$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho$$

$$\nabla \cdot \bar{B} = 0$$

↖ REAL

$\cos(\omega t)$

$e^{j\omega t}$

$$\frac{\partial}{\partial t} e^{j\omega t} = j\omega e^{j\omega t}$$

COMPLEX VECTOR $\bar{E}(\bar{r})$

$$\bar{E}(\bar{r}) = E_0 \bar{u} e^{-jkz}$$

$$= E_0 \bar{u} e^{-j\bar{k}\cdot\bar{r}}$$

REAL ELECTRIC FIELD : $\bar{E}(\bar{r}, t)$ (↙ x, y, z)

COMPLEX ELECTRIC FIELD : $\dot{\bar{E}}(\bar{r})$

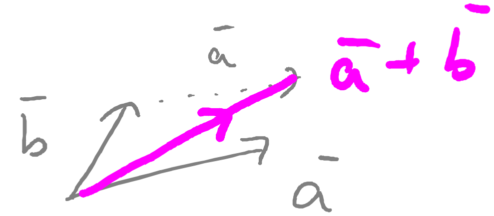
$$\bar{E}(\bar{r},t) = Re\{\bar{E}(\bar{r})\, e^{j\omega t}\}$$

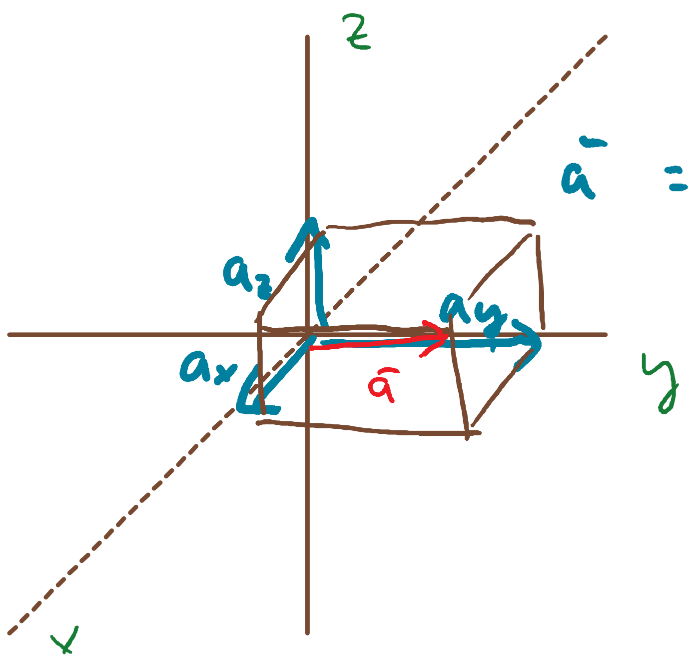
~~$e^{-j\omega t}$~~

REAL VECTORS

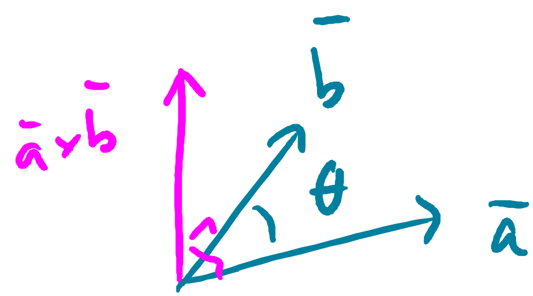
$\bar{a}$, $\bar{b}$

$\bar{a} + \bar{b}$





$$\bar{a} = a_x \bar{u}_x + a_y \bar{u}_y + a_z \bar{u}_z$$



$\bar{a}\ \bar{b} \quad |\bar{a}||\bar{b}|\sin\theta$

$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$$

$$|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\bar{E} = |\bar{E}|\,\bar{u}$$

$$\bar{u} = \frac{\bar{E}}{|\bar{E}|}$$

$$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{u}_x & \bar{u}_y & \bar{u}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$$

$$\bar{a}\times\bar{b} = -\bar{b}\times\bar{a}$$

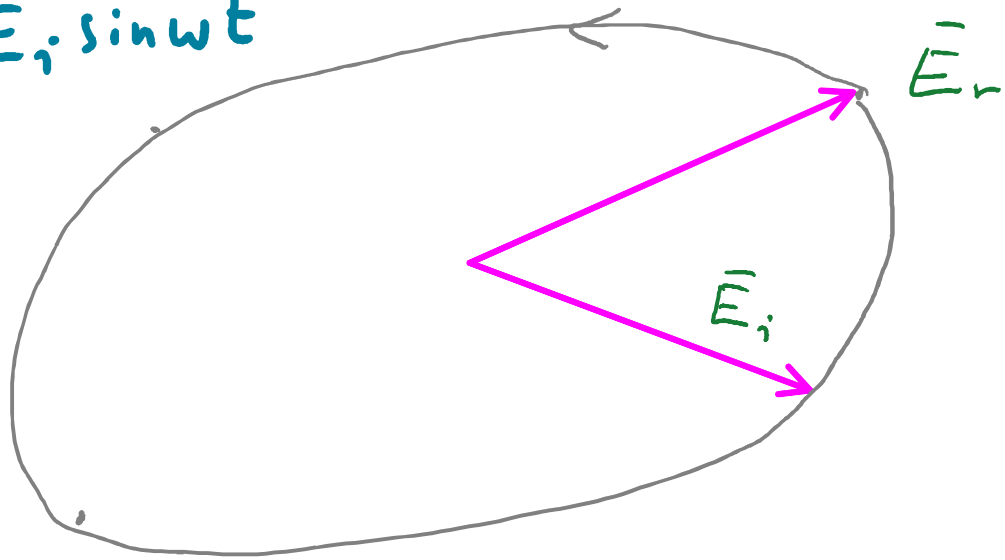
$$\bar{a}\cdot\bar{b}\times\bar{c} = \bar{a}\times\bar{b}\cdot\bar{c}$$

