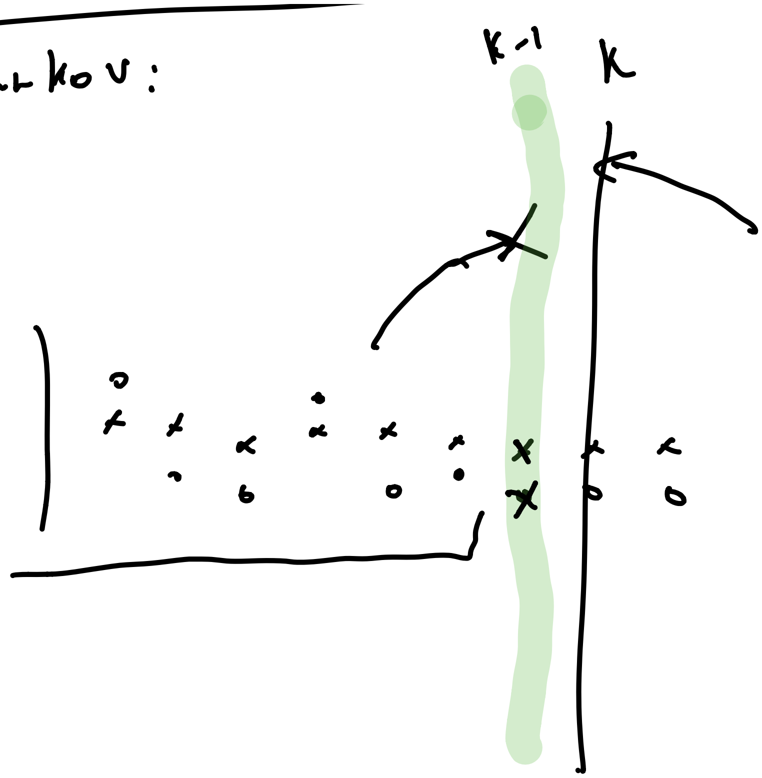
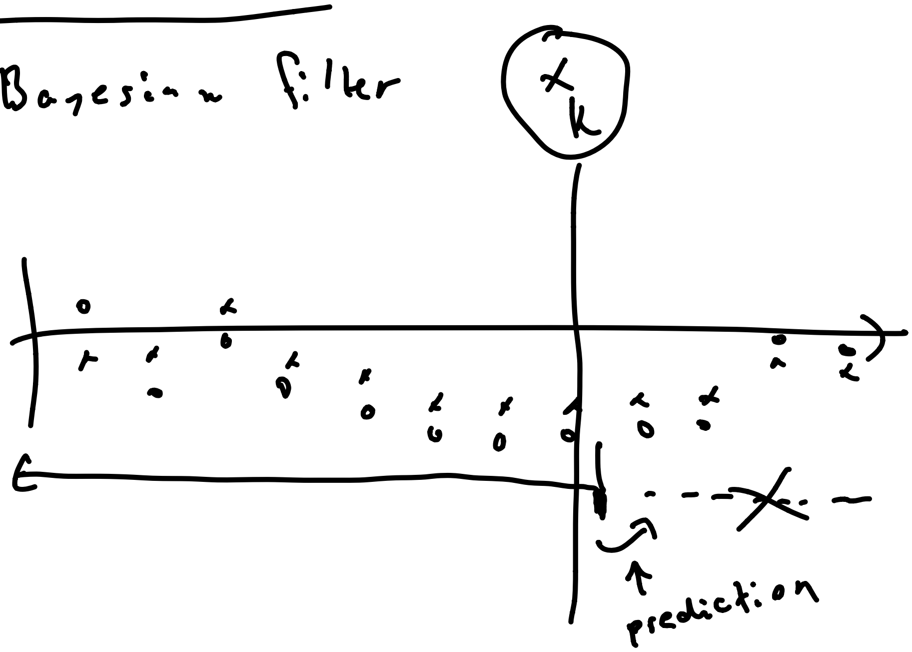
Markov:



Bayesian filter



$$p(A,B|C) = p(A|B,C)\, p(B|C)$$

$$p(B|A,C)\, p(A|C)$$

$$p(\underline{A}\mid B, C) = \frac{\cdots}{\underbrace{\int p(B\mid A, C)\, p(A\mid C)\, dA}_{Z}}$$

$$|cP| = c^n |P| \quad , \quad P \in \mathbb{R}^{n\times n}$$

const. matrix

KF:

$$p(x_{k-1}\mid y_{1:k-1}) = N(x_{k-1}\mid m_{k-1}, P_{k-1})$$

$$p(x_k\mid x_{k-1}, y_{1:k-1}) = N(x_k \mid A_{k-1} x_{k-1}, Q_{k-1})$$

$$p(x_k \mid y_{1:k-1}) = ?$$

mapping:

$$x \sim x_{k-1}, \quad y \sim x_k$$

$$p\left(\begin{pmatrix} x_{k-1} \\ x_k \end{pmatrix} \middle| y_{1:k-1}\right) = N\left(\begin{bmatrix} x_{k-1} \\ x_k \end{bmatrix} \middle| \begin{pmatrix} m_{k-1} \\ A_{k-1} m_{k-1} \end{pmatrix}, \begin{pmatrix} P_{k-1} & P_{k-1} A_{k-1}^T \\ A_{k-1} P_{k-1} & A_{k-1} P_{k-1} A_{k-1}^T + Q_{k-1} \end{pmatrix}\right)$$

$$p(x_k \mid y_{1:k-1}) = N(x_k \mid A_{k-1} m_{k-1}, A_{k-1} P_{k-1} A_{k-1}^T + Q_{k-1})$$

$$\underbrace{\qquad\qquad}_{m_k^-} \quad \underbrace{\qquad\qquad\qquad}_{P_k^-}$$

prediction step

$$p(x_k \mid y_{1:k-1}) = N(x_k \mid m_k^-, P_k^-)$$

$$p(y_k \mid x_k, y_{1:k-1}) = N(y_k \mid H_k x_k, R_k)$$

$$x \sim x_k, \quad y \sim y_k$$

$$p\left(\begin{pmatrix} x_k \\ y_k \end{pmatrix} \middle| \, y_{1:k-1}\right) = N\left(\begin{pmatrix} \overset{a}{m_k^-} \\ \underset{b}{H_k m_k^-} \end{pmatrix}, \begin{pmatrix} \overset{A}{P_k^-} & \overset{C}{P_k^- H_k^T} \\ \underset{C^T}{H_k P_k^-} & \underset{B}{H_k P_k^- H_k^T + R_k} \end{pmatrix}\right)$$

$$K_k = P_k^- H_k^T S_k^{-1}$$

$$p(x_k \mid \overbrace{y_k, y_{1:k-1}}^{y_{1:k}})$$

$$= N\Big(x_k \,\Big|\, m_k^- + \underbrace{P_k^- H_k^T (\underbrace{H_k P_k^- H_k^T + R_k}_{S_k})^{-1}}_{K_k} (y_k - H_k m_k^-),$$

$$P_k^- - \underbrace{P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} H_k P_k^-}_{= K_k S_k K_k^T} \Big)$$

$$\Downarrow$$

$$m_k = m_k^- + K_k (y_k - H_k m_k^-)$$

$$P_k = P_k^- - K_k S_k K_k^T$$

$$S_k = H_k P_k^- H_k^T + R_k$$

$$K_k = P_k^- H_k^T S_k^{-1}$$