Aalto University
School of Electrical Engineering

ELEC-E8126: Robotic Manipulation
Motion control

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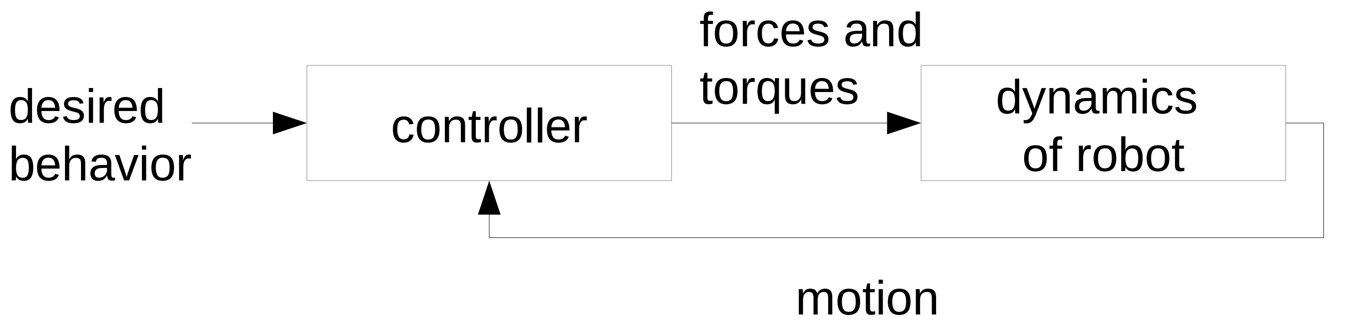
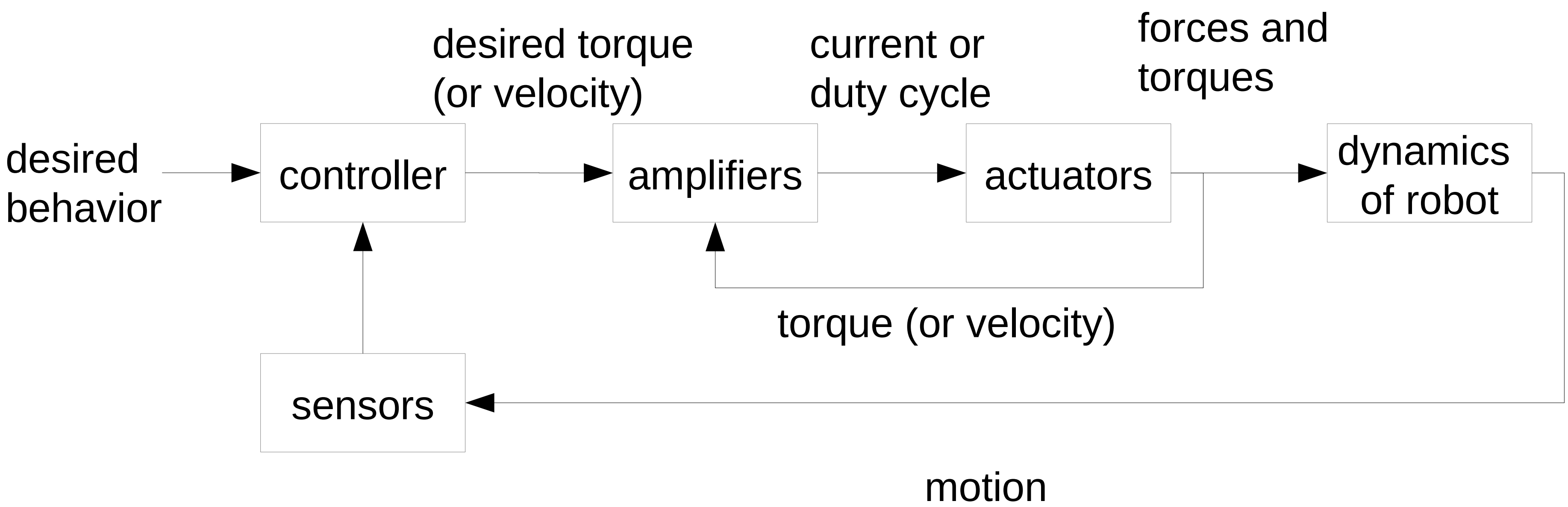
Learning goals

- Understand basic approaches of robot motion control.
- Understand structure of dynamics of serial kinematic chains such as robot arms.

