$$
\left\{
\begin{array}{l}
\int g(x)\, p(x \mid y_{1:T})\, dx \\[2ex]
x^{(i)} \sim p(x \mid y_{1:T}) \\
E[g(x) \mid y_{1:T}] \approx \frac{1}{N} \sum g(x^{(i)})
\end{array}
\right.
$$

$$
\begin{aligned}
&\int g(x)\, p(x \mid y_{1:T})\, dx \\
&= \int g(x)\, \frac{\pi(x \mid y_{1:T})}{\pi(x \mid y_{1:T})}\, p(x \mid y_{1:T})\, dx \\
&= \int \underbrace{\left[ g(x) \cdot \frac{p(x \mid y_{1:T})}{\pi(x \mid y_{1:T})} \right]}_{\tilde{g}(x)} \pi(x \mid y_{1:T})\, dx \\
&= \int \tilde{g}(x)\, \pi(x \mid y_{1:T})\, dx
\end{aligned}
$$

$$x^{(i)} \sim \pi(x \mid y_{1:T})$$

$$
\begin{aligned}
E[g \mid y_{1:T}] &= E_\pi[\tilde{g}(x) \mid y_{1:T}] \approx \frac{1}{N} \sum_i \tilde{g}(x^{(i)}) \\
&= \frac{1}{N} \sum_i g(x^{(i)})\, \frac{p(x^{(i)} \mid y_{1:T})}{\pi(x^{(i)} \mid y_{1:T})} \\
&= \sum_i \underbrace{\left[ \frac{1}{N} \frac{p(x^{(i)} \mid y_{1:T})}{\pi(x^{(i)} \mid y_{1:T})} \right]}_{\tilde{w}^{(i)}} g(x^{(i)}) \\
&= \sum \tilde{w}^{(i)} g(x^{(i)})
\end{aligned}
$$

$$p(x|y_{1:T}) = \frac{p(y_{1:T}|x)\,p(x)}{\int p(y_{1:T}|x)\,p(x)\,dx}$$

$$= Z \cdot p(y_{1:T}|x)\,p(x)$$

---

$$\int g(x)\,p(x|y_{1:T})\,dx$$

$$= \int g(x)\,\frac{p(y_{1:T}|x)\,p(x)}{\int p(y_{1:T}|x')\,p(x')\,dx'}\,dx$$

$$= \frac{\int g(x)\,p(y_{1:T}|x)\,p(x)\,dx}{\int 1 \cdot p(y_{1:T}|x)\,p(x)\,dx}$$

$$\int g(x)\,p(y_{1:T}|\alpha)\,p(x)\,dx$$

$$= \int g(x)\,\frac{\pi(x|y_{1:T})}{\pi(x|y_{1:T})}\,p(y_{1:T}|x)\,p(x)\,dx$$

$$= \int \underbrace{\left[\frac{p(y_{1:T}|x)\,p(x)}{\pi(x|y_{1:T})}\,g(x)\right]}_{\tilde{g}(x)} \pi(x|y_{1:T})\,dx$$

$$\approx \frac{1}{N}\sum_i \tilde{g}(x^{(i)}), \quad x^{(i)} \sim \pi(x^{(i)}|y_{1:T})$$

$$= \sum_i \frac{1}{N}\underbrace{\frac{p(y_{1:T}|x^{(i)})\,p(x^{(i)})}{\pi(x^{(i)}|y_{1:T})}}_{w^{*(i)}}\,g(x^{(i)})$$

$$\Rightarrow \frac{\int g(x)\, p(y_{1:T} \mid x)\, p(x)\, dx}{\int 1 \cdot p(y_{1:T} \mid x)\, p(x)\, dx}$$

