Problem 4.1: Radio receiver architecture

Figure 1 shows the architecture of a superheterodyne receiver which is the most popular receiver architecture in microwave receivers. The block diagram includes the following components:

- 1 = resonator-type antenna, 2 = transmission line, 3 = low-noise amplifier (LNA),
- 4 = image-rejection filter (band-pass filter), 5 = mixer, 6= local oscillator,
- 7 = channel-selection filter (band-pass filter), 8 = intermediate frequency amplifier,
- 9 = demodulator.

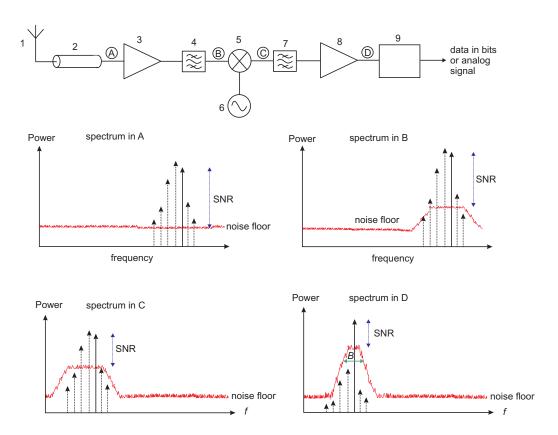


Figure 1: A superheterodyne receiver architecture and power spectrum observed at different points of the receiver.

The principled spectrum (power as a function of frequency) in locations A-D is shown. In the spectrum figures, the vertical arrows show signals with finite bandwidth (the solid arrow is our "wanted" signal). The red random line is the noise.

Use the source of your selection and answer the small questions using your own words. Give a reference to your source.

a) Why the signal coming from the antenna cannot be just directly demodulated – i.e., why the components 3-8 are needed?

- b) What is the function of the mixer (component 5)? Why the mixer is needed in receiver? Is a mixer linear or non-linear component?
- c) What is the function of the image-rejection filter (component 4)?
- d) What is "noise"? Where does the noise arise to the receiver system?
- e) Why the signal-to-noise-ratio (see SNR in figures) gets smaller after each component?
- f) In the spectrum in D, what is/means the horizontal green arrow with symbol B?

Problem 4.2: Noise

A real, noisy resistor at the physical temperature T_{phys} can be modelled as a noiseless resistor and a (thermal) noise voltage source v_{rms} in series as shown Fig. 2, where df is infinitesimally small bandwidth, f is the frequency, h is the Planck's constant and k is the Boltzmann's constant.

- a) Show that up to the millimetre wave frequencies $hf \ll kT_{\rm phys}$ in the room temperature.
- b) For the Taylors polynomial $e^x \approx 1 + x$ holds when x is small. Using the approximation, show that the formula for the available power P_n from a resistor at the noise bandwidth B is

$$P_{\rm n} = kT_{\rm phys}B. \tag{2.1}$$

- c) The thermal noise of a resistor is called white noise. Describe a definition of white noise, and justify why the noise (2.1) is white.
- d) From (2.1), identify two effective methods to reduce the noise level at the wireless receiver. Do you know where the identified technique is used in practice, e.g., which application and why it is critical to reduce the noise level?
- e) Assume an antenna of 150 K equivalent noise temperature. Explain briefly, what the equivalent noise temperature of an antenna means. Calculate the noise power for a radio system of 60 MHz bandwidth that appears at the output of the antenna. The impedance of the antenna is 50 Ω .

noisy noiseless resistor
$$+$$
 at T_{phys} noise source
$$\begin{cases} R \\ R \end{cases} = \begin{cases} R \\ V_{rms} = \sqrt{\frac{4Rhfdf}{e^{hf/kT_{phys}} - 1}} \end{cases}$$

Figure 2: Equivalent circuit model of a noisy resistor consisting of a voltage source and a noiseless resistor.

Problem 4.3: Noise temperature and noise figure

A horn antenna and a single side-band receiver front-end shown in Fig. 3 are used to receive a signal in a radio frequency signal at 60 GHz. The first stage of the receiver is a low-noise amplifier (LNA) connected to the antenna. The LNA forwards signals to a small cable to a mixer etc. as seen in the figure. The bandwidth of the receiver is defined at an intermediate (IF) frequency output by using a bandpass filter which has negligible loss. The signal received by the antenna is $S_i = 1$ nW.

- a) In problem 4.2, the antenna noise temperature was 150 K, while the same is 290 K in Fig. 3. Describe, what causes the difference of antenna noise temperature T_A .
- b) Calculate the equivalent noise temperature $T_{\rm R}$ of the receiver (defined with respect to the input of the LNA) and the system noise temperature $T_{\rm S}$.
- c) Calculate the noise figure F of the receiver.
- d) Calculate the signal-to-noise ratio (SNR) at the IF output, when the SNR at the LNA input is 30 dB.
- e) Calculate the noise bandwidth B_n of the IF filter, given the other parameters specified above.

