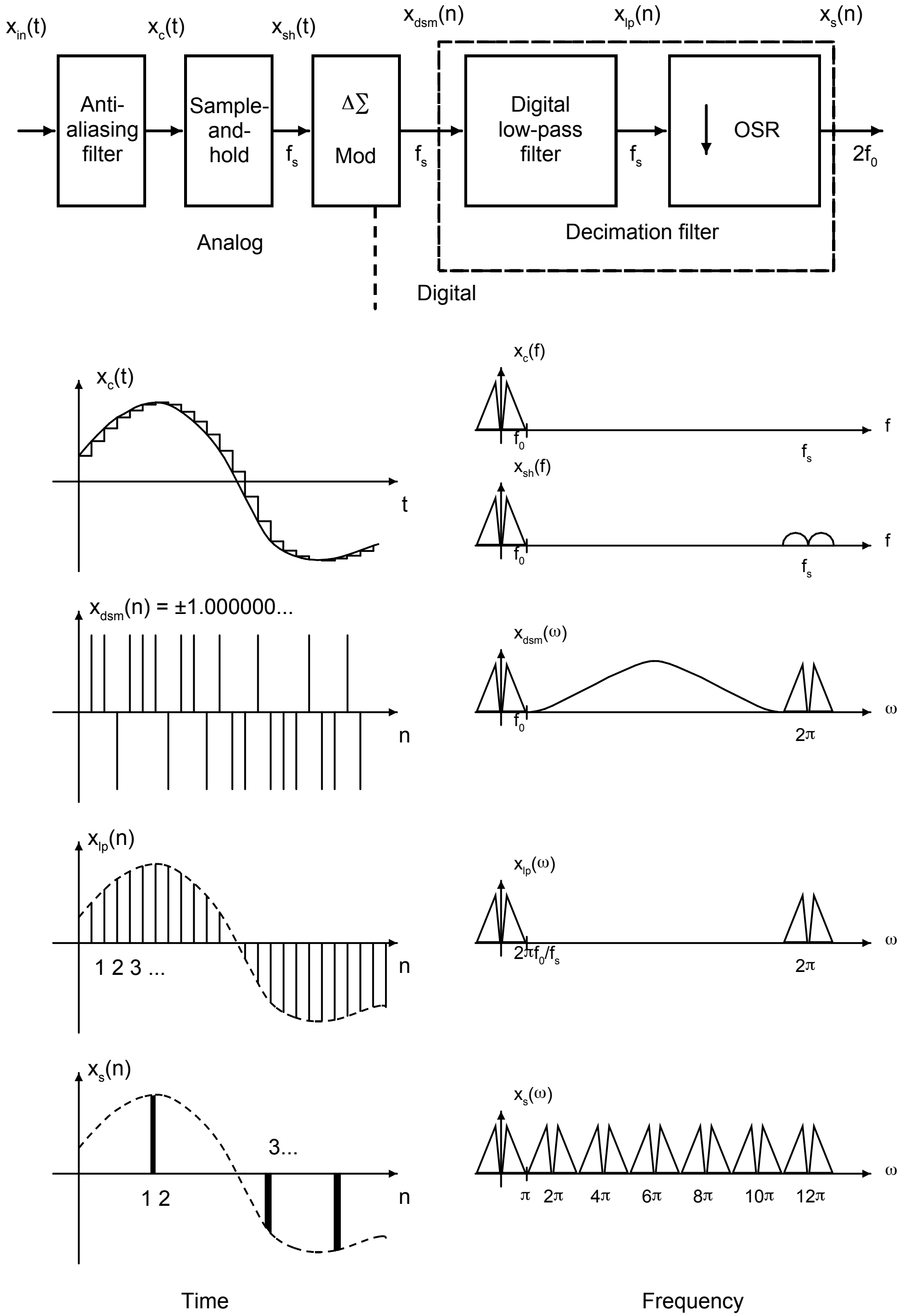
Sigma-delta modulator

Sample-hold

