KJR-C2003 Virtausmekaniikan perusteet, kaavakokoelma, VK1

Bernoulli

$$p + \frac{1}{2}\rho V^2 + \rho g z = \text{vakio virtaviivalla}$$

Taseyhtälöt

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial B_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V_n A b + \sum_{\text{out}} \rho V_n A b$$

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b d\cancel{V} + \int_A \rho b \vec{v} \cdot \vec{n} dA$$

$$\frac{\partial M_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V_n A + \sum_{\text{out}} \rho V_n A = 0$$

$$\frac{\partial (M\vec{v})_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V_n A \vec{v} + \sum_{\text{out}} \rho V_n A \vec{v} = \sum \vec{F}_{\text{cv}}$$

$$\frac{\partial (M\vec{r}\times\vec{v})_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V_n A (\vec{r}\times\vec{v}) + \sum_{\text{out}} \rho V_n A (\vec{r}\times\vec{v}) = \sum (\vec{r}\times\vec{F})_{\text{cv}}$$

$$\begin{aligned}\frac{\partial (Me)_{\text{cv}}}{\partial t} &+ \sum_{\text{out}} \left(\breve{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho A V_n \\ &- \sum_{\text{in}} \left(\breve{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho A V_n = \dot{Q}_{\text{net}} + \dot{W}_{\text{shaft}}\end{aligned}$$

$$\left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{\text{out}} = \left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{\text{in}} - \left(\breve{u}_{\text{out}} - \breve{u}_{\text{in}} - \frac{\dot{Q}_{\text{net}}}{\dot{m}}\right) + \frac{\dot{W}_{\text{shaft}}}{\dot{m}}$$

Pyörimisliike

$$(\vec{r}\times\vec{v})_z = r v_\theta, \quad (\dot{m} r v_\theta)_{\text{out}} - (\dot{m} r v_\theta)_{\text{in}} = T_{\text{shaft}}, \quad (\dot{m} r \omega v_\theta)_{\text{out}} - (\dot{m} r \omega v_\theta)_{\text{in}} = \dot{W}_{\text{shaft}}$$

Differentiaaliyhtälöt

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z$$

$$\sigma_{xx} = -p + 2\mu\frac{\partial u}{\partial x}, \ \sigma_{yy} = -p + 2\mu\frac{\partial v}{\partial y}, \ \sigma_{zz} = -p + 2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \ \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \ \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z$$