PHYS-E0436 – Modern Optics (Fundamentals of photonics)

Topics:

- ✓ Fourier optics
- ✓ Electromagnetic optics
- ✓ Polarization optics
- ✓ Statistical optics
- ✓ Nonlinear optics
- ✓ Ultrafast optics

11 meetings: 1 hour exercise session + 1 hour lecture

Home work: Reading the book (~30 pages) and solving 4 problems [~8 hours per week]

On the exercise sessions, the students explain solutions of two home exercises each [must be checked in 1162, Micronova, each Wednesday]

Grading: Exercise solution presentations – max **15** points
Home exercises – max **35** points
Final exam – max **70** points

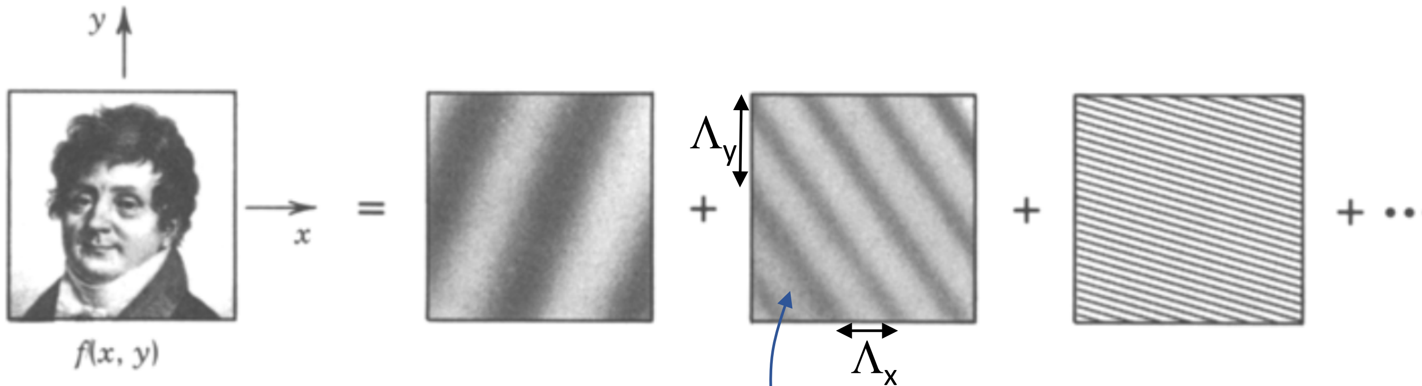
Syllabus

Date	Topics
1.3	**Introduction to the course** *Introduction to Fourier optics I*
8.3	Exercises to Fourier optics I *Introduction to Fourier optics II*
15.3	Exercises to Fourier optics II *Introduction to Electromagnetic optics I*
22.3	Exercises to Electromagnetic optics I *Introduction to Electromagnetic optics II* *Introduction to Polarization optics I*
29.3	Exercises to Electromagnetic optics II and Exercises to Polarization optics I *Introduction to Polarization optics II*
5.4	Exercises to Polarization optics II *Introduction to Statistical optics*
12.4	Exercises to Statistical optics *Introduction to Nonlinear optics I*
19.4	Easter
26.4	Exercises to Nonlinear optics I *Introduction to Nonlinear optics II*
3.5	Exercises to Nonlinear optics II *Introduction to Ultrafast optics I*
10.5	Exercises to Ultrafast optics I *Introduction to Ultrafast optics II*
17.5	Exercises to Ultrafast optics II
24.5	Exam