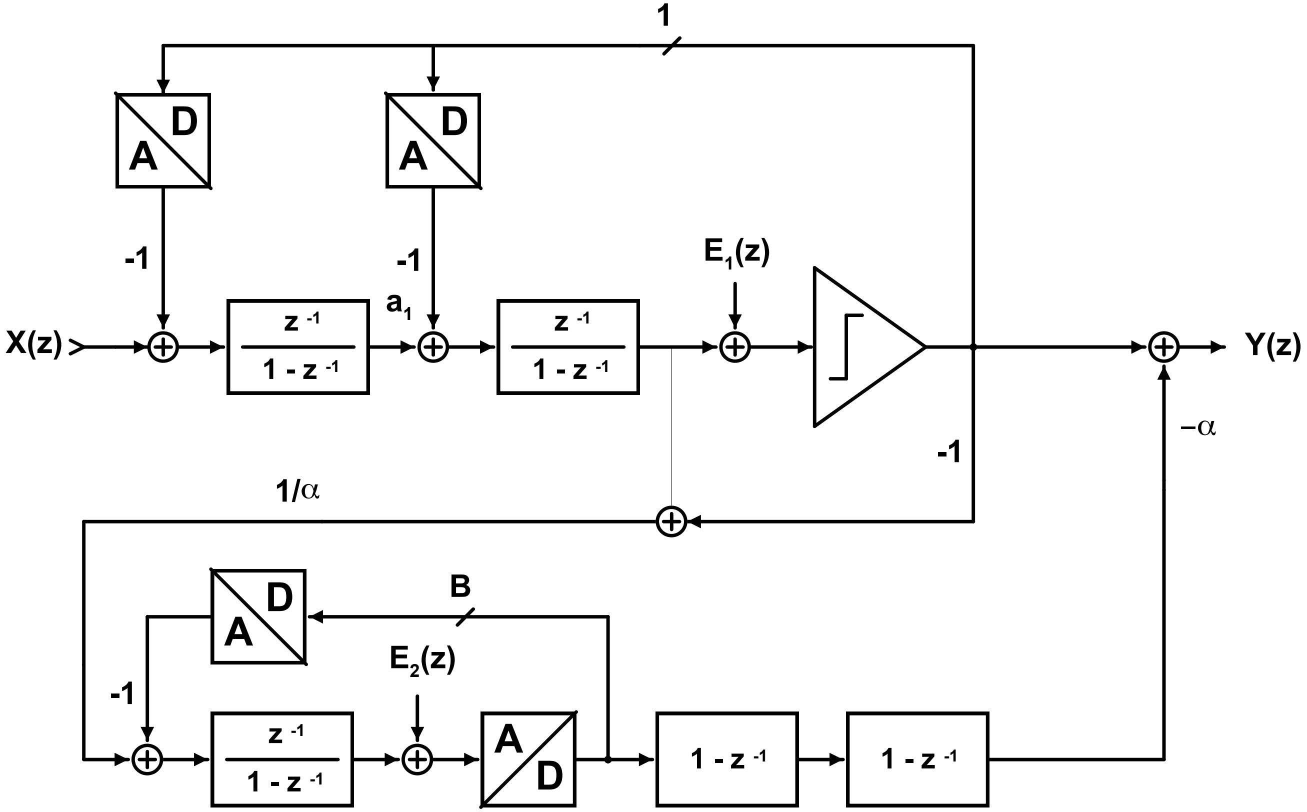
Delta-signal modulator

Low-pass filter

