GIS-E3010 Least-Squares Methods in Geoscience

Lecture 5, activation 4

<u>Space intersection</u>

Collinearity equations

$$\begin{cases} x = -c\dfrac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} = f_x \\ y = -c\dfrac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} = f_y \end{cases}$$

Can be developed in the form of

$$\begin{cases} (xr_{31} + cr_{11})X + (xr_{32} + cr_{12})Y + (xr_{33} + cr_{13})Z = (xr_{31} + cr_{11})X_0 + (xr_{32} + cr_{12})Y_0 + (xr_{33} + cr_{13})Z_0 \\ (yr_{31} + cr_{21})X + (yr_{32} + cr_{22})Y + (yr_{33} + cr_{23})Z = (yr_{31} + cr_{21})X_0 + (yr_{32} + cr_{22})Y_0 + (yr_{33} + cr_{23})Z_0 \end{cases}$$

The solution is $\hat{x} = \left(A^T A\right)^{-1} A^T y$, and no iteration is needed (a linear system)

Write (symbolically) the contents of a solution vector x.

Write (symbolically) the contents of a design matrix *A* and an observation vector *y* for one corresponding point observation (one image observation creates 2 equations i.e. corresponding observation from two images establish 4 rows). Indicate left image observations with a subscript *L* and the right image observations with a subscript *R*. At the right side of "="-sign none of scalars have connection to any unknown parameters i.e. everything belongs to the "observations" (inside the y vector). Because this is a linear case (each unknown parameter is independent from each other) linearization (creation of a Jacobian matrix) gives directly the scalar part before the unknown parameter. For example (a left image observation with subscripts),

$$\frac{f_x}{\partial X} = x_L r_{31L} + c_L r_{11L}$$

Solution:

$$x = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$A = \begin{bmatrix} x_L r_{31L} + c_L r_{11L} & x_L r_{32L} + c_L r_{12L} & x_L r_{33L} + c_L r_{13L} \\ y_L r_{31L} + c_L r_{21L} & y_L r_{32L} + c_L r_{22L} & y_L r_{33L} + c_L r_{23L} \\ x_R r_{31R} + c_R r_{11R} & x_R r_{32R} + c_R r_{12R} & x_R r_{33R} + c_R r_{13R} \\ y_R r_{31R} + c_R r_{21R} & y_R r_{32R} + c_R r_{22R} & y_R r_{33R} + c_R r_{23R} \end{bmatrix}$$

$$y = \begin{bmatrix} (x_L r_{31L} + c_L r_{11L}) X_{0L} + (x_L r_{32L} + c_L r_{12L}) Y_{0L} + (x_L r_{33L} + c_L r_{13L}) Z_{0L} \\ (y_L r_{31L} + c_L r_{21L}) X_{0L} + (y_L r_{32L} + c_L r_{22L}) Y_{0L} + (y_L r_{33L} + c_L r_{23L}) Z_{0L} \\ (x_R r_{31R} + c_R r_{11R}) X_{0R} + (x_R r_{32R} + c_R r_{12R}) Y_{0R} + (x_R r_{33R} + c_R r_{13R}) Z_{0R} \\ (y_R r_{31R} + c_R r_{21R}) X_{0R} + (y_R r_{32R} + c_R r_{22R}) Y_{0R} + (y_R r_{33R} + c_R r_{23R}) Z_{0R} \end{bmatrix}$$