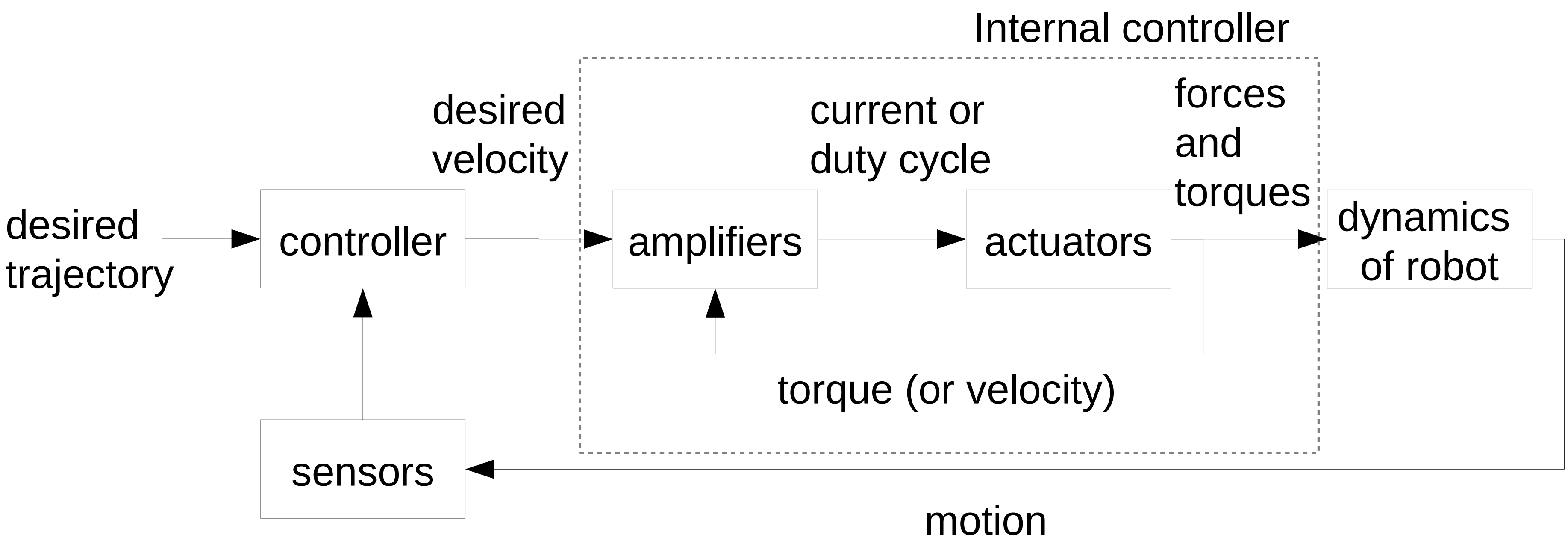
Control – general structure



Control – typical real structure

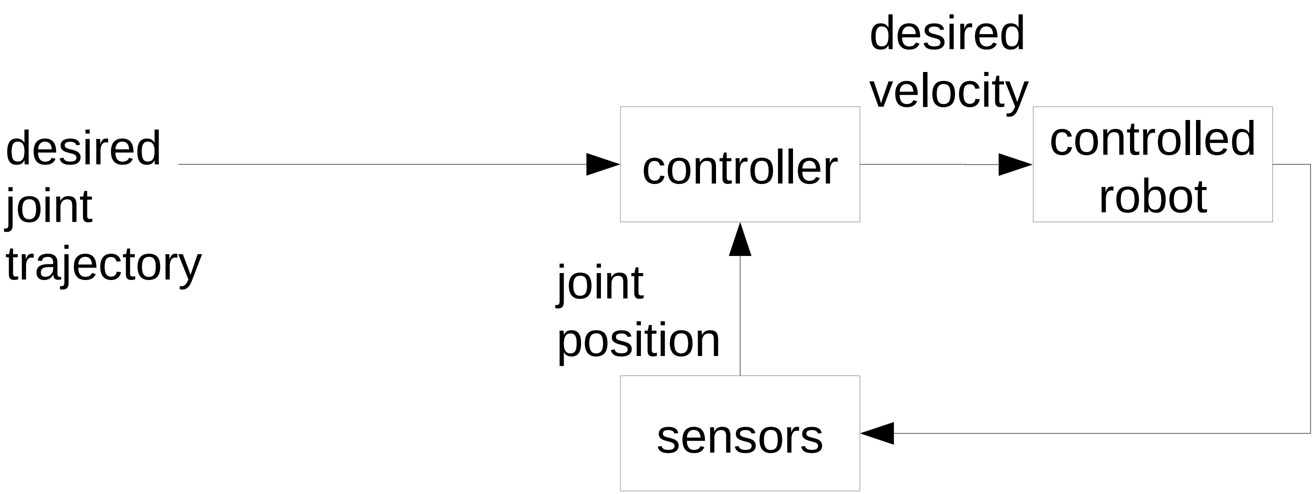


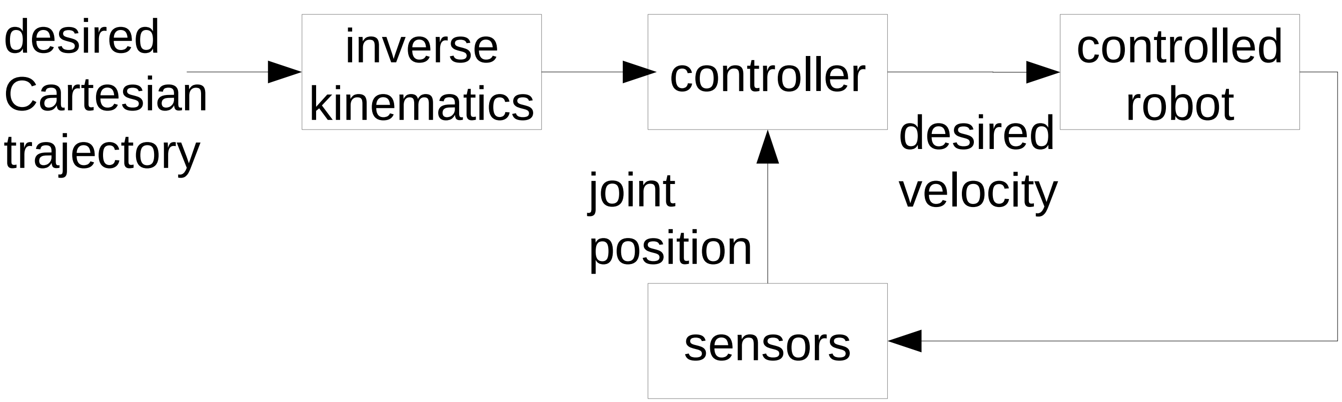
