1. *Let's study a heterojunction laser for optical telecommunication. The laser threshold current is 100 mA and the electron lifetime is 6 ns. a) What is the response time of the laser if the injection current is raised abruptly from 0 to 1.3 A? b) If the length of a pulse should be at least 0.4 ns, what is the maximum frequency at which the laser can be pulsed if the laser is unbiased (minimum injection current = 0 A) and the maximum injection current is 1.3 A? Assume that the pause time between pulses is also 0.4 ns. c) What is the maximum frequency, if the laser is biased with a current of 80 mA?*

a) Response time of the laser is $t_d = \tau \ln \frac{I}{I - I_{th}} = 0.48 \text{ ns}$.

b) The cycle time is the sum of the delay and the pulse width $t_{cycle} = t_d + t_{pulse} + t_{pause} = 1.28 \text{ ns}$.

The maximum frequency is then $f_{\max} = \frac{1}{t_{cycle}} \approx 780 \text{ MHz}$.

c) If the laser is biased below threshold, the delay is

$$t_d = \tau \ln \left( \frac{J - \frac{qdn_i}{\tau}}{J - \frac{qdn_f}{\tau}} \right) = \tau \ln \left( \frac{J - J_{bias}}{J - J_{th}} \right) = 0.10 \text{ ns} \quad \Rightarrow \quad t_{cycle} = 0.9 \text{ ns} \quad \Rightarrow \quad f_{\max} \approx 1.1 \text{ GHz}.$$

2. *A GaAs laser has a resonance frequency of $f_r$ = 3 GHz. The other relevant parameters of the laser are $\tau_p$ = 2 ps, $\Gamma$ = 0.9 and $\bar{\varphi}$ = $10^{18}$ cm$^{-3}$. The photon lifetime in the cavity is $\tau_p$ = 2 ps. Let us also assume that the thickness of the active layer is d = 0.1 µm. a) Calculate the differential gain in the laser. b) Calculate the angular frequency ($\omega = \omega_{max}$) at which the transfer function $H(\omega)$ attains the maximum. c) Calculate the ratio of the transfer function $H(\omega_{max})$ to its direct current value $H(0)$.*

a) Transfer function is $h\nu \frac{\Delta N_{p0}}{\Delta J_0} = \frac{\tau_p / qd}{\left(1 - \frac{(\omega\tau_p)^2}{\tau_p \Omega \bar{\varphi}}\right) + j\omega\tau_p}$, where $\Omega = \Gamma \frac{c}{n_r} \frac{\partial g}{\partial n}$. At the resonance angular frequency $\omega_r$ the real part of the denominator is zero: $1 - \frac{(\omega_r \tau_p)^2}{\tau_p \Omega \bar{\varphi}} = 0 \quad \Rightarrow$

$$(2\pi f_r \tau_p)^2 = \tau_p \Omega \bar{\varphi} \quad \Rightarrow \quad \Omega = \frac{(2\pi f_r)^2 \tau_p}{\bar{\varphi}} \quad \text{and}$$

$$\frac{\partial g}{\partial n} = \frac{\Omega n_r}{\Gamma c} = \frac{4\pi^2 f_r^2 \tau_p n_r}{\Gamma c \bar{\varphi}} = 9.5 \cdot 10^{-12} \frac{\text{cm}^3}{\text{m}} = 9.5 \cdot 10^{-14} \frac{\text{cm}^{-1}}{\text{cm}^{-3}} .$$

b) The amplitude of the transfer function is $H(\omega)=\left|h\nu\frac{\Delta N_{p0}}{\Delta J_0}\right|=\frac{\tau_p/qd}{\left[\left(1-\omega^2/\omega_r^2\right)^2+\left(\omega\tau_p\right)^2\right]^{1/2}}$.

