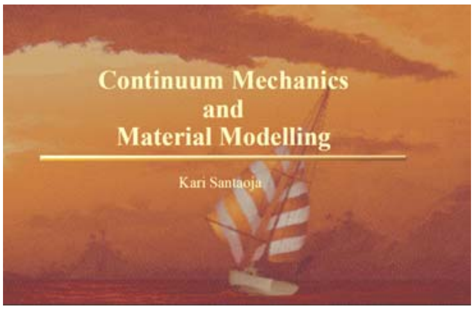
Continuum Mechanics and Material Modelling 2019
MEC-E8002 P (5 cr)

Letter to the students; January 28 / February 20, 2019

The following comments are based on the observations I have made during the years I have lectured the predecessors of this course and the calculation hour we had on Fridays.



Please, avoid using notations which are not tensor notations but refer to other fields of mathematics. Notation $[\cdot]$ stands for matrix not for a tensor. Notation 1 is not a second-order identity tensor but it is a scalar "one". Changes after January 28 in red.

Notation

$$\frac{\delta}{\delta x} \tag{1}$$

is not a partial derivative operator. The partial derivative operator reads

$$\frac{\partial}{\partial x}\,. \tag{2}$$

Boldface letters are difficult to write by hand. Therefore, when writing by hand, I use the following notations:

$$\text{second-order tensor} \quad \underline{1} \qquad \text{and} \qquad \text{fourth-order tensor} \quad \underline{\underline{I}}\,. \tag{3}$$

The idea of the tensor notations is that the mathematical derivation is carried out without substitution of the numbers 1,2,3 for the values of the indices. Thus, do not write

$$\vec{\nabla}(\vec{x}) = \vec{i}_1 \frac{\partial}{\partial x_1} + \vec{i}_2 \frac{\partial}{\partial x_2} + \vec{i}_3 \frac{\partial}{\partial x_3}\,. \tag{4}$$

Values for the indices are sometimes substituted when the obtained results are interpreted.

Please, remember that Einstein summation convention does not utilise summation symbol Σ. Matrices do not belong to tensor algebra.

There are some typos in my lecture notes. Usually they are fully harmless but some of them require corrections. Thus, at least the following errors have to be taken into account.

(1) On page 43 I wrote: The second term on the right side of Derivative (64) takes... In Derivative (65) such a "second term" does not exist, since ... I should have written as follows: The second terms on the right side of Derivatives (66) and (67) take ... In Derivative (68) such a "second term" does not exist ...

(2) On page 56 I wrote: Usually authors write $\vec{t}(\vec{x},t)$ instead of writing $\vec{t}(\vec{x},t,\vec{n})$. This does not cause any problems since the surface traction vector $\vec{t}$ is used for the definition of for the stress tensor $\boldsymbol{\sigma}$ which includes the unit normal vector $\vec{n}$ [see Definition (17)]. Thus, the same practice is followed here. This was a mistake. One should always write

$$\vec{t}(\vec{x},t,\vec{n}) \qquad \text{or} \qquad \vec{T}(\vec{X},t,\vec{N})\,.$$

(3) Equation $(3.29)_1$ should be

$$\vec{H} := \int_m (\vec{r} \times \vec{v})\ \mathrm{d}m$$

(3.5) In Equation (3.31) the integration differentials $\mathrm{d}A$ and $\mathrm{d}V$ should be replaced with $\mathrm{d}a$ and $\mathrm{d}v$.

(4) On page 118 in Figure 1 and in the adjacent text I tell what is the positive direction of the heat flux vector $\vec{q}$. It is incorrect. The heat flux vector $\vec{q}$ does not have a positive direction.