Joint velocity control



Joint velocity control

Assume internal controller tracks velocity accurately.

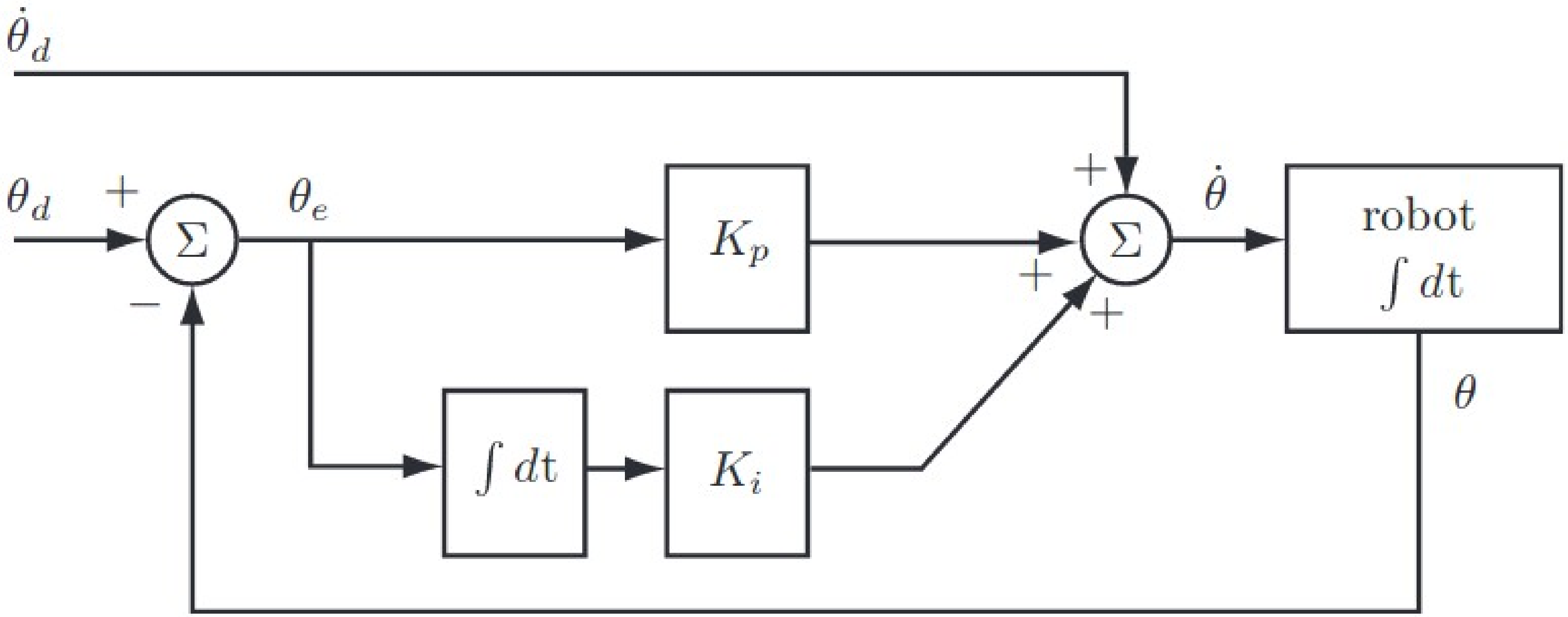




These may be the case with an industrial robot.