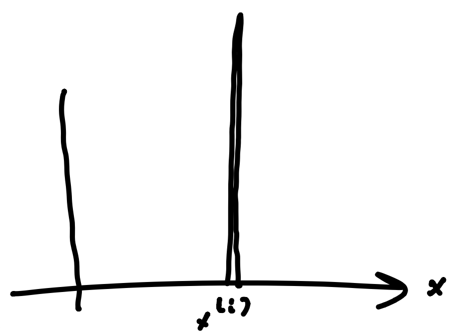
$$\approx \frac{\sum_i w^{*(i)} g(x^{(i)})}{\sum_i w^{*(i)}}$$

$$= \sum_i \underbrace{\left( \frac{w^{*(i)}}{\sum_i w^{*(i)}} \right)}_{\text{just } w^{*(i)}\text{'s normalized to } 1} g(x^{(i)})$$

---

$$\delta(x - x^{(i)}) = \begin{cases} \infty, & x = x^{(i)} \\ 0, & \text{otherwise} \end{cases}$$

but




for any function $g(x)$:

$$\int g(x)\, \delta(x - x^{(i)})\, dx = g(x^{(i)})$$

$$p(x \mid y_{1:T}) = \sum_i w^{(i)} \delta(x - x^{(i)})$$

$$E[g(x) \mid y_{1:T}] = \int g(x)\, p(x \mid y_{1:T})\, dx$$

$$= \int g(x) \sum_i w^{(i)} \delta(x - x^{(i)})\, dx$$

$$= \sum_i w^{(i)} \underbrace{\int g(x)\, \delta(x - x^{(i)})\, dx}$$

$$= \sum_i w^{(i)} g(x^{(i)})$$

$$\boxed{p(x_{0:k}|y_{1:k})}$$

$$= p(x_{0:k}|y_k, y_{1:k-1})$$

$$= \frac{p(y_k|x_{0:k}, y_{1:k-1})\, p(x_{0:k}|y_{1:k-1})}{\int \cdots \, dx_{0:k}}$$

$p(A,B) = p(A|B)\,p(B)$

$$\propto \underbrace{p(y_k|x_{0:k}, y_{1:k-1})}\, \underbrace{p(x_{0:k}|y_{1:k-1})}$$

$$= p(y_k|x_k)\, p(x_k|x_{0:k-1}, y_{1:k-1})\, p(x_{0:k-1}|y_{1:k-1})$$

$$= p(y_k|x_k)\, p(x_k|x_{k-1}) \cdot \boxed{p(x_{0:k-1}|y_{1:k-1})}$$

recursion!

---

$$x_{0:k}^{(i)} \sim \pi(x_{0:k}^{(i)}|y_{1:k})$$

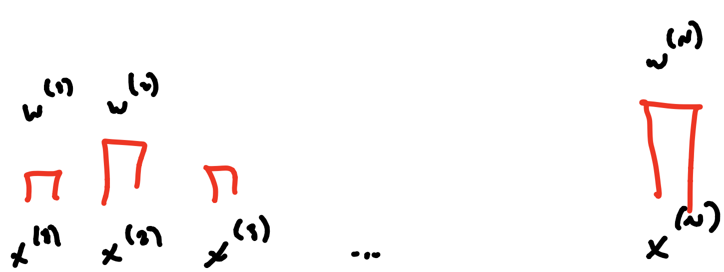
$$w_k^{*(i)} = \frac{p(y_k|x_k^{(i)})\, p(x_k^{(i)}|x_{k-1}^{(i)})\, p(x_{0:k-1}|y_{1:k-1})}{\pi(x_{0:k}^{(i)}|y_{1:k})}$$

$\propto w_k^{(i)}$

$$= \frac{p(y_k|x_k^{(i)})\, p(x_k^{(i)}|x_{k-1}^{(i)})}{\pi(x_k^{(i)}|x_{0:k-1}^{(i)}, y_{1:k})} \cdot \underbrace{\frac{p(x_{0:k-1}|y_{1:k-1})}{\pi(x_{0:k-1}|y_{1:k-1})}}_{\propto w_{k-1}^{*(i)}}$$

$\propto w_{k-1}^{(i)}$

---




resample from the distribution

$$[w^{(1)}, \dots, w^{(n)}]$$

$$\downarrow$$