1. *a) Derive the expression of the extraction efficiency for a planar LED b) Calculate the extraction efficiency of a surface emitting planar light diode when the refractive index of the semiconductor is 3.6 c) Calculate the extraction efficiency of a LED with a dome-shaped encapsulant if the refractive index of the encapsulant is 1.6.*

a) Extraction efficiency is limited by the total reflection from the semiconductor-air interface. When $\theta = 90°$ the critical angle θ_C is

$$n_1 \sin\theta = n_1 = n_2 \sin\theta_C \quad \Rightarrow \quad \sin\theta_C = \frac{n_1}{n_2} .$$

The light rays coming with a smaller angle will get out of the component. The amount of these photons with respect to the total emission is (because spontaneous emission is isotropic)

$$\frac{A_{em}}{A} \approx \frac{\pi y^2}{4\pi r^2} = \frac{\pi r^2 \sin^2\theta_C}{4\pi r^2} = \frac{1}{4}\sin^2\theta_C = \frac{1}{4}\left(\frac{n_1}{n_2}\right)^2 ,$$

Where the outcoming emission surface area has been approximated by planar circle.

Back reflection also limits the amount of outcoming light. The Fresnel formulas give the amplitude reflection coefficient r and the intensity reflection coefficient R in case of perpendicular reflection

$$r_{\theta_2 \approx 0} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \approx \frac{n_1 - n_2}{n_1 + n_2} \quad \Rightarrow \quad R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 .$$

This equation holds for the angles $\theta < \theta_C$.

Transmission coefficient $T = 1 - R = \dfrac{4n_1 n_2}{(n_2 + n_1)^2}$ and extraction efficiency

$$\eta_{ex} = T\frac{A_{em}}{A} = \frac{1}{4}\left(\frac{n_1}{n_2}\right)^2 \frac{4n_1 n_2}{(n_2 + n_1)^2} = \left(\frac{n_1}{n_2}\right)^2 \frac{n_1 n_2}{(n_2 + n_1)^2} .$$

b) $n_2 = 3.6 \quad \Rightarrow \quad \dfrac{A_{em}}{A} = 0.019 \quad , \quad T = 0.681 \quad \Rightarrow \quad \eta_{ex} = T\dfrac{A_{em}}{A} = 1.3\%$.

c) Here one has to take into account the transmission coefficient in the interface between air and dome

$$n = 1.6 \quad \Rightarrow \quad \frac{A_{em}}{A} = 0.049 \quad , \quad T_1 = 0.852 \quad , \quad T_2 = 0.947 \quad \Rightarrow \quad \eta_{ex} = T_1 \frac{A_{em}}{A} T_2 = 4.0\% .$$

2. *The p and n sides of a GaAs LED have a doping concentration of 10^{18} cm^{-3}. The emission of light is caused mainly by the injection of electrons into the p-side. There is a recombination center in the active region with a time constant of 5×10^{-9} s. Assume that the lifetime of the electrons and the holes is the same and that D_e = 120 cm^2 s^{-1}, D_h = 0.01 D_e. What is the injection efficiency with bias voltage of 1 V, if the coefficient of band-to-band radiative recombination is B_r = 7.2×10^{-10} cm^3 s^{-1}?*

Diode current equation for the electron current in p-side

$I_e = AqD_e \frac{n_{p0}}{L_e}\left(e^{qV_f/k_BT}-1\right) = AqD_e \frac{n_i^2}{p_{p0}L_e}\left(e^{qV_f/k_BT}-1\right) = AqD_e \frac{n_i^2}{N_A L_e}\left(e^{qV_f/k_BT}-1\right)$, because in the p-side $n_{p0}p_{p0} = n_i^2$. Hole current to the n-side is

$$I_h = AqD_h \frac{p_{n0}}{L_h}\left(e^{qV_f/k_BT}-1\right) = AqD_h \frac{n_i^2}{n_{n0}L_h}\left(e^{qV_f/k_BT}-1\right) = AqD_h \frac{n_i^2}{N_D L_h}\left(e^{qV_f/k_BT}-1\right).$$

Recombination current according to Shockley-Hall-Read–theory: $I_{rec} = \frac{Aq}{\tau_{nr}} n_i W\left(e^{qV_f/2k_BT}-1\right)$.

