

Exercise problems of topic 3: Analysis of microwave circuits

Write your answers clearly, so that the answer proceeds logically and includes necessary intermediate steps and sufficient explanations. Your answer should be understandable without oral explanations, too. See further instructions for systematic problems solving in MyCourses.

The exercise problem answers are to be returned during the contact sessions to the course teachers either handwritten (on paper) or typescripted (shown on screen). For other return methods, contact the teachers.

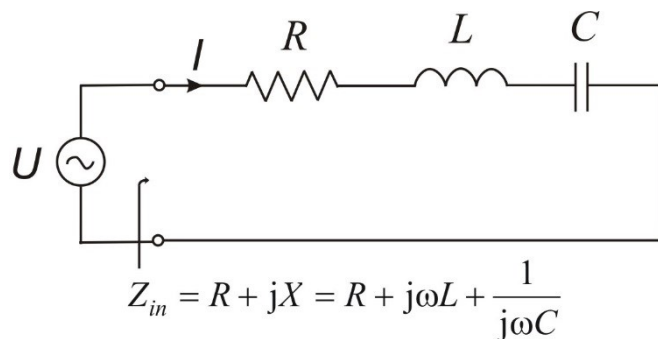
Return your answers one by one when a teacher is free. You may also ask help and instruction.

Be prepared to explain and justify your answer to the teacher. The purpose of this returning method is to enhance your learning through two-way communication and constructive feedback given by the teacher. The teacher will grade your answer in the scale of 0-3 points.

Note that at least two (2) of the problems must be returned latest on Thu 14 February and two (2) more latest on Thu 21 February. If you cannot meet this, you lose a chance to earn those points. However, if you have a good reason not to meet the DL, contact the teachers well in advance. The optimal return speed is about three (3) returned problems per week ☺

Exercise problem 3.1. In this problem you will learn the *electrical resonator theory*. Solve and answer the following small problems.

The figure shows a lumped-element series RLC resonator circuit, which is the simplest form of an electrical resonator. Hence, it is perfect for learning the basics of electrical resonators.



- a. Let us define $U = 1$ V. Plot (e.g., with Matlab or GNU Octave) the magnitude of the electrical current $I = U/Z_{in}$ as a function of angular frequency $\omega = 0.5 \dots 1.5$ rad/s in three different cases:

- 1) $L = 1$ H, $C = 1$ F, $R = 2$ Ω
- 2) $L = 10$ H, $C = 1/10$ F, $R = 1$ Ω
- 3) $L = 100$ H, $C = 1/100$ F, $R = 0.5$ Ω .

The problem continues on the next page!

- b. Based on the three current curves of part a., what particular you notice that happens at the angular frequency $\omega = 1$? How is this frequency is called (let us mark it ω_0)? Derive the expression of ω_0 as a function of L and C .
- c. Read from the literature of your choice, what the (unloaded) *quality factor*

$$Q = \text{angular resonant frequency} \cdot \frac{\text{energy stored}}{\text{energy lost per time unit}} = \omega_0 \frac{W}{P_l}$$

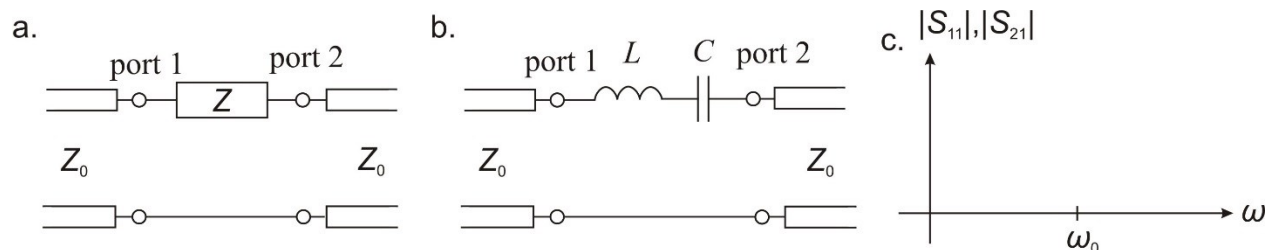
means. Starting from the above definition of the quality factor, show that the unloaded quality factor of the series RCL circuit can calculated from the expression

$$Q_0 = \frac{\omega_0 L}{R}.$$

- d. Calculate the unloaded quality factor Q_0 in each case 1)-3) of part b. Based on the observations of the curves of a., describe how the numeric value of the unloaded quality factor affects the bandwidth of the curves.