Chapter 4

FOURIER OPTICS I

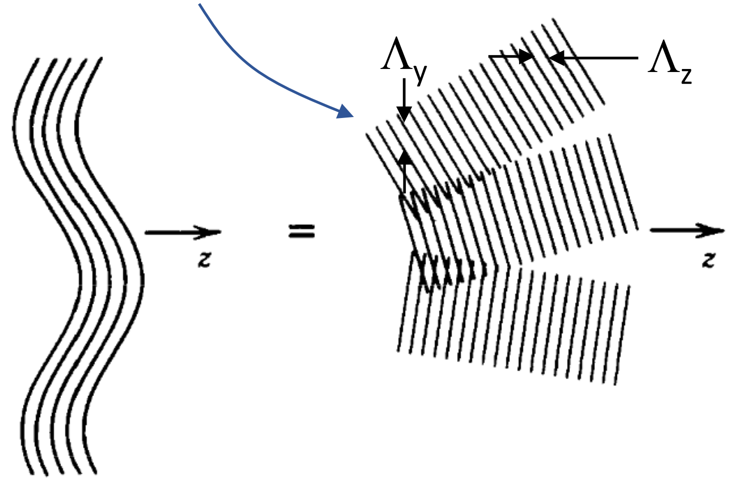
The principle of Fourier optics



$$F(x,y) = Ae^{-j2\pi(x/\Lambda_x + y/\Lambda_y)}$$

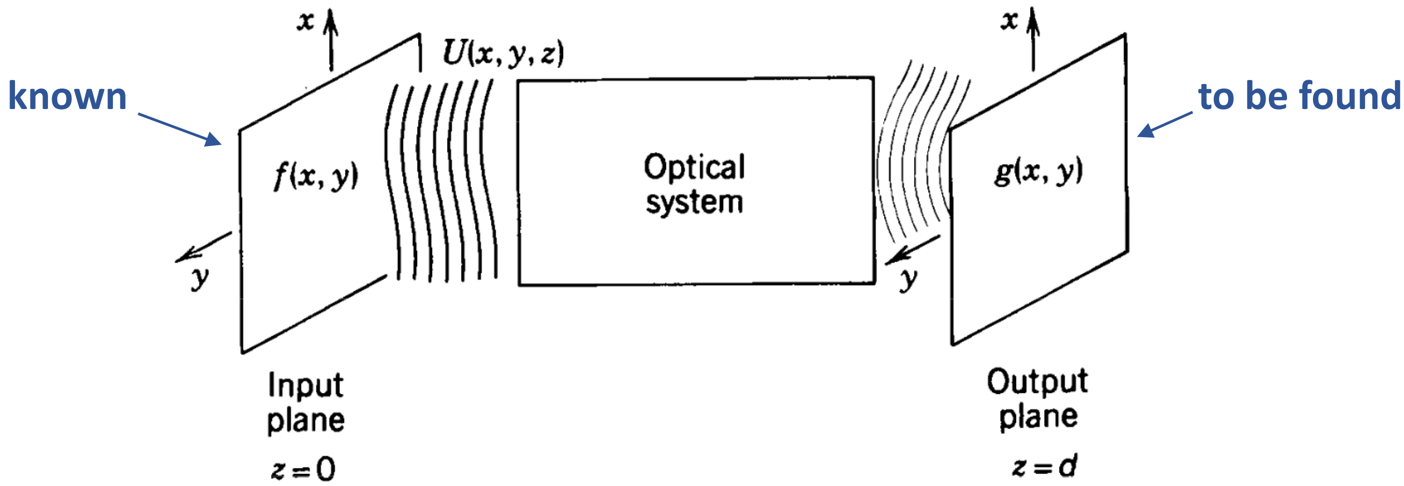
2D picture: An arbitrary function $f(x, y)$ may be analyzed as a sum of harmonic functions of different spatial frequencies and complex amplitudes.

$$F(x,y,z) = Ae^{-j2\pi(x/\Lambda_x + y/\Lambda_y + z/\Lambda_z)}$$



3D picture: The principle of Fourier optics: an arbitrary wave in free space can be analyzed as a superposition of plane waves.

What type of problems can Fourier optics solve?



Three-step solution: **(1)** Expand the input field into plane waves as

$$\underbrace{f(x,y)}_{\text{IFT}(F)} = \iint_{-\infty}^{\infty} \underbrace{F(\nu_x,\nu_y)}_{\text{FT}(f)} \exp\left[-j2\pi(\nu_x x + \nu_y y)\right] d\nu_x \, d\nu_y,$$

spatial frequency $\nu_i = 1/\Lambda_i$

i.e, calculate two-dimensional **FT**(f), **(2)** propagate the obtained plane waves to the output plane, and **(3)** sum the propagated waves to obtain the output field. If the system is free space, the field at any z is

$$\underbrace{g(x,y,z)}_{\text{IFT}(F)} = \iint_{-\infty}^{\infty} \underbrace{F(\nu_x,\nu_y)}_{\text{FT}(g)} \exp\left[-j(2\pi\nu_x x + 2\pi\nu_y y)\right] \exp(-jk_z z) \, d\nu_x \, d\nu_y$$