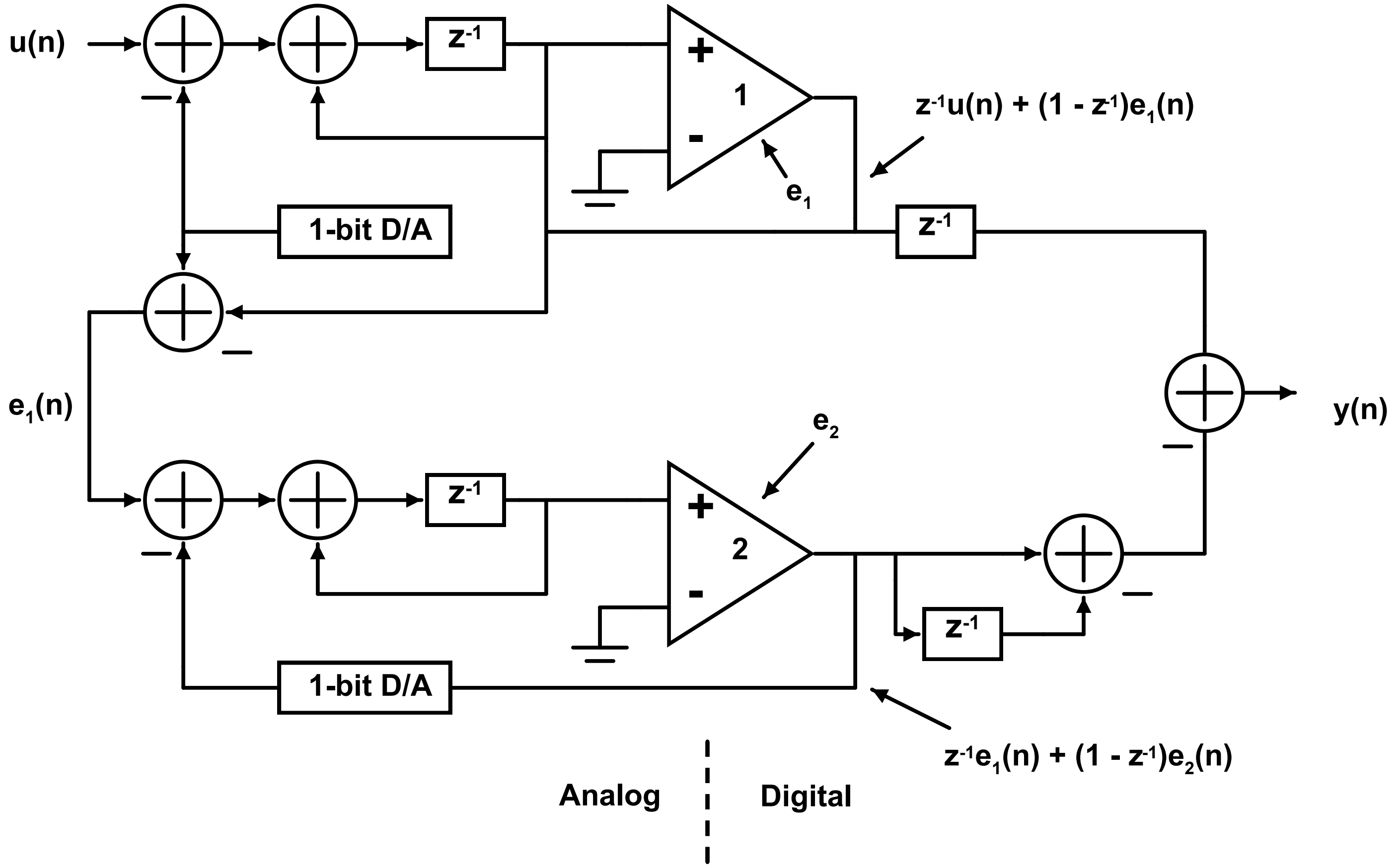
Decimation filter



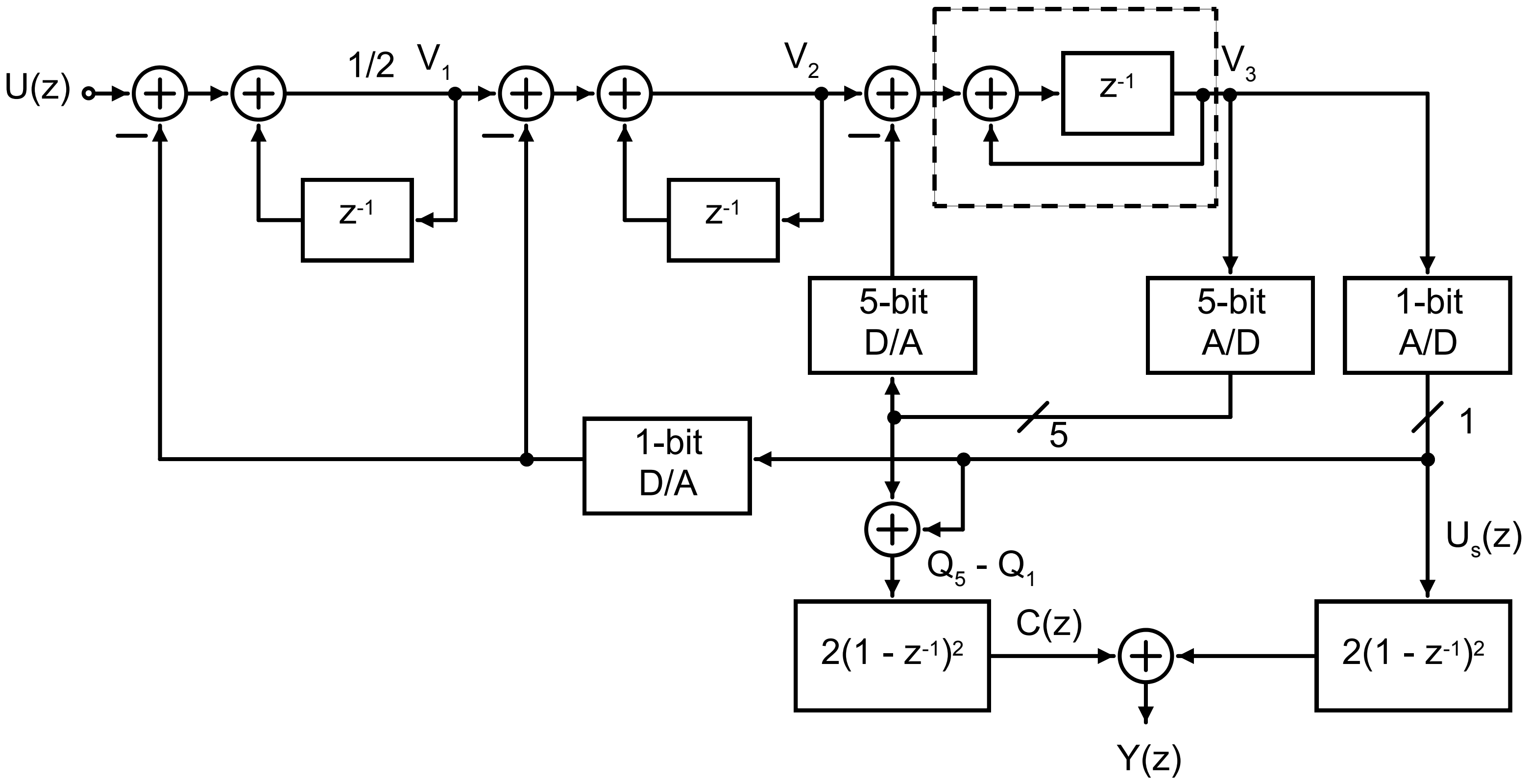
MASH-multibit $\Sigma\Delta$ –modulator with one-bit quatization in the first stage



