GIS-E3010 Least-Squares Methods in Geoscience
Lecture 7, activation 1

Derive a minimum norm inner constraint matrix *E*, when we use a 3D transformation. If we later add inner constraints $E^T\ddot{\Delta}=0$ to the normal equation, we can remove the datum defect. The equation of 3D transformation (image observation are invariants to this transformation) is

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} dt_X \\ dt_Y \\ dt_Z \end{bmatrix} + \begin{bmatrix} 0 & -d\chi & d\beta \\ d\chi & 0 & -d\alpha \\ -d\beta & d\alpha & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + d\lambda \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

In which t_X,t_Y and t_Z are translations

Tip: Write matrix-vector form open as a group of equations. Then build the correction vector $\ddot{\Delta}$ using corrections of unknown parameters:

$$\ddot{\Delta} = \begin{bmatrix} dt_X & dt_Y & dt_Z & d\alpha & d\beta & d\chi & d\lambda \end{bmatrix}^T$$

Then build matrix *E* as a coefficient matrix for this vector. You can think that you have to make partial derivation of functions with respect to all corrections of unknown parameters. After you have completed matrix *E* for one point, extend it to cover two points.

Solution:

E-matrix for one point

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} dt_X \\ dt_Y \\ dt_Z \end{bmatrix} + \begin{bmatrix} 0 & -d\chi & d\beta \\ d\chi & 0 & -d\alpha \\ -d\beta & d\alpha & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + d\lambda \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} =$$

$$\Leftrightarrow \begin{cases} dX = dt_X - d\chi Y + d\beta Z + d\lambda X \\ dY = dt_Y + d\chi X - d\alpha Z + d\lambda Y \\ dZ = dt_Z - d\beta X + d\alpha Y + d\lambda Z \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y & X \\ 0 & 1 & 0 & -Z & 0 & X & Y \\ 0 & 0 & 1 & Y & -X & 0 & Z \end{bmatrix} \begin{bmatrix} dt_X \\ dt_Y \\ dt_Z \\ d\alpha \\ d\beta \\ d\chi \\ d\lambda \end{bmatrix} = E\ddot{\Delta}$$

E-matrix for two points:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_1 & -Y_1 & X_1 \\ 0 & 1 & 0 & -Z_1 & 0 & X_1 & Y_1 \\ 0 & 0 & 1 & Y_1 & -X_1 & 0 & Z_1 \\ 1 & 0 & 0 & 0 & Z_2 & -Y_2 & X_2 \\ 0 & 1 & 0 & -Z_2 & 0 & X_2 & Y_2 \\ 0 & 0 & 1 & Y_2 & -X_2 & 0 & Z_2 \end{bmatrix}$$