Two-dimensional FT and IFT

In rectangular coordinates

FT:

$$F(\nu_x, \nu_y) = \iint_{-\infty}^{\infty} f(x, y) \exp[j2\pi(\nu_x x + \nu_y y)] \, dx \, dy,$$

IFT:

$$f(x, y) = \iint_{-\infty}^{\infty} F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)] \, d\nu_x \, d\nu_y,$$

(spatial frequencies)

In polar coordinates for *circularly symmetric field profiles*

FT:

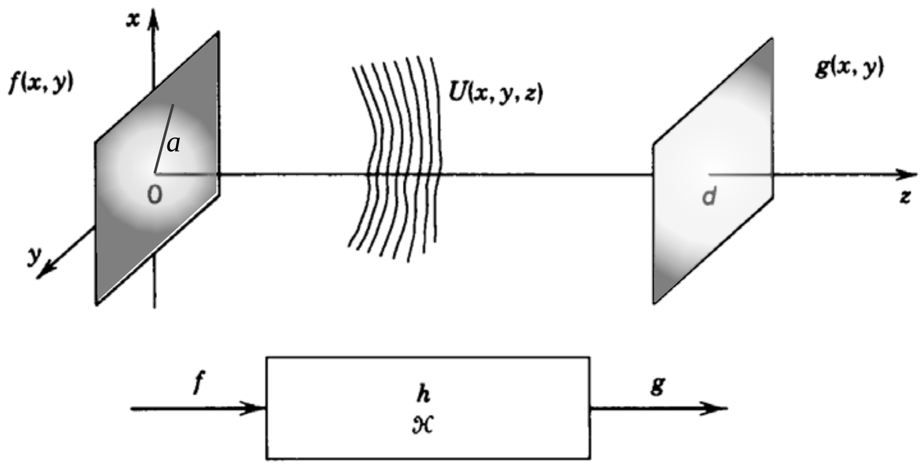
$$G(\rho) = \int_0^{\infty} g(r) \, 2\pi r \, J_0(2\pi r\rho) \, dr, \qquad r = \sqrt{x^2 + y^2}$$

IFT:

$$g(r) = \int_0^{\infty} G(\rho) \, 2\pi\rho \, J_0(2\pi r\rho) \, d\rho, \qquad \rho = \sqrt{\nu_x^2 + \nu_y^2}$$

(radial spatial frequency)

Propagation in free space (diffraction)



Any linear system can be characterized either by an ***impulse-response function***

$$h(x, y)$$

or by a ***transfer function***

$$H(\nu_x, \nu_y).$$

point-spread function

plane-wave propagator

$$g(x,y) = \iint_{-\infty}^{\infty} h(x-x', y-y')\, f(x', y')\, dx'\, dy' \quad - \quad \text{convolution } h * f$$

$$\Rightarrow \quad G(\nu_x, \nu_y) = H(\nu_x, \nu_y) F(\nu_x, \nu_y).$$

The intensity is $I(x, y) = |g(x, y)|^2$.

The transfer function of free space is

$$H(\nu_x, \nu_y) = e^{-jk_z d} = \exp\left(-j2\pi d \sqrt{\frac{1}{\lambda^2} - \nu_x^2 - \nu_y^2}\right).$$

Diffraction in Fresnel approximation

For small propagation angles ($4d^3 >> a^4/\lambda$), the Taylor expansion yields

$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)} \approx k - \frac{k_x^2 + k_y^2}{2k} = k - \pi\lambda(\nu_x^2 + \nu_y^2).$$

$$\Rightarrow \; H(\nu_x, \nu_y) = e^{-jk_z d} \approx \exp\left[-jkd + j\pi\lambda d(\nu_x^2 + \nu_y^2)\right]$$

The **IFT** of $H(\nu_x, \nu_y)$ is the impulse-response function

$$h(x, y) \approx \frac{j}{\lambda d} \exp\left[-jk\left(d + \frac{x^2 + y^2}{2d}\right)\right]$$

a paraboloidal wave instead of spherical, $\frac{e^{-jkr}}{r}$

The diffracted field $g\,(x, y)$ is found from

$$g(x, y) = \iint_{-\infty}^{\infty} F(\nu_x, \nu_y) H(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y) d\nu_x d\nu_y]$$

or directly from the convolution integral (**Fresnel diffraction integral**)

$$g(x, y) = \frac{j}{\lambda d} \iint_{-\infty}^{\infty} \exp\left[-jk\left(d + \frac{(x - x')^2 + (y - y')^2}{2d}\right)\right] f(x', y') dx' dy'.$$