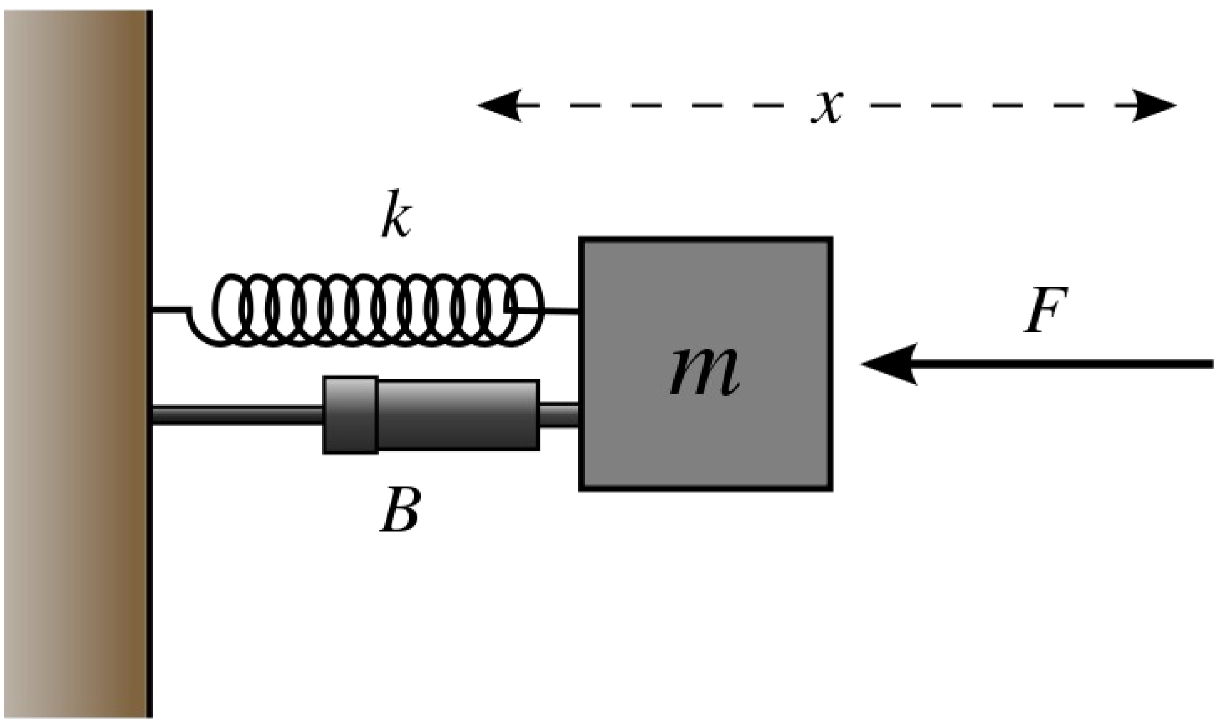
Give an example controller!

Practical controller: PI with feedforward



What kind of dynamics does this system have?

How does the system below behave?



Cartesian space control

$$\begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} R^{\mathrm{T}}(t)R_d(t) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_d(t) \\ \dot{p}_d(t) \end{bmatrix} + K_p X_e(t) + K_i \int_0^t X_e(\mathrm{t})\, dt,$$

pseudoinverse

$$\dot{\theta} = J_{ee}^{+} \begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix}$$

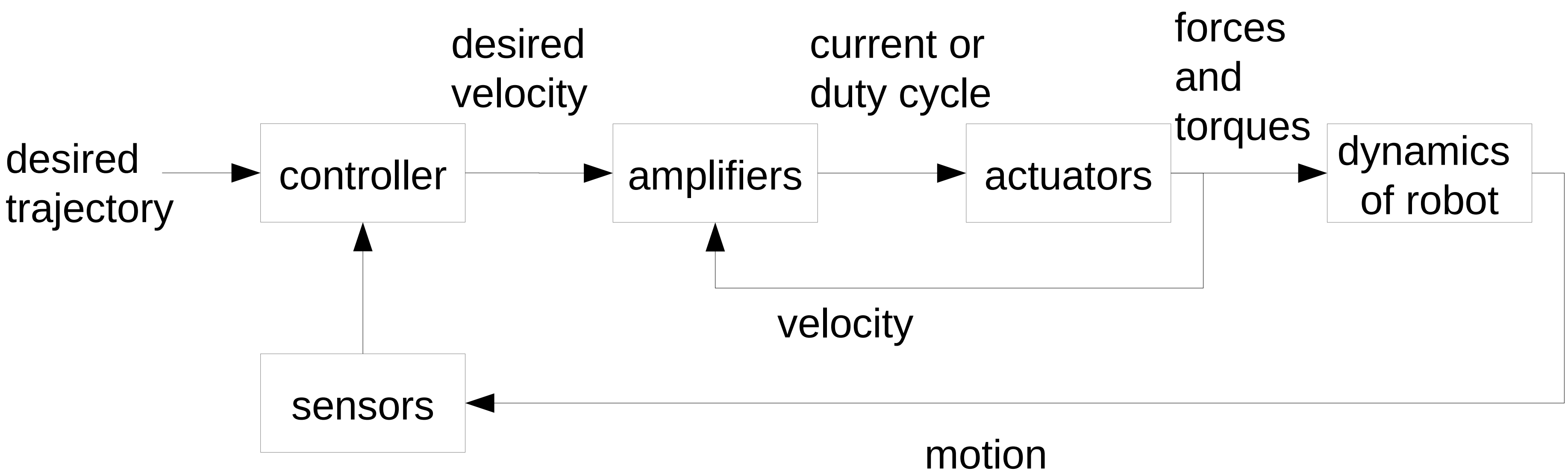
end-effector Jacobian

$$X_e(t) = \begin{bmatrix} \log(R^{\mathrm{T}}(d)R_d(t)) \\ p_d(t) - p(t) \end{bmatrix}$$

How does the Cartesian trajectory look like?