<u>bac - cab</u>

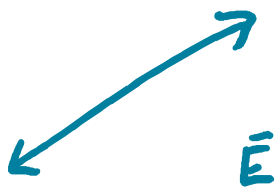
$$\bar{a}\times(\bar{b}\times\bar{c}) = \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b})$$

$$\bar{E} = \bar{E}_r + j\bar{E}_i$$

$$\bar{E}(t) = Re\left\{(\bar{E}_r + j\bar{E}_i)\, e^{j\omega t}\right\}$$
$$= \bar{E}_r \cos\omega t - \bar{E}_i \sin\omega t$$



LP



$\bar{E}_r = 0$ or $\bar{E}_i$ or $\bar{E}_r \times \bar{E}_i = 0$

CP $|\bar{E}_r| = |\bar{E}_i|$ $\quad \bar{E}_r \cdot \bar{E}_i = 0$

CONJUGATION:
$$\bar{E} = \bar{E}_r + j\bar{E}_i$$
$$E^* = \bar{E}_r - j\bar{E}_i$$

$\bar{E}\times\bar{E}$	$\bar{E}\times\bar{E}^*$
$\bar{E}\cdot\bar{E}$	$\bar{E}\cdot\bar{E}^*$

$\bar{E}\times\bar{E}$: 0

$\bar{E}\times\bar{E}^*$: 0 LP; $\neq 0$ CP

$\bar{E}\cdot\bar{E}$: $= 0$ CP

$\bar{E}\cdot\bar{E}^*$: $|\bar{E}|^2$

$\bar{E}$ complex vector

CP $\quad (\bar{u}_x + j\bar{u}_y)\cdot(\bar{u}_x + j\bar{u}_y) = 1 + j0 + j0 - 1 = 0$

$$(\bar{E}_r + j\bar{E}_i)\cdot(\bar{E}_r - j\bar{E}_i) = |\bar{E}_r|^2 - j\bar{E}_r\cdot\bar{E}_i + j\bar{E}_i\cdot\bar{E}_r + |\bar{E}_i|^2$$

$$= |\bar{E}_r|^2 + |\bar{E}_i|^2 = |\bar{E}|^2$$

$$(\bar{E}_r + j\bar{E}_i)\times(\bar{E}_r + j\bar{E}_i)$$

$$= j\bar{E}_r\times\bar{E}_i + j\underbrace{\bar{E}_i\times\bar{E}_r}_{-\bar{E}_r\times\bar{E}_i} = 0$$

LP $\quad j(\bar{u}_x + \bar{u}_y)$

$$= j(\bar{u}_x + \bar{u}_y)$$

$$j(\bar{u}_x + \bar{u}_y) \times \left(j(\bar{u}_x + \bar{u}_y)\right)^* = -jj(\bar{u}_x + \bar{u}_y) \times (\bar{u}_x + \bar{u}_y)$$

$$= 0$$

CP

$$(\bar{u}_x + j\bar{u}_y) \times (\bar{u}_x - j\bar{u}_y) = -j\bar{u}_z - j\bar{u}_z$$

REAL $\bar{a}, \bar{b}$

$$|\bar{a}\cdot\bar{b}|^2 + |\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2$$

$\cos^2\theta$ $\sin^2\theta$

$$(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d}) = \bar{a}\cdot\bar{b}\times(\bar{c}\times\bar{d})$$

$$= \bar{a}\cdot[\bar{c}\,\bar{b}\cdot\bar{d} - \bar{d}\,\bar{b}\cdot\bar{c})$$

$$= (\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{d}) - (\bar{a}\cdot\bar{d})(\bar{b}\cdot\bar{c})$$

($\bar{c} \to \bar{a}^*$, $\bar{d} \to \bar{b}^*$)

$$(\bar{a}\times\bar{b})\cdot(\bar{a}^*\times\bar{b}^*) = (\bar{a}\cdot\bar{a}^*)(\bar{b}\cdot\bar{b}^*) - (\bar{a}\cdot\bar{b}^*)(\bar{b}\cdot\bar{a}^*)$$

$(\bar{a}^* \cdot \ldots)^*$

$(\bar{a}\times \bar{b})\cdot(\bar{a}^*\times\bar{b}^*) = (\bar{a}\cdot\ldots)$

$\underbrace{(\bar{a}^*\times\bar{b}^*)}_{(\bar{a}\times\bar{b})^*}$ $\qquad$ $\underbrace{\ldots}_{(\bar{b}^*\cdot\bar{a})^* = (\bar{a}\cdot\bar{b}^*)^*}$

$$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2 - |\bar{a}\cdot\bar{b}^*|^2$$

$\bar{a}\cdot\bar{b} = 0 \qquad \perp$

$\bar{a}\times\bar{b} = 0 \qquad \parallel$

ASSUME:

$\bar{a}\times\bar{b} = 0$ and $\bar{a}\neq 0 \Rightarrow \bar{b} = \alpha\bar{a}$

$$\bar{a}^*\times\underbrace{(\bar{a}\times\bar{b})}_{=0} = \bar{a}(\bar{a}^*\cdot\bar{b}) - \bar{b}(\bar{a}\cdot\bar{a}^*)$$

$$\bar{b} = \frac{\bar{a}^*\cdot\bar{b}}{|\bar{a}|^2}\bar{a}$$