(5) The following does not belong to the material which is for the examination. Equations (3.40) and (3.42) should read

$$\frac{\mathrm{D}}{\mathrm{D}t} \int_{v(t)} \rho(\vec{x},t)\ \mathrm{d}v \;=\; \int_{v(t)} \left[\frac{\mathrm{D}\rho(\vec{x},t)}{\mathrm{D}t} + \rho(\vec{x},t)\ \vec{\nabla}(\vec{x})\cdot\vec{v}(\vec{x},t) \right] \mathrm{d}v\,.$$

and

$$\int_{v(t)} \left[\frac{\mathrm{D}\rho(\vec{x},t)}{\mathrm{D}t} + \rho(\vec{x},t)\ \vec{\nabla}(\vec{x})\cdot\vec{v}(\vec{x},t) \right] \mathrm{d}v \;=\; 0\,.$$

(6) Equation (11.15) on page 138 has one extra double-dot operator. It has to be cancelled. Equation (11.15) reads

$$\begin{aligned} \rho_0\, \dot{u}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathrm{i}},\boldsymbol{\alpha},s,h(\vec{X})) \;=\; & \rho_0\, \frac{\partial u(\ldots)}{\partial(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathrm{i}})} : (\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}) : \;+\; \rho_0\, \frac{\partial u(\ldots)}{\partial \boldsymbol{\alpha}} : \dot{\boldsymbol{\alpha}} \;+\; \rho_0\, \frac{\partial u(\ldots)}{\partial s}\, \dot{s} \\ & +\; \rho_0\, \frac{\partial u(\ldots)}{\partial h}\, \dot{h}\,. \end{aligned} \tag{15}$$

(6.5) The text starting the page 142 should read: Substitution of Derivatives (7) and (8) ...

(6.7) The second term in Equation (12.9) should be $-\,\boldsymbol{\beta}:\dot{\boldsymbol{\alpha}}$. Thus, there should be minus sign instead of plus sign.

(7) Equation (13.10) on page 146 should be

$$\Phi \;=\; \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}} \;-\; \boldsymbol{\sigma}:(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}) \;+\; \boldsymbol{\beta}:\dot{\boldsymbol{\alpha}} \;-\; \rho_0\, T\, \dot{s} \;+\; \rho_0\, T\, \dot{s} \;-\; \frac{\vec{\nabla} T}{T}\cdot\vec{q} \qquad (\,\geq\, 0\,)\,,$$

(8) When you apply tensor algebra, please be sure that you show all the steps needed for derivation. Furthermore, refer to the expressions given in Chapter 2 or in the appendices.

(9) After the today's lecture Fabio Bitetti (2018) told me that on pages 120 and 121 in Equations (8.22)...(8.25) there should be a quantity $\vec{q}^{\,\square}$ instead of $\vec{q}$. He is right. In the next version of the book the is a quantity $\vec{q}^{\,\square}$ called Piola-Kirchhoff type of heat flux vector. I was not bold enough to have it in the present version of the book, since I did not find it in any other book. Equation (27) is correct.

(9.2) In the second term on the right side of Equation (19.31) there is b/a. There should be b/c.

(9.5) After Equation (20.3) I wrote φ_{con}. I should have written φ_{ther}. The notation φ_{con} (conduction ?) is due to Truessdell, Toupin and Noll, but should be replaced by a more informative φ_{ther}.

(10) Equation $(20.5)_1$ should be

$$-\frac{\vec{\nabla} T}{T} \;=\; \mu_{\mathrm{ther}}\, \rho_0\, \frac{\partial}{\partial \vec{q}} \left(\frac{\vec{q}\cdot\vec{q}}{2\, \mu_{\mathrm{ther}}\, \rho_0\, \gamma(T)\, T} \right) \;=\; \frac{1}{\gamma(T)\, T}\, \frac{\partial}{\partial \vec{q}} \left(\tfrac{1}{2}\, \vec{q}\cdot\vec{q} \right). \tag{5}$$

The first quantity on the right side should be μ_{ther} instead of $1/2$.

(11) On page 202 in Equations (21.10) and (21.11) there should by the terms

$$\frac{1}{2\,\mu}\cdots$$

(12) On page 203 after Equation (20.17) one should write “It can be shown that Properties (16) and (17) ...”

Best Regards,

Kari Santaoja

Continuum Mechanics letter to the students 2019 01 28.wpd/Santaoja