Width of the depletion region: $W = \sqrt{\frac{2\varepsilon_r\varepsilon_0}{q}\left(\frac{1}{N_D}+\frac{1}{N_A}\right)\left|V_{bi}\right|}$.

Because $V_{bi} = \frac{k_BT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$ = 1.40 V $\Rightarrow$ W = 0.064 μm.

Diffusion lengths can be calculated from recombination time constants and diffusion constants:

$\tau_e = \frac{1}{B_r N_A} = 1.39$ ns $\Rightarrow$ $L_e = \sqrt{\tau_e D_e}$ = 4.1 μm.

$\tau_h = \tau_e$ $\Rightarrow$ $L_h = \sqrt{0.01\cdot\tau_e D_e}$ = 0.4 μm.

Let's mark $e^{qV_f/k_BT}-1 \approx e^{qV_f/k_BT} = C\left(V_f\right)$. Then injection efficiency is

$$\eta_{in} = \frac{I_e}{I_e+I_h+I_{rec}} = \frac{D_e \frac{n_i^2}{N_A L_e} C\left(V_f\right)}{D_e \frac{n_i^2}{N_A L_e} C\left(V_f\right) + D_h \frac{n_i^2}{N_D L_h} C\left(V_f\right) + \frac{n_i W}{\tau_{nr}} C\left(V_f\right)} =$$

$$= \frac{1}{1+\frac{D_h}{D_e}\frac{L_e}{L_h}+\frac{L_e}{D_e}\frac{N_A}{\tau_{nr}n_i}W\cdot C^{-1/2}\left(V_f\right)} = \frac{1}{1+0.1+9.4} = 9.5\%.$$

3. *Using the parameters of the previous exercise, calculate the spontaneous emission wavelength and the optical power of the LED at a bias voltage of 1 V assuming that the extraction efficiency is 10% and the surface of the diode is 1 mm^2.*

Injection current of the electrons is according to the previous exercise is given by $I_e = AqD_e \frac{n_{p0}}{L_e}\left(e^{qV_f/k_BT} - 1\right)$, where $n_{p0} = \frac{n_i^2}{N_A}$, so that $I_e \approx 0.10$ mA. Total rate of the spontaneous emission is then $R_{tot} = \frac{I_e}{q} \approx 6.2\times 10^{14}$ s^{-1}, if one assumes that all charges injected to the p-side will recombine radiatively. The total power is obtained by multiplying the emission frequency with the photon energy (approximately the same as GaAs bandgap) and extraction efficiency $P_{opt} = \eta_e \cdot E_g \cdot R_{tot} \approx 0.014$ mW.

4. *Show that the intensity distribution of the radiation emitted by a planar LED can be expressed by the Lambertian distribution. Assume that the light source inside the semiconductor can be considered as a point source.*