Exercise problem 3.2. Let us study, how one can define scattering parameters also with a pencil and a paper.

- a. Derive a general (2 x 2) scattering parameter matrix of a series impedance Z (see Figure a.) when both ports are terminated with the reference impedance Z_0 . Let us assume that Z consists of reciprocal and lumped components.
- b. The impedance Z is a series LC circuit (Figure b.), the frequency variable is the angular frequency ω . Write the S-parameter matrix with the help of a.
- c. At which angular frequency ω_0 the circuit of b. is fully matched ($|S_{11}| = 0$) to the reference impedance Z_0 ? Roughly sketch the shape of the frequency responses of $|S_{11}|$ and $|S_{21}|$. You can, for example, consider $|S_{11}|$ and $|S_{21}|$ at the angular frequencies $\omega = 0$, ω_0 and ∞ .



Exercise problem 3.3. Solve and answer the following problems.

The impedance behavior of a real surface-mounted lumped capacitor is modelled as a series RLC equivalent circuit, see the figure below. The datasheet of a capacitor gives the following information:

- The nominal capacitance of the capacitor is $C = 3.0 \text{ pF}$,
- The resistor R models the resistive losses,
- The inductor L is the parasitic inductance,
- The self-resonant frequency of the capacitor is $f_r = 2.46 \text{ GHz}$ (in AWR series-resonant FR)
- The unloaded quality factor Q_o (defined at the resonant frequency f_r) is 43.

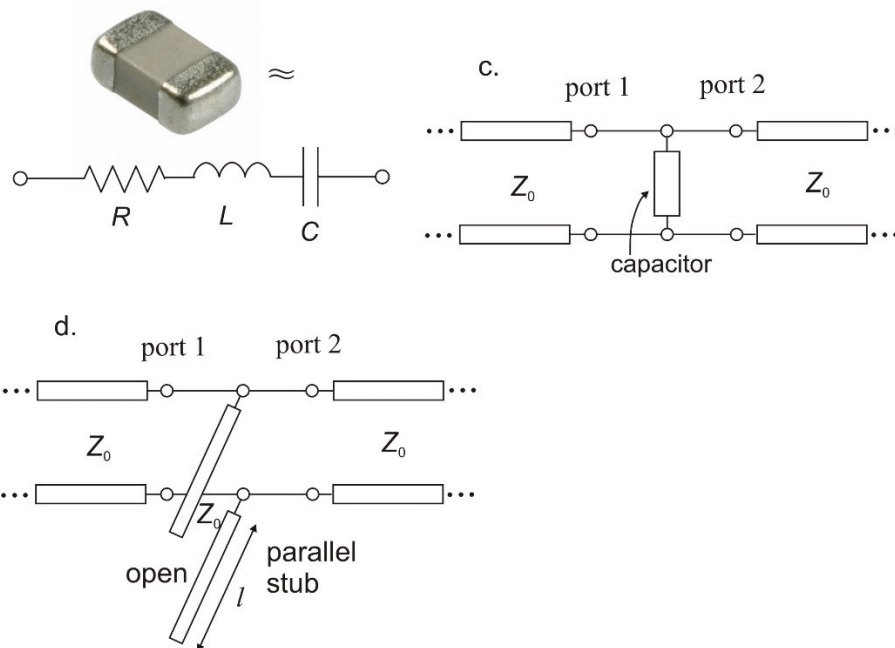
- Solve the component values L and R of the equivalent circuit of the capacitor.
- At which frequencies is the capacitor capacitive? When it is inductive? What do you learn from this problem?
- The capacitor is used as a filter to block a strong Wi-Fi signal at 2.4-2.5 GHz, see figure c. Hence, the capacitor is soldered between the lines of a transmission line ($Z_o = 50 \Omega$). At which frequency does the capacitor attenuate the most and what is the attenuation in decibels?

Hint: consider first, under which conditions this circuit might work as a band-stop filter.

- As an alternative approach, the band-stop filter might be implemented with an open single parallel stub (see figure d.). What would be the length l (in λ) of the open stub?