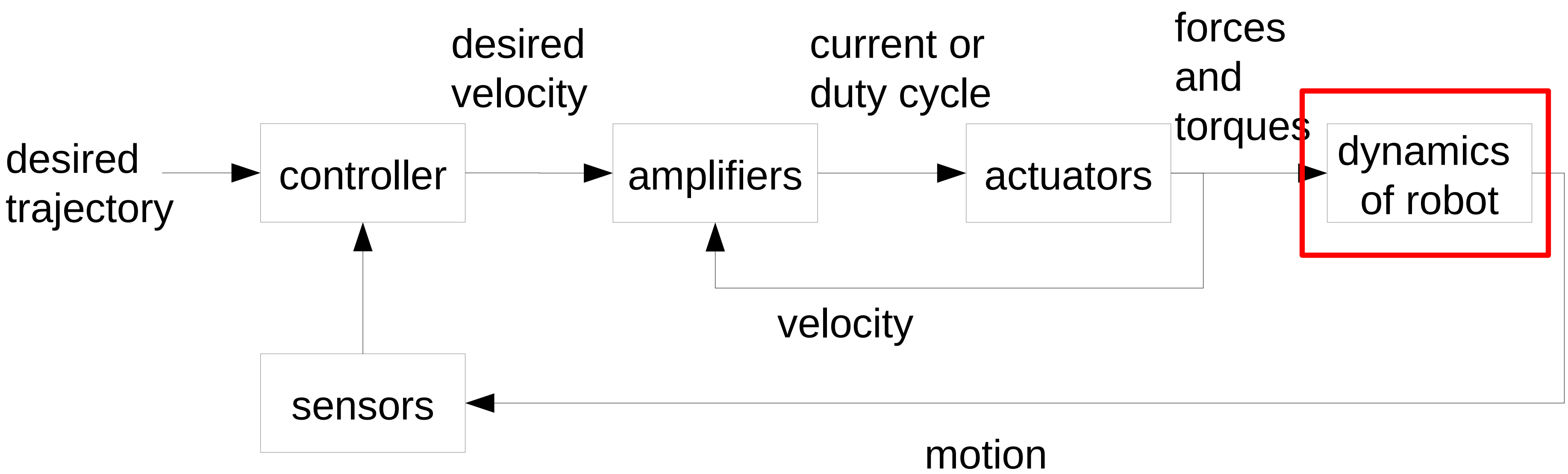
Toward torque control

Can this model control force interactions?



Toward torque control

Can this model control force interactions?



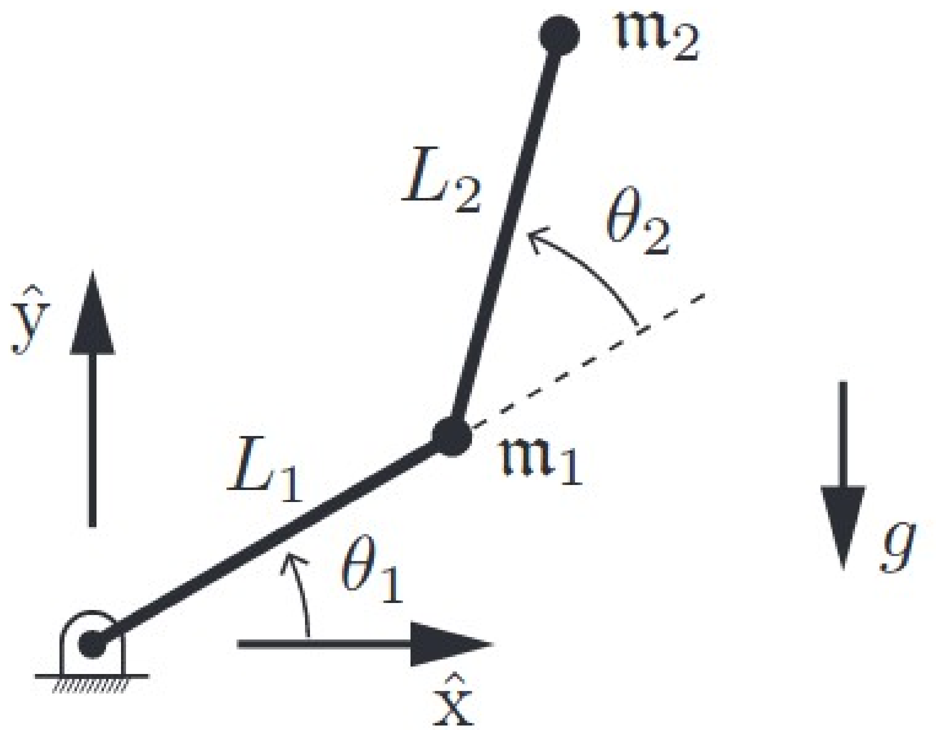
Dynamics

- Represent response to motor/joint torques
- Equation of motion

$$\underbrace{\boldsymbol{\tau}}_{\text{torques}} = \underbrace{M(\theta)}_{\text{mass matrix}}\ddot{\boldsymbol{\theta}} + \underbrace{h(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})}_{\text{gravity, friction, centripetal, Coriolis}}$$