Diffraction in Fraunhoffer approximation

If in addition, we have $d >> a^2/\lambda$, then

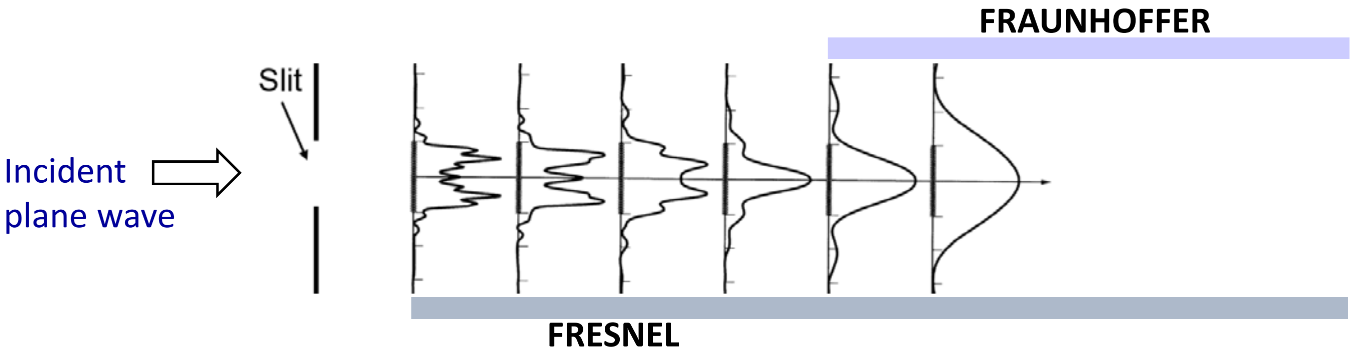
$$g(x,y) = \frac{j}{\lambda d}\iint_{-\infty}^{\infty} \exp\left[-jk\left(d + \frac{(x-x')^2+(y-y')^2}{2d}\right)\right] f(x',y')dx'dy'$$

$$\approx \frac{j}{\lambda d} e^{-jkd} e^{-j\frac{k}{2d}(x^2+y^2)} \underbrace{\iint_{-\infty}^{\infty} f(x',y')\exp\left[j2\pi\left(\frac{x}{\lambda d}x' + \frac{y}{\lambda d}y'\right)\right] dx'dy'}_{F\left(\frac{x}{\lambda d},\frac{y}{\lambda d}\right)}$$

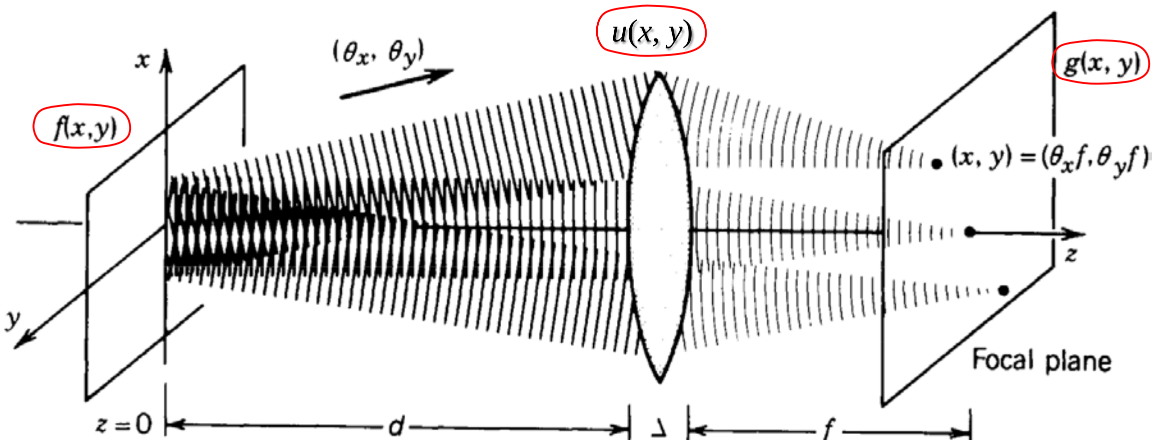
Neglecting also the quadratic phase factor, we obtain

