

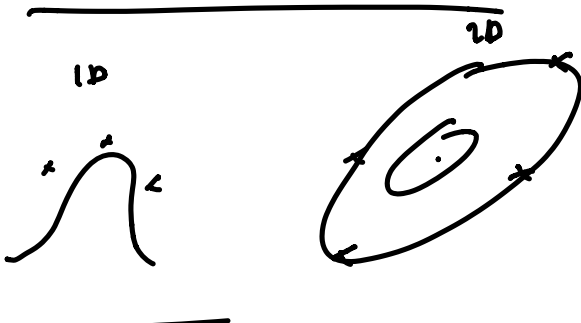
$$\begin{aligned}
 x^{(1)} &= \mu \\
 x^{(2)} &= \mu + 2 \\
 x^{(3)} &= \mu - 2
 \end{aligned}$$

$$\begin{aligned}
 \sum_i w^{(i)} x^{(i)} &= w^{(1)} \mu + w^{(2)} (\mu + 2) + w^{(3)} (\mu - 2) \\
 &= \mu
 \end{aligned}$$

$$\begin{aligned}
 \sum w^{(i)} (x^{(i)} - \mu)^2 &= w^{(1)} \cdot 0 + w^{(2)} \cdot 2^2 + w^{(3)} \cdot 2^2 \\
 &= 2^2
 \end{aligned}$$

$$\begin{aligned}
 w^{(1)} + w^{(2)} &= 1 \\
 w^{(1)} + w^{(2)} + w^{(3)} &= 1 \\
 w^{(1)} - w^{(2)} &= 0
 \end{aligned}$$

$$\begin{aligned}
 w^{(1)} &= 0 \\
 w^{(2)} &= \frac{1}{2} \\
 w^{(3)} &= \frac{1}{2}
 \end{aligned}$$



$$\begin{aligned}
 \sigma^2 &= 2 \cdot 2 \\
 P &= LL^T
 \end{aligned}$$

$$x \sim \mathcal{N}(\mu, P)$$

$$y = g(x) + q, \quad q \sim \mathcal{N}(0, Q)$$

$$\begin{aligned} E \left[\begin{pmatrix} x \\ g(x) + q \end{pmatrix} \right] &= E \left[\begin{pmatrix} x \\ \underbrace{g(x)}_{\mu_y} \end{pmatrix} \right] \\ &\approx \sum_i w^{(i)} \begin{pmatrix} x^{(i)} \\ g(x^{(i)}) \end{pmatrix} \\ &= \begin{pmatrix} \mu \\ \sum_i w^{(i)} g(x^{(i)}) \end{pmatrix} = \begin{pmatrix} \mu \\ \mu_y \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{Cov} \left[\begin{pmatrix} x \\ g(x) + q \end{pmatrix} \right] &= E \left[\begin{pmatrix} x - \mu \\ g(x) + q - \mu_y \end{pmatrix} \begin{pmatrix} \cdot \end{pmatrix}^T \right] \\ &= E \left[\begin{pmatrix} x - \mu \\ g(x) - \mu_y \end{pmatrix} \begin{pmatrix} \cdot \end{pmatrix}^T \right] + \begin{pmatrix} 0 & 0 \\ 0 & Q \end{pmatrix} \\ &\approx E \left[\begin{pmatrix} (x - \mu)(x - \mu)^T & (x - \mu)(g - \mu_y)^T \\ (g - \mu_y)(x - \mu)^T & (g - \mu_y)(g - \mu_y)^T \end{pmatrix} \right] + \begin{matrix} \downarrow \\ Q \end{matrix} \\ &\approx \sum w^{(i)} \left[\begin{matrix} (x^{(i)} - \mu)(x^{(i)} - \mu)^T & (x^{(i)} - \mu)(g(x^{(i)}) - \mu_y)^T \\ \leftarrow \begin{matrix} (x^{(i)} - \mu) \\ (g(x^{(i)}) - \mu_y) \end{matrix} & (g(x^{(i)}) - \mu_y)(g(x^{(i)}) - \mu_y)^T \end{matrix} \right] \\ &= \begin{pmatrix} P & \sum w^{(i)} (x^{(i)} - \mu)(g(x^{(i)}) - \mu_y)^T \\ \leftarrow \begin{matrix} (x^{(i)} - \mu) \\ (g(x^{(i)}) - \mu_y) \end{matrix} & \sum w^{(i)} (g(x^{(i)}) - \mu_y)(g(x^{(i)}) - \mu_y)^T + Q \end{pmatrix} \end{aligned}$$