KJR-C2003 Virtausmekaniikan perusteet, kaavakokoelma, tentti

Bernoulli

$$p + \frac{1}{2}\rho V^2 + \rho g z = \text{vakio virtaviivalla}$$

Taseyhtälöt

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial B_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V A b + \sum_{\text{out}} \rho V A b$$

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b d\mkern-9mu V + \int_A \rho b \vec{v} \cdot \vec{n} dA$$

$$\frac{\partial M_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V A + \sum_{\text{out}} \rho V A = 0$$

$$\frac{\partial (M\vec{v})_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V A \vec{v} + \sum_{\text{out}} \rho V A \vec{v} = \sum \vec{F}_{\text{cv}}$$

$$\frac{\partial (M\vec{r} \times \vec{v})_{\text{cv}}}{\partial t} - \sum_{\text{in}} \rho V A (\vec{r} \times \vec{v}) + \sum_{\text{out}} \rho V A (\vec{r} \times \vec{v}) = \sum (\vec{r} \times \vec{F})_{\text{cv}}$$

$$\begin{aligned} \frac{\partial (Me)_{\text{cv}}}{\partial t} &+ \sum_{\text{out}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho A V \\ &- \sum_{\text{in}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho A V = \dot{Q}_{\text{net}} + \dot{W}_{\text{shaft}} \end{aligned}$$

$$\left(\frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{out}} = \left(\frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{in}} - \left(\check{u}_{\text{out}} - \check{u}_{\text{in}} - \frac{\dot{Q}_{\text{net}}}{\dot{m}} \right) + \frac{\dot{W}_{\text{shaft}}}{\dot{m}}$$

Pyörimisliike

$$(\vec{r} \times \vec{v})_z = r v_\theta, \quad (\dot{m} r v_\theta)_{\text{out}} - (\dot{m} r v_\theta)_{\text{in}} = T_{\text{shaft}}, \quad (\dot{m} r \omega v_\theta)_{\text{out}} - (\dot{m} r \omega v_\theta)_{\text{in}} = \dot{W}_{\text{shaft}}$$

Differentiaaliyhtälöt

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right) = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho g_x$$

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho g_y$$

$$\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z$$

$$\sigma_{xx} = -p + 2\mu\frac{\partial u}{\partial x}, \; \sigma_{yy} = -p + 2\mu\frac{\partial v}{\partial y}, \; \sigma_{zz} = -p + 2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \; \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \; \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y$$

$$\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z$$

Potentiaali- ja virtafunktio

$$u = \frac{\partial \phi}{\partial x},\ v = \frac{\partial \phi}{\partial y},\ w = \frac{\partial \phi}{\partial z}, \qquad v_r = \frac{\partial \phi}{\partial r},\ v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta},\ v_z = \frac{\partial \phi}{\partial z}$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}, \qquad v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta},\ v_\theta = -\frac{\partial \psi}{\partial r}$$

$$\phi = Ux = Ur\cos\theta, \quad \psi = Uy = Ur\sin\theta, \quad u = U, \quad v = 0, \quad \text{(homogeenivirtaus)}$$

$$\phi = \frac{m}{2\pi}\ln r, \quad \psi = \frac{m}{2\pi}\theta, \quad v_r = \frac{m}{2\pi r}, \quad v_\theta = 0 \quad \text{(lähde/nielu)}$$

$$\phi = \frac{\Gamma}{2\pi}\theta, \quad \psi = -\frac{\Gamma}{2\pi}\ln r, \quad v_r = 0, \quad v_\theta = \frac{\Gamma}{2\pi r} \quad \text{(pyörre)}$$

$$\phi = \frac{K\cos\theta}{r}, \quad \psi = -\frac{K\sin\theta}{r}, \quad v_r = -\frac{K\cos\theta}{r^2}, \quad v_\theta = -\frac{K\sin\theta}{r^2} \quad \text{(dipoli)}$$

Dimensioanalyysi

$$u_1 = f(u_2, u_3, \ldots, u_k), \qquad \Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

Putkivirtaukset

$$h_\mathrm{L} = \sum f \frac{\ell}{D}\frac{V^2}{2g} + \sum K_\mathrm{L}\frac{V^2}{2g}$$

Rajakerros

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy, \qquad \theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy, \qquad \tau_\mathrm{w} = \rho U^2 \frac{d\theta}{dx}$$

Pumput ja turbiinit

$$C_\mathrm{H} = \frac{g h_\mathrm{p}}{\omega^2 D^2} = \phi_1\left(\frac{Q}{\omega D^3}\right), \qquad C_\mathscr{P} = \frac{\dot{W}_\mathrm{shaft}}{\rho \omega^3 D^5} = \phi_2\left(\frac{Q}{\omega D^3}\right), \qquad \eta = \frac{\rho g Q h_\mathrm{p}}{\dot{W}_\mathrm{shaft}} = \phi_3\left(\frac{Q}{\omega D^3}\right)$$