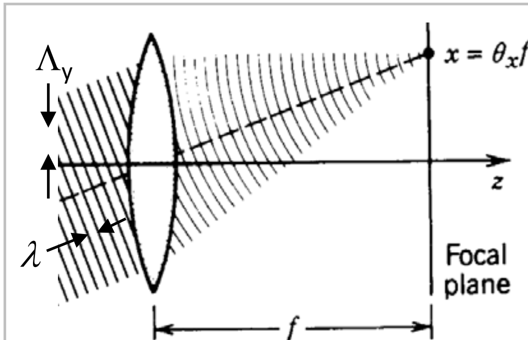
$$g(x,y) \approx h_0 F\left(\frac{x}{\lambda d},\frac{y}{\lambda d}\right)$$

where $h_0 = \frac{j}{\lambda d} e^{-jkd}$.



Fourier transform using a lens





$$\frac{\lambda}{\Lambda_x} = \frac{x}{f} \Rightarrow \nu_x \equiv \frac{1}{\Lambda_x} = \frac{x}{\lambda f}$$

Hence, the lens maps the spatial frequencies of the field to the coordinates (x, y), i.e.,

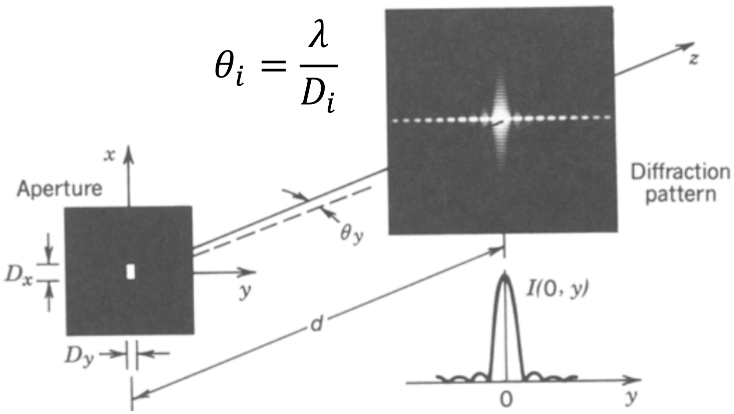
$$g(x,y) \propto F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right).$$

The transmission coefficient of the lens is $t_l = e^{j\frac{k}{2f}(x^2+y^2)}$. The Fresnel approximation gives $g(x,y) = \frac{j}{\lambda f} e^{-j\frac{k}{2f}(x^2+y^2)} U\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$. Due to propagation over the distance d,

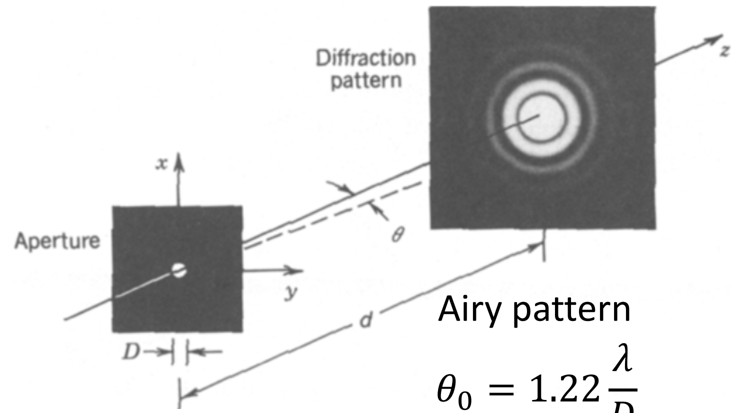
$$U\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right) = F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right) H_d\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right) \Rightarrow g(x,y) = h_0^{(f)} H_0^{(d)} e^{j\pi\frac{(x^2+y^2)(d-f)}{\lambda f^2}} F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right).$$

Examples of diffraction patterns

Fraunhoffer diffraction in free space



$$\theta_i = \frac{\lambda}{D_i}$$



Airy pattern

$$\theta_0 = 1.22\frac{\lambda}{D}$$

Fresnel diffraction of focused light (Fourier transform by a lens)