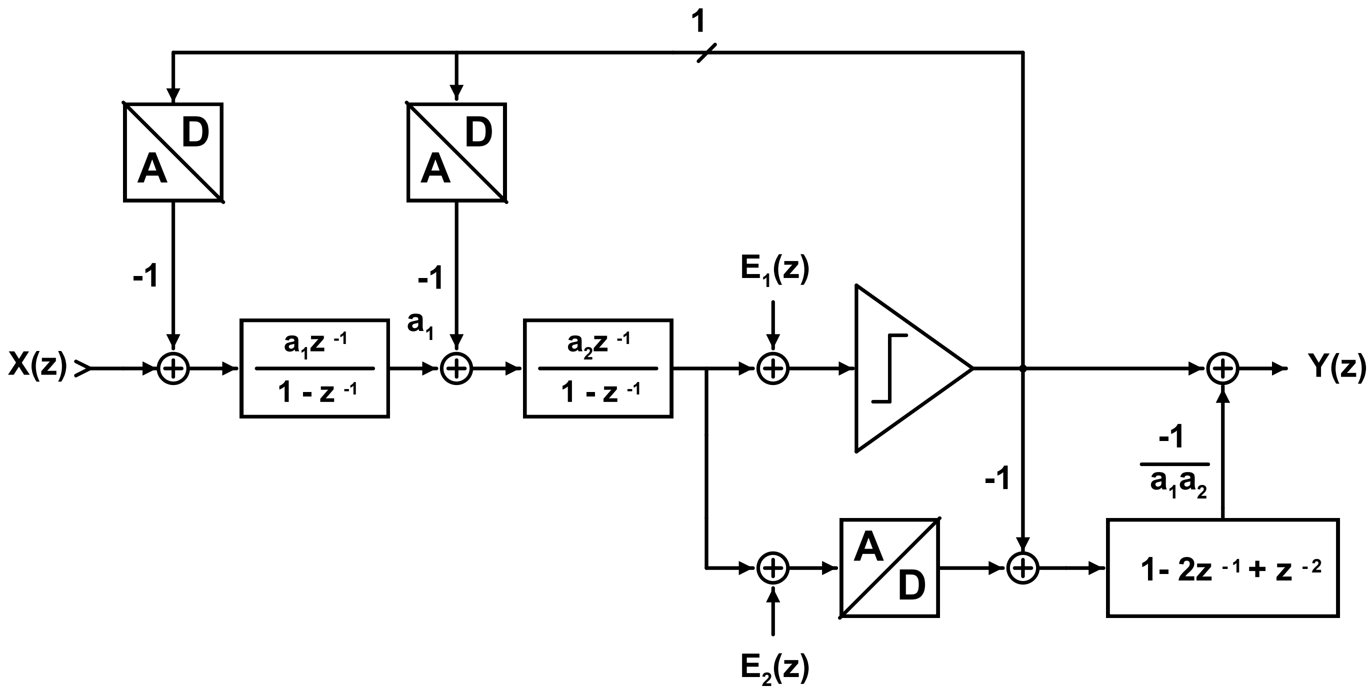
Second-order MASH modulator with cascaded first order modulators