Hint: see also Problem 2.1 where the impedance of an open transmission line was derived.

- Simulate the filter of part c. with AWR. Use the component called “CHIPCAP” in AWR and give the parameters C , f_r and Q_o as defined in this problem. Do you get the same result as in c. part? If not, try to explain why.

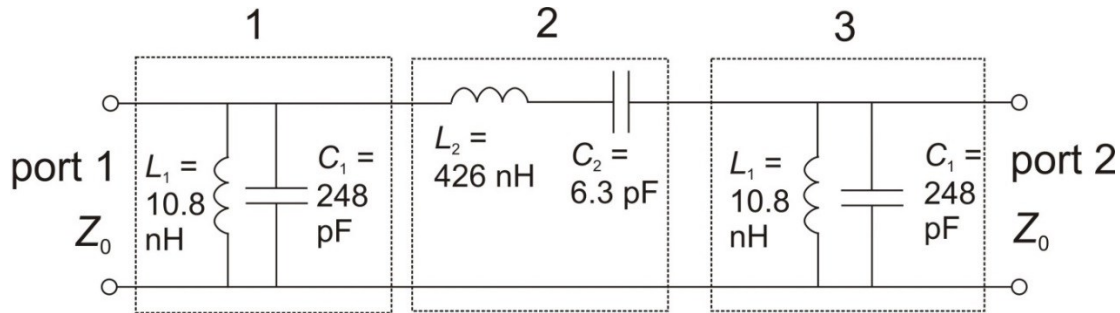


Exercise problem 3.4. Solve and answer the following problems.

- a. Let us make a circuit simulator for the band-pass filter shown below. Plot $|S_{11}|$ and $|S_{21}|$ [dB] as a function of frequency in the frequency range of 80 - 120 MHz to the same Cartesian coordinate system.

Hint: Use the ABCD matrices, see Pozar book Chapter 4.4. Calculate the total ABCD matrix of the whole filter and it to the S-parameter matrix. The frequency response is triple-resonant.

- b. Calculate the resonant frequency of each resonator blocks 1-3. Compare them with the resonant frequencies of the frequency response of $|S_{11}|$ plotted in a. part. What interesting (also surprising?) findings you can make?
- c. Optional part: if you like/want, you can simulate the filter with AWR and compare with the a. part result.



Exercise problem 3.5. The output power of a matched generator is 5 dBm. That power is split into two ports with a resistive power splitter whose S-parameter ($Z_0 = 50 \Omega$) matrix (see Pozar Chapter 7.2) is

$$S = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

- a. Both output ports of the power splitter are terminated with identical dipole antennas, whose radiation efficiency is 0.79 and input impedance $Z_L = 70 \Omega$. Assume that the isolation between the antennas is very high. Calculate the power radiated (dBm) by one antenna? How about the whole system (dBm)?

Hint: a. part can be solved with the help of the matrix equation: $[b_n] = [S] \cdot [a_n]$, in which the antenna reflection coefficient ρ_L is written in terms of a_2 and b_2 .)

- b. Model and simulate the circuit system with AWR. Find the circuit schematic of the power splitter in Pozar Chapter 7.2. The antenna impedance (resistance) can be modelled as the loss R_{loss} and radiation resistance R_{rad} in series.

Exercise problem 3.6. Solve and answer the following problem.

An amplifier consists of two 90 degrees hybrids and two transistors. The hybrids are modelled with the S-parameter matrix shown below. For the transistors, S_{11a} and S_{11b} denote the input reflection coefficient and S_{21a} and S_{21b} the voltage transmission (amplification). All the S-parameters are defined with respect to $Z_0 = 50 \Omega$. Assume that the transistors are unilateral.

- a. Define the expressions for the input reflection coefficient ρ_{in} (function of S_{11a} and S_{11b}) and the power gain G (function of S_{21a} and S_{21b}) of the amplifier.

$$\rho_{in}(S_{11a}, S_{11b}) = \frac{b_1}{a_1}, \quad G(S_{21a}, S_{21b}) = \left| \frac{b_7}{a_1} \right|^2$$

- b. How do the expressions for ρ_{in} and G change if the transistors are identical – i.e., the transistor S-parameters satisfy $S_{11a} = S_{11b}$ and $S_{21a} = S_{21b}$?

