GIS-E3010 Least-Squares Methods in Geoscience

Lecture 6, activation 1

We are examining the structure of the design matrix *A* in the case of bundle block adjustment.

Collinearity equations:

$$\begin{cases} x = -c\dfrac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} = f_x \\ y = -c\dfrac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} = f_y \end{cases}$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \cos\varphi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\varphi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\varphi\cos\kappa \\ -\cos\varphi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\varphi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\varphi\sin\kappa \\ \sin\varphi & -\sin\omega\cos\varphi & \cos\omega\cos\varphi \end{bmatrix}$$

We have a case in which we have 2 images and 2 ground points. The ground point 1 is observed from both images but the point 2 is observed only from the 2nd image. The purpose is to identify those elements that have some values in the A matrix (the rest contain the value 0). At first we define the order of our unknowns to be solved:

$$\Delta = \begin{bmatrix} d\omega_1 & d\varphi_1 & d\kappa_1 & dX_{01} & dY_{01} & dZ_{01} & d\omega_2 & d\varphi_2 & d\kappa_2 & dX_{02} & dY_{02} & dZ_{02} & X_1 & Y_1 & Z_1 & X_2 & Y_2 & Z_2 \end{bmatrix}^T$$

When one image observation establishes 2 equations, how many rows A-matrix will have in this case?

A is a Jacobian matrix, which means that each column contains the result from the partial derivation with respect to corresponding unknown parameter (so you can see the order from the vector above). In this case, just put any mark (e.g. ✓) to the element of matrix *A* if there is some value (i.e. when you make partial derivation you'll find that unknown parameter from the collinearity equation). You'll need as many columns for your matrix *A* than you have unknown parameters.

Solution:

A=

Relates to the orientation of image 1						*Relates to the orientation of image 2*						*Relates to the ground point 1*			*Relates to the ground point 2*		
✓	✓	✓	✓	✓	✓	0	0	0	0	0	0	✓	✓	✓	0	0	0
✓	✓	✓	✓	✓	✓	0	0	0	0	0	0	✓	✓	✓	0	0	0
0	0	0	0	0	0	✓	✓	✓	✓	✓	✓	✓	✓	✓	0	0	0
0	0	0	0	0	0	✓	✓	✓	✓	✓	✓	✓	✓	✓	0	0	0
0	0	0	0	0	0	✓	✓	✓	✓	✓	✓	0	0	0	✓	✓	✓
0	0	0	0	0	0	✓	✓	✓	✓	✓	✓	0	0	0	✓	✓	✓

E.g. the 1st row corresponds to the image observation of ground point 1 measured from the image 1. This row includes partial derivatives with respect to all unknown parameters from the first (upper) collinearity equation. The 2nd row is otherwise exactly the same than the first row, but derivation happens to the second (lower) collinearity equation.