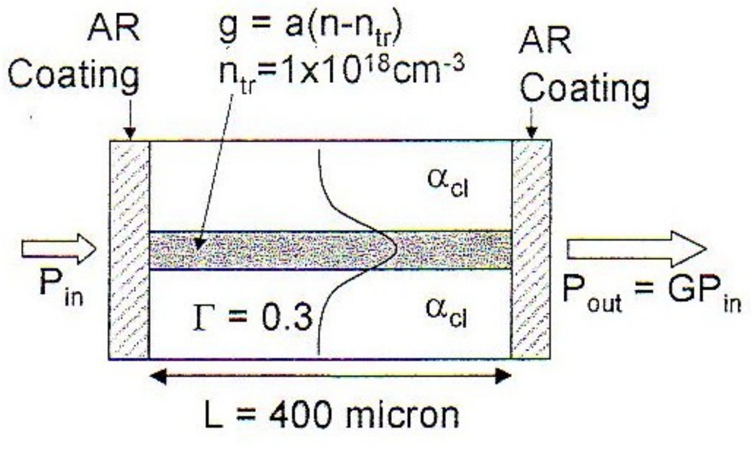
$H(\omega)$ achieves the maximum when the denominator $\left(1-\omega^2/\omega_r^2\right)^2+\left(\omega\tau_p\right)^2$ achieves the minimum $\Rightarrow$ $\frac{d\left(\left(1-\omega^2/\omega_r^2\right)^2+\left(\omega\tau_p\right)^2\right)}{d\omega}=-2\left(1-\frac{\omega^2}{\omega_r^2}\right)\frac{2\omega}{\omega_r^2}+2\tau_p^2\omega=0 \quad\Rightarrow\quad 1-\frac{\omega^2}{\omega_r^2}=\frac{\tau_p^2\omega_r^2}{2}$

$\Rightarrow \quad \omega=\omega_{\max}=\sqrt{\omega_r^2-\frac{\tau_p^2\omega_r^4}{2}}\cong\omega_r$. The maximum of the amplitude is then

$H(\omega_{\max})=\left|h\nu\frac{\Delta N_{p0}}{\Delta J_0}\right|_{\omega=\omega_r}=\frac{\tau_p/qd}{\left[\tau_p^2\omega_r^2\right]^{1/2}}=\frac{\tau_p/qd}{\tau_p\omega_r}=\frac{1}{qd\omega_r}$. The amplitude at the zero frequency is

$H(0)=\frac{\tau_p}{qd} \quad\Rightarrow\quad \frac{H(\omega_{\max})}{H(0)}=\frac{1}{\omega_r\tau_p}=26.5.$

3. *A Fabry-Perot semiconductor laser (spatially single-mode) can be turned into a semiconductor optical amplifier (SOA) by preventing the feedback by coating the facets with antireflection (AR) coatings (sketched below). Estimate the carrier density needed for an overall optical gain of G = 20 dB, using the SOA parameters: an active region length of 400 µm, carrier density at transparency $n_{tr}=1\times10^{18}$ cm$^{-3}$, and optical confinement factor 0.3. Assume small signal conditions with no output power or gain saturation. Gain coefficient is given by $g=a(n-n_{tr})$ with a gain constant of $a=2.5\times10^{-16}$ cm$^2$. Losses in the cavity are mainly caused by free carrier absorption in the doped cladding regions; use the absorption coefficient of $\alpha_{clad}$ =50 cm$^{-1}$ for the cladding layers.*




Gain of the SOA: $G=\frac{P_{out}}{P_{in}}=e^{g_{eff}L}$, where now $g_{eff}=\Gamma g-(1-\Gamma)\alpha_{clad} \quad\Rightarrow$

$G=e^{\Gamma gL}e^{-(1-\Gamma)\alpha_{clad}L} \qquad\Rightarrow\quad \ln G=\Gamma gL-(1-\Gamma)\alpha_{clad}L$

$\Rightarrow \quad g=\frac{\ln G}{L\Gamma}+\frac{(1-\Gamma)}{\Gamma}\alpha_{clad}=a(n-n_{tr})$

$$\Rightarrow \quad n = \frac{\ln G}{aL\Gamma} + \frac{(1-\Gamma)}{a\Gamma}\alpha_{clad} + n_{tr} = 3.0\times10^{18}\ \text{cm}^{-3}.$$