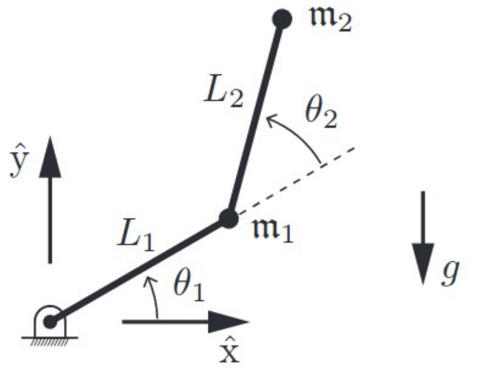
$$\boldsymbol{F} = m\,\boldsymbol{a} + m\,\boldsymbol{g} = m\,\ddot{\mathbf{x}} + m\,\boldsymbol{g}$$

Example: 2R robot under gravity



$$\boldsymbol{\tau}=M(\theta)\ddot{\boldsymbol{\theta}}+\boldsymbol{c}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})+\boldsymbol{g}(\boldsymbol{\theta})$$

Example: 2R robot



$$\boldsymbol{\tau} = M(\theta)\ddot{\boldsymbol{\theta}} + \boldsymbol{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta})$$

$$M(\boldsymbol{\theta}) = \begin{bmatrix} m_1 L_1^2 + m_2(L_1^2 + 2L_1 L_2 \cos\theta_2 + L_2^2) & m_2(L_1 L_2 \cos\theta_2 + L_2^2) \\ m_2(L_1 L_2 \cos\theta_2 + L_2^2) & m_2 L_2^2 \end{bmatrix}$$

$$\boldsymbol{c}(\dot{\boldsymbol{\theta}}, \boldsymbol{\theta}) = \begin{bmatrix} -m_2 L_1 L_2 \sin\theta_2(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2) \\ m_2 L_1 L_2 \dot{\theta}_1^2 \sin\theta_2 \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{\theta}) = \begin{bmatrix} (m_1 + m_2) g L_1 \cos\theta_1 + m_2 g L_2 \cos(\theta_1 + \theta_2) \\ m_2 g L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

What do the terms mean?

Forward dynamics with contact force

What's this? And why?

$$\boldsymbol{\tau} = M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + J^T(\theta)\boldsymbol{F}_{tip}$$

contact force

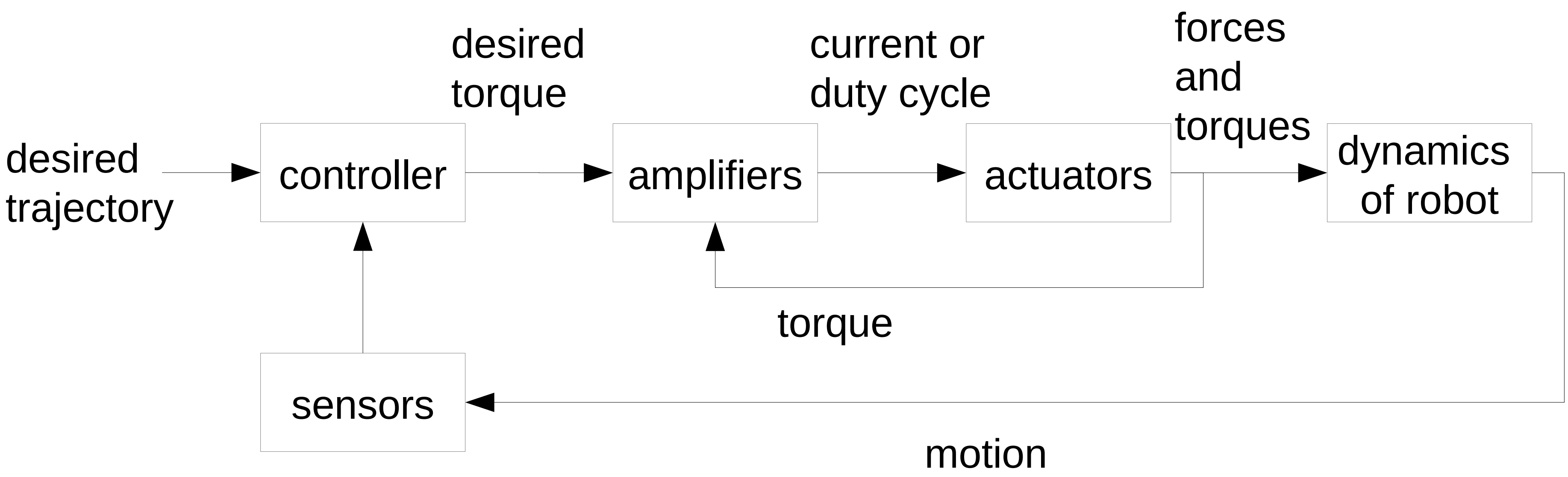
Solve:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \boldsymbol{\tau} - \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) - J^T(\theta)\boldsymbol{F}_{tip}$$

linear system of equations

Torque control

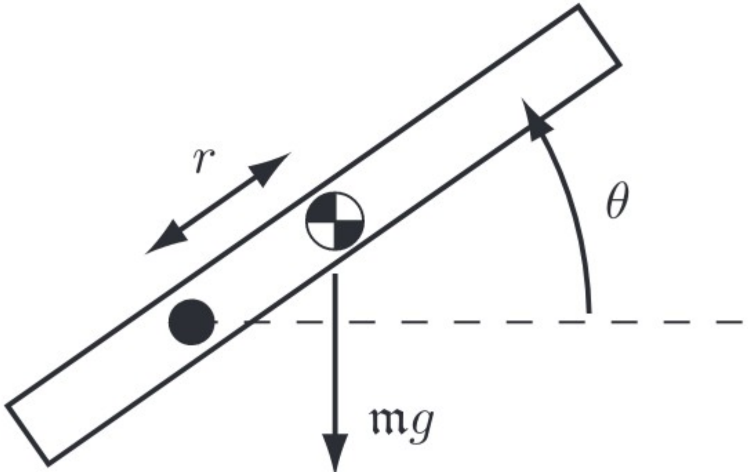
Can this model control force interactions?