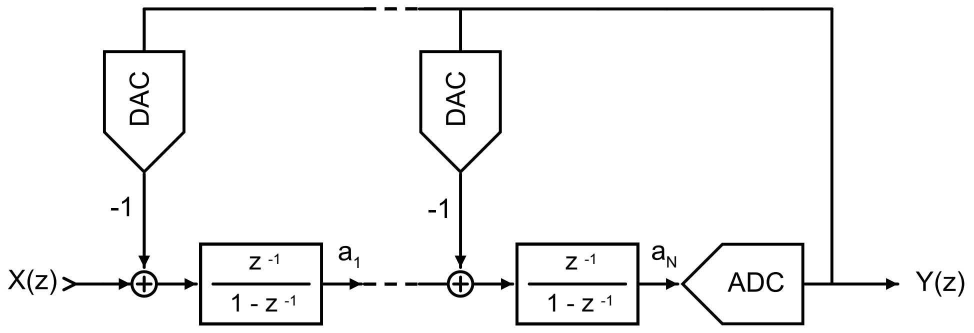
Third-order modulator multibit feedback



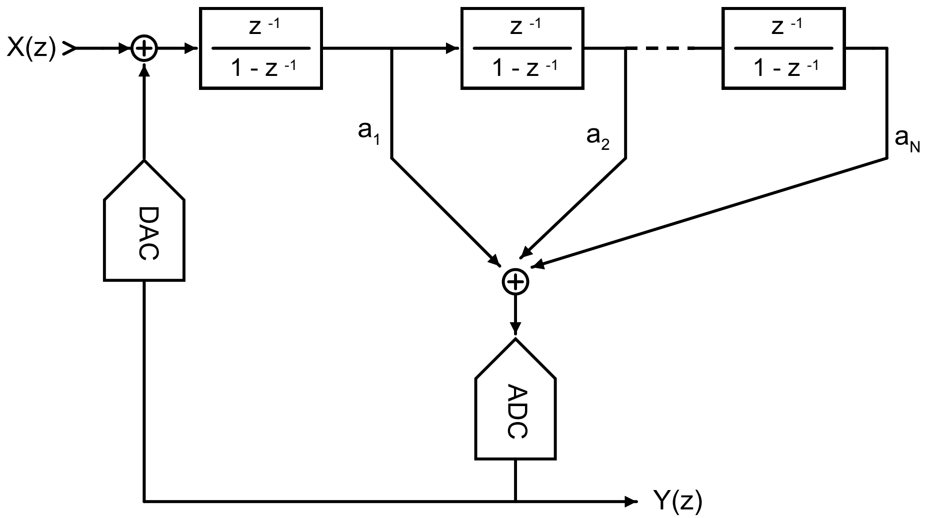
Leslie-Singh topology



Multiple feedback and feedforward ΣΔ -modulators



(a)



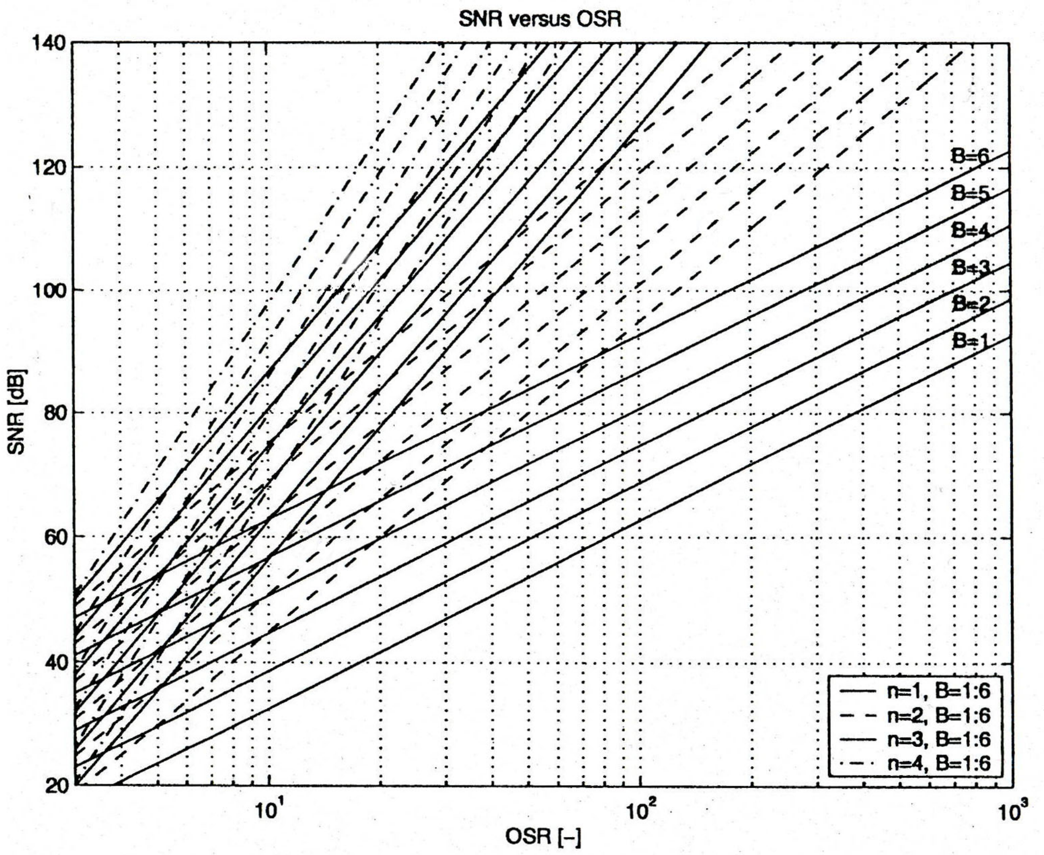
(b)

ΔΣ Design Overview (OSR)