Inside the semiconductor the light intensity is isotropic, i.e., $I(\theta_i) = \frac{P}{4\pi r^2} = I_0$. If a point source is at the distance of d from the surface, the light ray travelling to direction θ_i has travelled a distance of $r = d/\cos\theta_i$ when reaching the air-semiconductor interface. Energy flux through a differential surface element is $dP_i = I(\theta_i)\cdot r d\theta_i \cdot r d\varphi$. Respectively, out far away from the surface the energy flux is $dP_O = I(\theta_O)\cdot R d\theta_O \cdot R d\varphi = dP_i$ $\Rightarrow$ $I(\theta_O) = I(\theta_i)\frac{r^2}{R^2}\frac{d\theta_i}{d\theta_O} = \frac{P_{tot}}{4\pi r^2}\frac{r^2}{R^2}\frac{d\theta_i}{d\theta_O} = \frac{P_{tot}}{4\pi R^2}\frac{d\theta_i}{d\theta_O} = I_O\frac{d\theta_i}{d\theta_O}$, where $I_O = \frac{P_{tot}}{4\pi R^2}$. The relation between the angles is obtained from Snell's Law $n_i \sin\theta_i = n_O \sin\theta_O$ $\Rightarrow$ $n_i \cos\theta_i \, d\theta_i = n_O \cos\theta_O \, d\theta_O$ $\Rightarrow$ $\frac{d\theta_i}{d\theta_O} = \frac{n_O}{n_i}\frac{\cos\theta_O}{\cos\theta_i}$. Let's consider the interface between GaAs ($n_i = 3.6$) and air ($n_0 = 1.0$), then $\theta_i < 16.1°$, so the denominator gets values $0.96 < \cos\theta_i < 1$ $\Rightarrow$ $I(\theta_O) \approx I'_O \cos\theta_O$.

5. *Let's consider the coupling efficiency of a LED into an optical fiber. Assume that the refractive indices of the core and cladding material of the fiber are 1.52 and 1.48, respectively. a) Light entering the fiber in angles larger than the critical angle θ_a does not couple into the traveling modes. Calculate θ_a. b) Derive an approximation for the critical angle θ_a, if the refractive index difference between the core and the cladding is small. c) Calculate the coupling efficiency into a fiber of a planar surface-emitting LED with an intensity distribution of $I = I_0 \cos\theta$ outside the LED.*

a) The angle for total internal reflection θ_c at the interface of core (n_1) and cladding (n_2) is $n_1 \sin\theta_c = n_2 \sin\theta_2$, where $\theta_2 = 90 \Rightarrow n_1 \sin\theta_c = n_2$.
Then the light beam coming from air to the core has to bend to an angle of $\theta_1 = 90° - \theta_C$.
At the interface of core and air (n_0) one gets $n_0 \sin\theta_a = n_1 \sin\theta_1$, so that one has to solve the equation pair

$$\begin{cases} n_0 \sin\theta_a = n_1 \sin\theta_c \\ n_1 \sin\theta_c = n_2 \end{cases} \Rightarrow n_0^2 \sin^2\theta_a + n_2^2 = n_1^2 \sin^2\theta_c + n_1^2 \cos^2\theta_c = n_1^2$$

$$\Rightarrow n_0^2 \sin^2\theta_a = n_1^2 - n_2^2 \Rightarrow \theta_a = \arcsin\sqrt{\frac{n_1^2 - n_2^2}{n_0^2}} \approx 20.3$$

b) $n_2 = n_1 - \Delta n \Rightarrow n_1^2 - n_2^2 = 2n_1\Delta n - (\Delta n)^2 \approx 2n_1\Delta n$, when $\Delta n << n_1, n_2$

$$\Rightarrow \theta_a = \arcsin\sqrt{\frac{2n_1\Delta n}{n_0^2}} \approx 20.4$$

c) Coupling efficiency is the intensity of the light entering travelling modes inside the fiber divided by the intesity extracted out the component:

$$\eta_c = \frac{\int_0^{\theta_a} I(\theta)\sin\theta\, d\theta}{\int_0^{\pi/2} I(\theta)\sin\theta\, d\theta} = \frac{\int_0^{\theta_a} I_0 \cos\theta \sin\theta\, d\theta}{\int_0^{\pi/2} I_0 \cos\theta \sin\theta\, d\theta} = \frac{\sin^2\theta_a - \sin^2 0}{\sin^2\frac{\pi}{2} - \sin^2 0} = \sin^2\theta_a = \frac{n_1^2 - n_2^2}{n_0^2} = 12\% .$$