Single joint torque control

- Dynamics with (simple) friction

$$\tau = M\ddot{\theta} + mgr\cos\theta + b\dot{\theta}$$

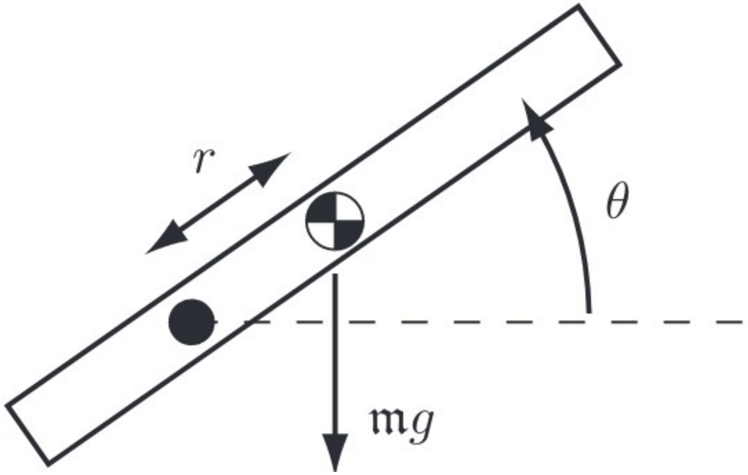


Assume you have a trajectory to follow.
Propose a controller. Or several.

Single joint torque control

- Dynamics with (simple) friction

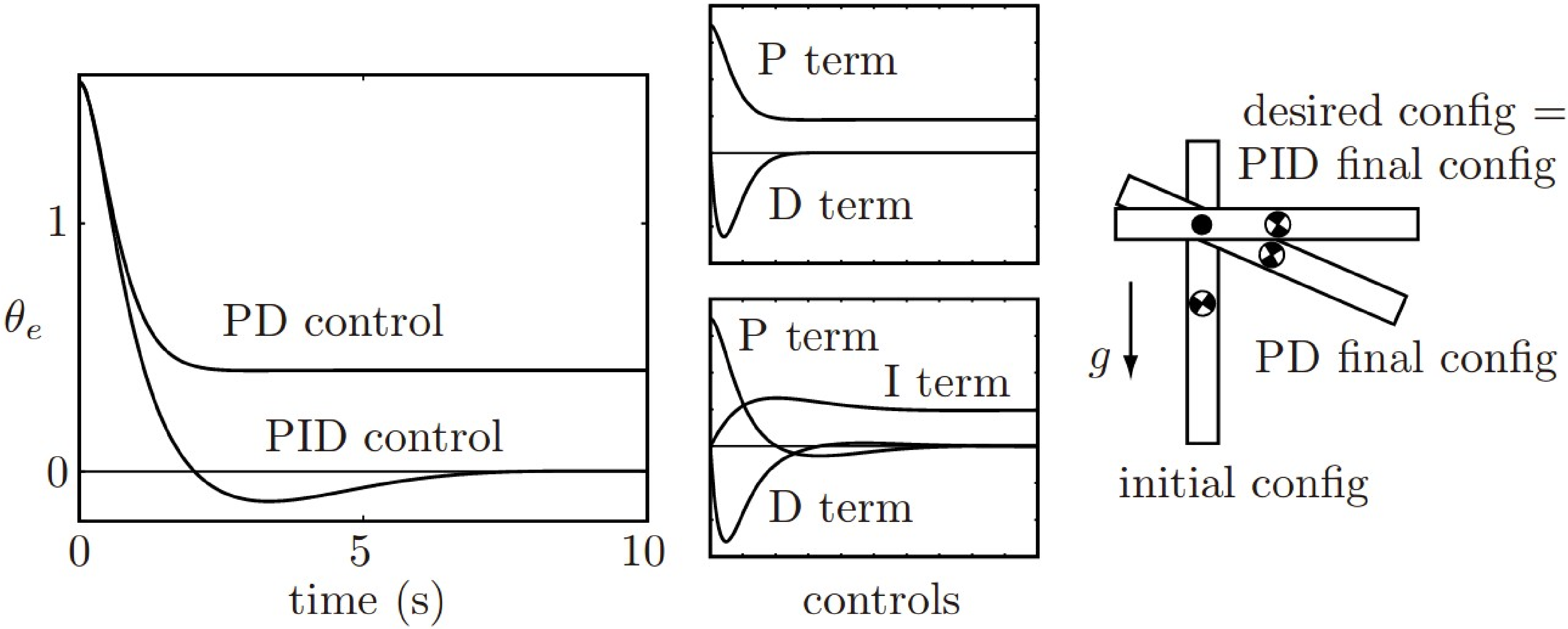
$$\tau = M\ddot{\theta} + m\,g\,r\cos\theta + b\,\dot{\theta}$$



Assume you have a trajectory to follow.
Propose a controller.

