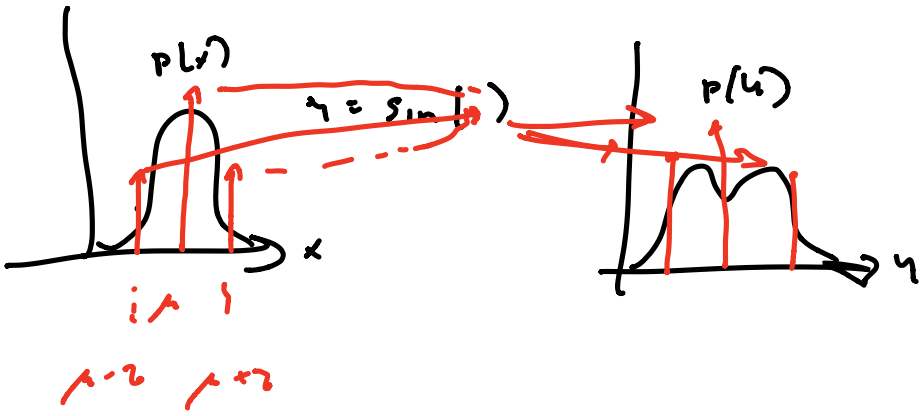

μ 1

$\mu-2$ $\mu+2$

$$x^{(0)} = \mu$$
$$x^{(1)} = \mu + 2$$
$$x^{(2)} = \mu - 2$$

$$\sum_i w^{(i)} x^{(i)} = w^{(0)}\mu + w^{(1)}(\mu+2) + w^{(2)}(\mu-2)$$
$$= \mu$$

$$\sum w^{(i)}(x^{(i)} - \mu)^2 = w^{(0)}\cdot 0 + w^{(1)}\cdot 2^2 + w^{(2)} 2^2$$
$$= 2^2$$

$$w^{(1)} + w^{(2)} = 1 \qquad w^{(0)} = 0$$
$$w^{(0)} + w^{(1)} + w^{(2)} = 1 \qquad w^{(1)} = \frac{1}{2}$$
$$w^{(1)} - w^{(2)} = 0 \qquad w^{(2)} = \frac{1}{2}$$

$$2^2 = 2\cdot 2$$
$$P = LL^T$$

$$x \sim N(\mu, P)$$

$$y = g(x) + q \quad , \quad q \sim N(0, Q)$$

$$E\left[\begin{pmatrix} x \\ g(x)+q \end{pmatrix}\right] = E\left[\begin{pmatrix} x \\ \underbrace{g(x)}_{\hat{g}(x)} \end{pmatrix}\right]$$

$$\approx \sum_i w^{(i)} \begin{pmatrix} x^{(i)} \\ g(x^{(i)}) \end{pmatrix}$$

$$= \begin{pmatrix} \mu \\ \sum_i w^{(i)} g(x^{(i)}) \end{pmatrix} = \begin{pmatrix} \mu \\ \mu_y \end{pmatrix}$$

$$\mathrm{Cov}\left[\begin{pmatrix} x \\ g(x)+q \end{pmatrix}\right]$$

$$= E\left[\begin{pmatrix} x - \mu \\ g(x)+q - E[g(x)+q] \end{pmatrix}(\cdot)^T\right]$$

$$= E\left[\begin{pmatrix} x-\mu \\ g(x) - E[g(x)] \end{pmatrix}(\cdot)^T\right] + \begin{pmatrix} 0 & 0 \\ 0 & Q \end{pmatrix}$$

$$\approx E\left[\begin{pmatrix} (x-\mu)(x-\mu)^T & (x-\mu)(g-\mu_y)^T \\ (g(x)-\mu_y)(x-\mu)^T & (g-\mu_y)(g-\mu_y)^T \end{pmatrix}\right] + \begin{pmatrix} 0 & 0 \\ 0 & Q \end{pmatrix}$$

$$\approx \sum w^{(i)} \begin{bmatrix} (x^{(i)}-\mu)(x^{(i)}-\mu)^T & (x^{(i)}-\mu)(g(x^{(i)})-\mu_y)^T \\ (\cdot)^T & (g(x^{(i)})-\mu_y)\cdot(g(x^{(i)})-\mu_y)^T \end{bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & Q \end{pmatrix}$$

$$= \begin{pmatrix} P & \sum w^{(i)}(x^{(i)}-\mu)(g(x^{(i)})-\mu_y)^T \\ (\cdot)^T & \sum w^{(i)}(g(x^{(i)})-\mu_y)(g(x^{(i)})-\mu_y)^T + Q \end{pmatrix}$$