$p(u_{0:k} | y_{1:k}) =$

$= p(u_{0:k} | y_k, y_{1:k-1})$

$= \dfrac{p(y_k | u_{0:k}, y_{1:k-1})\, p(u_{0:k} | y_{1:k-1})}{\int \;-''-\; du_{0:k}}$

$\propto p(y_k | u_{0:k}, y_{1:k-1})\, p(u_{0:k} | y_{1:k-1})$

$= p(y_k | u_{0:k}, y_{1:k-1})\, p(u_k | u_{0:k-1}, y_{1:k-1})$

$\cdot\; p(u_{0:k-1} | y_{1:k-1})$

$p(x_{k-1} | u_{0:k-1}, y_{1:k-1}) = N(x_{k-1} | m_{k-1}, P_{k-1})$

$p(x_k | u_{0:k}, y_{1:k}) = p(x_k | y_k, u_{0:k}, y_{1:k-1})$

$\propto p(y_k | x_k, u_{0:k}, y_{1:k-1})\, p(x_k | u_{0:k}, y_{1:k-1})$

$= p(y_k | x_k, u_k)\, p(x_k | u_{0:k}, y_{1:k-1})$

$\Rightarrow$ KF update prediction

hier:

$p(x_k | u_{0:k}, y_{1:k-1})$

$= \int p(x_k, x_{k-1} | u_{0:k}, y_{1:k-1})\, dx_{k-1}$

$= \int p(x_k | x_{k-1}, u_{0:k}, y_{1:k-1})$

$\quad p(x_{k-1} | u_{0:k}, y_{1:k-1})\, dx_{k-1}$

$= \int p(x_k | x_{k-1}, u_k)$

$\quad p(x_{k-1} | u_{0:k-1}, y_{1:k-1})\, dx_{k-1}$

$\Rightarrow$ KF prediction

mixing:

$$p(x_k, u_k \mid u_{0:k-1}, y_{1:k-1})$$

$$= \int p(x_k, u_k, x_{k-1} \mid u_{0:k-1}, y_{1:k-1})\, dx_{k-1}$$

$$= \int p(x_k, u_k \mid x_{k-1}, u_{0:k-1}, y_{1:k-1}) \cdot p(x_{k-1} \mid u_{0:k-1}, y_{1:k-1})\, dx_{k-1}$$

$$= \int p(x_k, u_k \mid x_{k-1}, u_{k-1})\, p(x_{k-1} \mid u_{0:k-1}, y_{1:k-1})\, dx_k$$

$$= N\left(\begin{pmatrix} u_k \\ x_k \end{pmatrix} \middle| \begin{bmatrix} g \\ f \end{bmatrix} + \begin{bmatrix} B \\ A \end{bmatrix} m_{k-1}, \begin{bmatrix} B \\ A \end{bmatrix} P_{k-1} \begin{bmatrix} B \\ A \end{bmatrix}^T + Q_{k-1} \right)$$

$$p(x_k \mid u_k, u_{0:k-1}, y_{1:k-1})$$

= as in slides

KF: $$\frac{p(y_k \mid x_k)\, p(x_k \mid y_{1:k-1})}{p(y_k \mid y_{1:k-1})}$$

$$p(y_k \mid y_{1:k-1})$$

$$= \int \underbrace{p(y_k \mid x_k)}_{N(y_k \mid H x_k, R)}\ \underbrace{p(x_k \mid y_{1:k-1})}_{N(x_k \mid m_k^-, P_k^-)}$$

$$= N(y_k \mid H m_k^-, H P_k^- H^T + R)$$