4. *a) Calculate the change in refractive index in GaAs for an applied electric field of* $2\times10^{5}$ *V/cm. b) Calculate the value of* $V_\pi$ *for a GaAs modulator when the wavelength of the incoming light is 1.1 μm. The waveguide is 1 μm thick and 1.5 mm long. c) A phase shift of* $\pi$ *is needed to produce in an electro-optical modulator having a length of 150 μm. Calculate the linear electro-optical coefficient the active material has to have in order to achieve the required phase shift with a bias voltage of 10 V. The refractive index of the material is about 3.5, the thickness of the active layer is 1 μm and the operation wavelength is 1.55 μm.*

a) $\Delta\left(\frac{1}{n_r^2}\right) = -\frac{2\Delta n_r}{n_r^3} = r_{ij}^{l}E_z,\quad n_r = 3.6,\ r_{ij} = 1.6\times10^{-12}\,\text{m/V} \quad\Rightarrow\quad |\Delta n_r| = 4.7\times10^{-4}$

b) $V_\pi = \frac{\lambda d}{l r_{ij}^{l} n_{r0}^{3}} = 11.8\ \text{V}$

c) The phase shift is given by $\phi = \frac{2\pi}{\lambda}Ln_r^3 r_{41}\frac{V}{d}$, so the electro-optical coefficient is directly $r_{41} = \frac{\lambda d\phi}{2\pi Ln_r^3 V} = \frac{\lambda d}{2Ln_r^3 V}$ = $1.2\times10^{-11}$ m V$^{-1}$. (This is 10 times larger than the coefficient for GaAs. So the actual device made of GaAs would be 1.5 mm long.

5. *The absorption coefficient due to excitonic absorption in a quantum well is given by*

$$\alpha_{ex}(\ \omega) \cong \frac{2.9\times10^{3}}{\Delta\varepsilon L_z}\exp\left[-\frac{(\varepsilon^{ex} - \ \omega)^2}{\sqrt{q}\,(\Delta\varepsilon)^2}\right],$$

*where* $\Delta\varepsilon$ *is the linewidth of the absorption peak and the well width* $L_z$ *is in Å. Calculate the on-off ratio of a 100Å GaAs/*$Al_{0.2}Ga_{0.8}As$ *QCSE modulator under an applied field of 100 kV/cm. The active region consists of 100 periods of the MQW. The heavy-hole exciton resonance has a linewidth of 2.5 meV and the incident photon energy coincides with the exciton energy at zero bias.*

At zero field, the absorption coefficient $\alpha(0)$ is given by

$$\alpha(0) = \frac{2.9\times10^{3}}{100\times2.5\times10^{-3}} = 1.2\times10^{4}\ \text{cm}^{-1} \quad (\ \omega = \varepsilon^{ex})$$

At the transverse bias of 100 kV/cm, the shift in the heavy-hole exciton resonance is given by

$$|\Delta\varepsilon^{ex}| \cong -3\times10^{-20}\left(m_e^* + m_{hh}^*\right)E^2L_z^4\ (\text{eV}) = 3\times10^{-20}\times0.51\times\left(100\times10^{3}\right)^2\times100^4 = 15.3\ \text{meV}$$

The absorption coefficient is now $\alpha(E) = \frac{2.9\times10^{3}}{100\times2.5\times10^{-3}}\exp\left[-\frac{(15.3)^2}{\sqrt{2}\,(2.5)^2}\right] = 5.8\times10^{-8} \cong 0$

The on-off ratio is given by the ratio of the transmitted light

$$\frac{T(E)}{T(0)} = \frac{e^{-\alpha(E)d}}{e^{-\alpha(0)d}},\ \text{where } d = 100\ \text{Å}\times100 = 1\ \mu\text{m} \quad\Rightarrow\quad \frac{T(E)}{T(0)} = 3.3$$