Does your controller converge to zero error if desired state is constant?

PID convergence



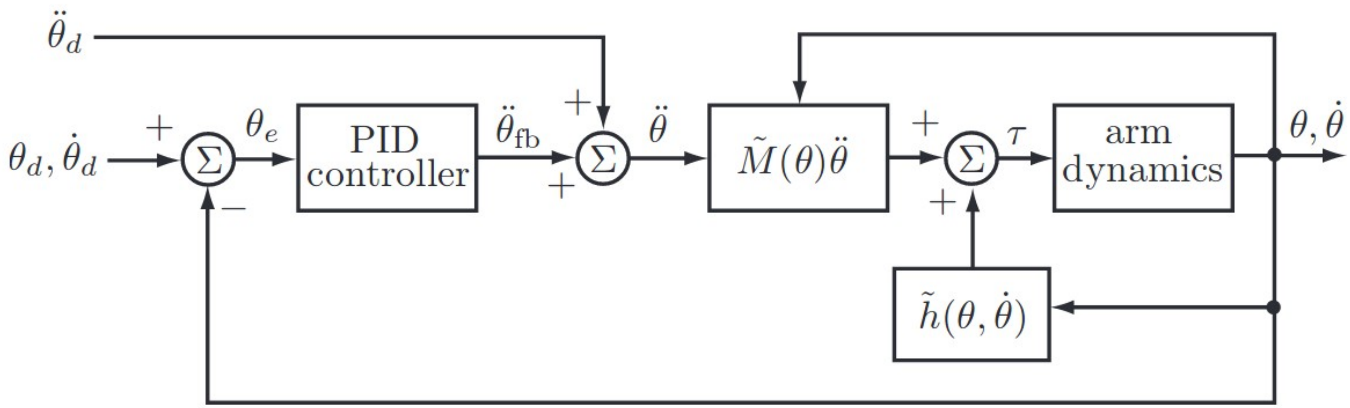
Would PID eliminate steady state error if desired rotational velocity is constant?

Inverse dynamics / computed torque control

dynamics compensation

$$\tau = \hat{M}(\theta)\Big(\ddot{\theta}_d + \underbrace{K_p\,\theta_e + K_i \int \theta_e + K_d\,\dot{\theta}_e}_{\text{PID feedback}}\Big) + \hat{h}(\theta, \dot{\theta})$$

feedforward



Inverse dynamics

- Problem: Calculate right hand side of

$$\boldsymbol{\tau} = M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

- Informally: If I want to follow a certain trajectory, how high torques do I need to apply at joints.

- Solution: Calculate M and **h** by Newton-Euler algorithm.

Cartesian space dynamics

- If Jacobian is invertible, dynamics can be expressed in Cartesian space as

$$F = M_C(\boldsymbol{\theta})\ddot{\mathbf{x}} + \boldsymbol{h}_C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$$

Cartesian force

Cartesian acceleration

Cartesian dynamics parameters can be calculated using joint space dynamics + Jacobian. E.g.

$$M_C(\boldsymbol{\theta}) = J^{-T} M(\boldsymbol{\theta}) J^{-1}$$

- Furthermore, if inverse kinematics is unique, dynamics can be expressed in Cartesian space as

$$F = M_C(\boldsymbol{x})\ddot{\mathbf{x}} + \boldsymbol{h}_C(\boldsymbol{x}, \dot{\boldsymbol{x}})$$

Compare to joint space!

Cartesian control

$$\boldsymbol{F} = M_C(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{h}_C(\boldsymbol{x}, \dot{\boldsymbol{x}})$$

- Inverse dynamics controller can then be written also in Cartesian space.

$$\boldsymbol{\tau} = J^T(\boldsymbol{\theta})\underbrace{\left(M_C(\boldsymbol{x})\left(\ddot{\boldsymbol{x}}_{\boldsymbol{d}} + K_p \boldsymbol{x}_{\boldsymbol{e}} + K_i \int \boldsymbol{x}_{\boldsymbol{e}} + K_d \dot{\boldsymbol{x}}_{\boldsymbol{e}}\right) + \boldsymbol{h}_{\boldsymbol{C}}(\boldsymbol{x}, \dot{\boldsymbol{x}})\right)}_{\text{Cartesian force}}$$

Compare to

$$\tau = \hat{M}(\theta)(\ddot{\theta}_d + K_p \theta_e + K_i \int \theta_e + K_d \dot{\theta}_e) + \hat{h}(\theta, \dot{\theta})$$

Represents forces in Cartesian space.
May be useful when looking at force control.

Summary

- Accurate motion control requires knowledge (model) of robot dynamics.

- Good recipe: inverse dynamics + PID + feedforward (computed torque control).

Next time: Control with external forces

- Readings:
  - Lynch & Park, Chapter 11.5-11.6