- Theoretical SNR of ΔΣ modulator:

$$SNR \approx 20\log_{10}\left[2^B\sqrt{1.5\pi(2n+1)}\left(\frac{OSR}{\pi}\right)^{(2n+1)/2}\right]$$

n=order
B=ADC resolution
OSR=oversampling ratio

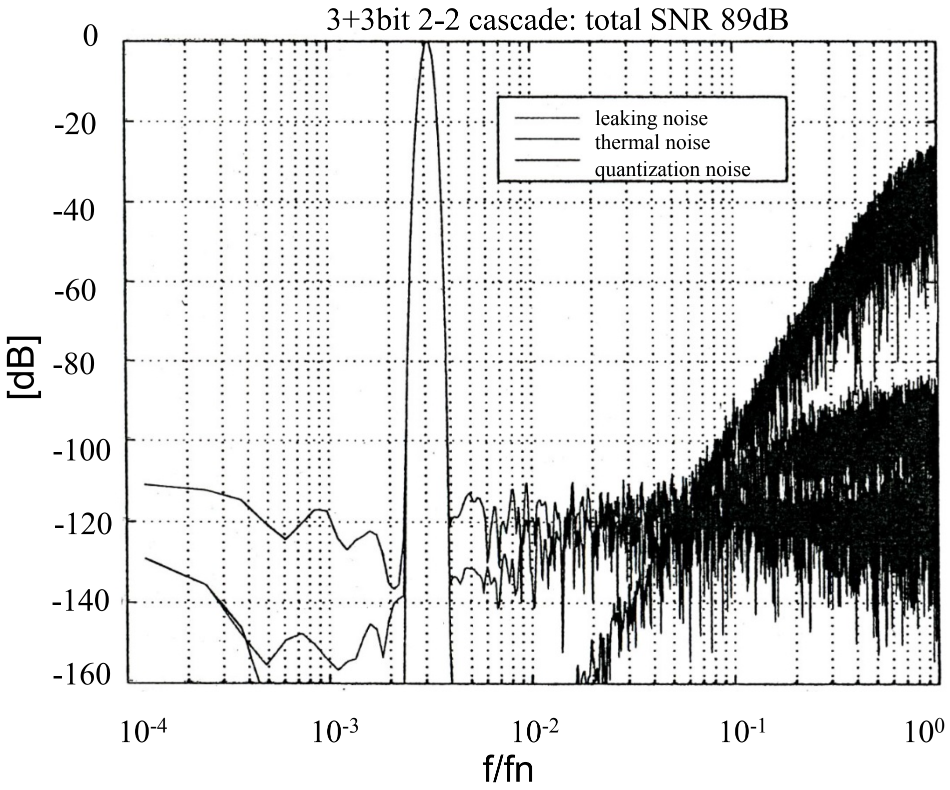


Order of the modulator, cascade

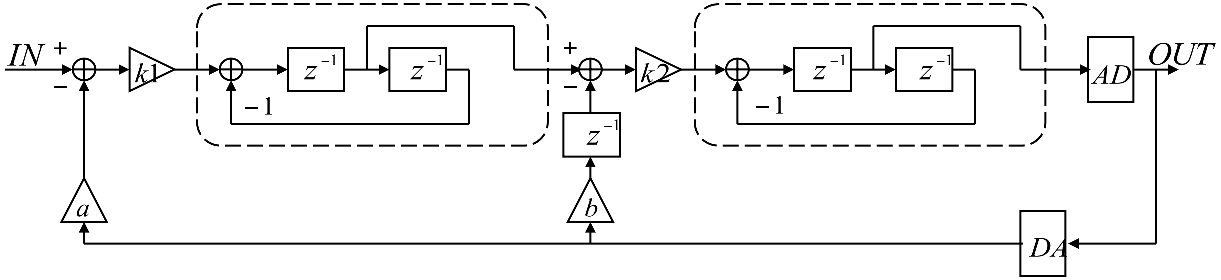
- Modulator order can be increased by cascading 1st or 2nd order modulators while maintaining the stability

- Coefficient matching between the modulators becomes very strict

- 1st modulator's amplifiers' finite gain generates "leaking" noise to the output
  - Can be reduced by adding more bits to the first modulator



The 4th order band-pass sigma-delta modulator



- Specifications:
  - 14b resolution over 250kHz bandwidth (OSR=150)
  - 80MHz sampling frequency
  - 20 MHz IF frequency
  - internal resolution 1-2 bits
- Implementation:
  - using a new SC-resonator topology
  - amplifier GBW=500MHz, A=70dB
- Accuracy limitation:
  - resonator imperfections increases the noise floor