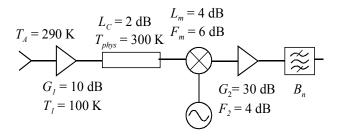


Figure 3: A single side-band receiver frontend.

Problem 4.4: Antenna gains

The gain of an antenna to a given direction (typically in the main lobe) can be defined by measuring the radiated power density (W/m^2) in the far-field region. The power density radiated by a horn antenna is 4.0 mW/m^2 in the main lobe at the distance of 2.0 m. The feed power of the antenna is 3.5 mW. The measurement was performed at 7.2 GHz. The size of the aperture of the horn is $10 \times 9 \text{ cm}^2$ (width and height). Assume that the impedance matching is "very good".

- a) Explain using your own words, what does the gain (unit dBi) of an antenna mean. Does an antenna with a higher gain radiate more power (unit W) than another antenna with a smaller gain (assume the resistive losses are the same).
- b) In the above setting, is the power density measured in the far-field region or not? Justify your answer.
- c) Estimate the gain of the antenna in dBi.

Problem 4.5: Friis' transmission formula

- a) Suppose a rectangular aperture antenna of $4 \times 4\lambda^2$ in size, where λ is the wavelength of a carrier. Gain of the aperture antenna shows G = 22 dB under perfect antenna matching and the radiation efficiency $\eta_{\rm rad} = 0.9$. Derive aperture efficiency $\eta_{\rm ap}$ of the antenna. What kind of physical observation applies to explain the efficiency less than 1?
- b) When there is an isotropic transmit antenna that radiates energy to the entire solid angle, i.e., to all the direction in space uniformly, derive the power density $[W/m^2]$ of the field in vacuum at a distance R from the antenna; the total power input to the antenna is P_t and assume perfect matching and radiation efficiency, i.e., $\eta_{rad} = 1$, in this question. Calculate the total power available at distance R and compare it with the radiated power from the antenna.
- c) At distance R, an antenna with effective antenna aperture $A_{\rm e}$ receives the field from the transmit antenna. Express the power output at the receive antenna port $P_{\rm r}$ using $P_{\rm t}$, R and $A_{\rm e}$ assuming that the receive antenna does not have matching losses.
- d) What is a maximal gain G of an antenna with effective aperture size A_e ? Use the relationship to express the power output at the receive antenna port P_r using P_t , R and G assuming that the receive antenna does not have matching losses.
- e) Based on the results of c) and d), let us think about a case of next generation mobile radio communications systems called fifth-generation (5G). It will use higher carrier frequencies for over-the-air radio data transmission than the legacy systems; 5G and legacy systems typically consider above and below 6 GHz carrier frequencies, respectively. Researchers have discussed if the use of higher carrier frequency is advantageous compared to the legacy systems or not. The following is commonly spread understanding among researchers about antenna receiving power of 5G and legacy systems. Do you support the understanding or not? Tell reasons why you support it or not.

Understanding: According to the Friis' transmission formula, the higher frequency radios for 5G systems are inferior to lower frequency radios for legacy systems in terms of a received power at the antenna.

Problem 4.6: Link budget and link margin

A wireless backhaul link is established between rooftops of two buildings where cellular base stations are located. The link has a line-of-sight and clear of reflections or scattering from the environment. The link has the following properties.

- Carrier frequency: $f_c = 40 \text{ GHz}$,
- System bandwidth: B = 200 MHz,
- Distance between buildings: R = 400 m.
- Transmitter power: $P_{\rm t} = 20 \text{ dBm}$,
- Gain of the transmitter and receiver antennas for the link: $G_{\rm t}=G_{\rm r}=20~{\rm dBi},$
- Noise temperature of the receiver antenna: $T_{\rm A}=2000$ K,
- Noise temperature of the receiver block: $T_{\rm r} = 300$ K,
- Minimum signal-to-noise (SNR) ratio required at the receiver for reliable data transfer: $SNR_{\min} = 20 \text{ dB}$.

Answer the following questions.

- a) Calculate the equivalent isotropic radiated power (EIRP) of the transmitter.
- b) Derive the SNR, $S_{\rm o}/N_{\rm o}$, at the receiver. What is the link margin in this case?
- c) On a windy day, the antennas on the rooftop vibrate and their main beams towards the other side of the link are slightly misaligned, leading to a decrease of the received power by 10 dB. If we want the link to work robust even in windy days by limiting the distance between the transmitter and receiver, what is the maximum possible separation distance?