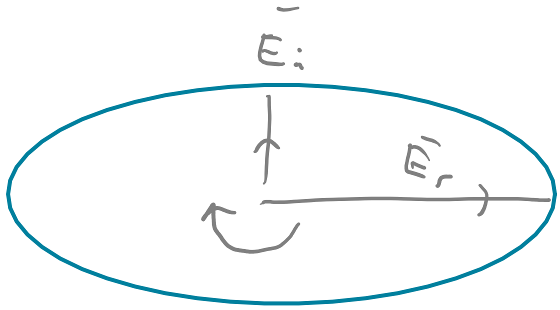
$$\bar{E} = \bar{E}_r + j\bar{E}_i$$

$$\bar{E}(t) = \mathrm{Re}\{\bar{E}e^{j\omega t}\} = \bar{E}_r \cos\omega t - \bar{E}_i \sin\omega t$$

IF $\bar{E}_r \cdot \bar{E}_i = 0 \Rightarrow \bar{E}\cdot\bar{E} = \text{real}$

$$|\bar{E}(t)|^2$$

$$= |\bar{E}_r|^2 \cos^2\omega t + |\bar{E}_i|^2 \sin^2\omega t$$



$\bar{a}$ general complex vector

$\bar{a}\cdot\bar{a} \neq \text{real}$

$$\bar{b} = \bar{a}e^{j\theta}$$

$$= \frac{|\sqrt{\bar{a}\cdot\bar{a}}|}{\sqrt{\bar{a}\cdot\bar{a}}}\bar{a}$$

$$\bar{b}\cdot\bar{b} = \frac{|\sqrt{\bar{a}\cdot\bar{a}}|^2}{\sqrt{\bar{a}\cdot\bar{a}}^2}\bar{a}\cdot\bar{a}$$

$$= |\sqrt{\bar{a}\cdot\bar{a}}|^2$$

(real)

$$|\bar a \cdot \bar b|^2 + |\bar a \times \bar b^*|^2 = |\bar a|^2 |\bar b|^2$$

$$\bar a \times (\bar a^* \times \bar b) = \bar a^* (\bar a \cdot \bar b) - \bar b \underbrace{\bar a \cdot \bar a^*}_{|\bar a|^2}$$

$$\bar b = -\frac{\bar a \times (\bar a^* \times \bar b)}{|\bar a|^2} + \bar a^* \frac{\bar a \cdot \bar b}{|\bar a|^2}$$

$\bar E \sim \bar h$

$\bar h$

$$|\bar h|^2 |\bar E|^2 = |\bar h \cdot \bar E|^2 + |\bar h \times \bar E^*|^2$$

$$\text{"polarization match"} = \frac{|\bar h \cdot \bar E|^2}{|\bar h|^2 |\bar E|^2} = 1 - \frac{|\bar h \times \bar E^*|^2}{|\bar h|^2 |\bar E|^2}$$

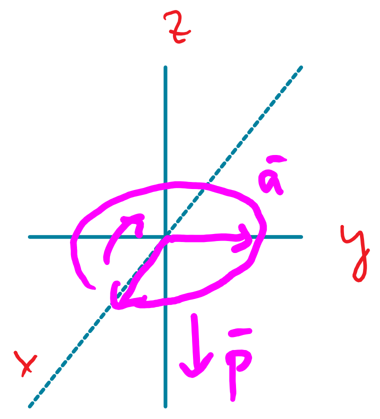
POLARIZATION VECTOR

$$\bar{p}(\bar{a}) = \frac{\bar{a} \times \bar{a}^*}{j|\bar{a}|^2} = \frac{(\bar{a}_r + j\bar{a}_i) \times (\bar{a}_r - j\bar{a}_i)}{j|\bar{a}|^2}$$

$$= \frac{-2j\,\bar{a}_r \times \bar{a}_i}{j|\bar{a}|^2} = -2\frac{\bar{a}_r \times \bar{a}_i}{|\bar{a}|^2}$$

$$\bar{a} = \bar{u}_x + j\bar{u}_y$$

$$\Rightarrow \bar{p}(\bar{a}) = -2\frac{\bar{u}_z}{1+1} = -\bar{u}_z$$



COMPLEX VECTOR BASES

$$(\bar a\times\bar b)\times(\bar c\times\bar d) = \bar c(\bar a\times\bar b\cdot\bar d) - \bar d(\bar a\times\bar b\cdot\bar c)$$
$$= \bar b(\bar a\cdot\bar c\times\bar d) - \bar a(\bar b\cdot\bar c\times\bar d)$$

$$\bar a\times(\bar b\times\bar c) = \bar b\,\bar a\cdot\bar c - \bar c\,\bar a\cdot\bar b$$
$$(\bar a\times\bar b)\times\bar c = \bar b\,\bar a\cdot\bar c - \bar a\,\bar b\cdot\bar c$$
$$-\bar c\times(\bar a\times\bar b) = \bar c\times(\bar b\times\bar a) = \bar b\,\bar c\cdot\bar a - \bar a\,\bar c\cdot\bar b$$

$$\bar d\,(\bar a\times\bar b\cdot\bar c) = +\bar a(\bar b\cdot\bar c\times\bar d) - \bar b(\bar a\cdot\bar c\times\bar d) + \bar c(\bar a\times\bar b\cdot\bar d)$$
$$= \bar a(\bar b\times\bar c\cdot\bar d) + \bar b(\bar c\times\bar a\cdot\bar d) + \bar c(\bar a\times\bar b\cdot\bar d)$$
$$= \bar a\,\bar b\times\bar c\cdot\bar d + \bar b\,\bar c\times\bar a\cdot\bar d + \bar c\,\bar a\times\bar b\cdot\bar d$$

$$\bar d = \underbrace{(\bar a\bar a' + \bar b\bar b' + \bar c\bar c')}_{\bar{\bar I}\ \text{UNIT DYADIC}}\cdot\bar d$$

$$\bar{\bar I}\cdot\bar f = \bar f$$
$$\bar f\cdot\bar{\bar I} = \bar f$$

$\bar a, \bar b, \bar c$

RECIPROCAL BASIS $\bar a', \bar b', \bar c'$

$$\bar a' = \frac{\bar b\times\bar c}{\bar a\times\bar b\cdot\bar c}\qquad \bar b' = \frac{\bar c\times\bar a}{\bar a\times\bar b\cdot\bar c}\qquad \bar c' = \frac{\bar a\times\bar b}{\bar a\times\bar b\cdot\bar c}$$

$$\bar{E}(\bar{r}) = \int \bar{\bar{G}}(\bar{r},\bar{r}')\cdot\bar{J}(\bar{r}')\,dv'$$

GREEN DYADIC

$$\bar{\bar{G}}(\bar{r},\bar{r}') = -j\omega\mu\left(\bar{\bar{I}} + \frac{\nabla\nabla}{k^2}\right)G(\bar{r},\bar{r}')$$

$$G(\bar{r},\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}$$

$$\bar{a}\bar{b} \neq \bar{a}\cdot\bar{b} \neq \bar{a}\times\bar{b} \neq \bar{a}+\bar{b}$$

DYAD

$i,j = x,y,z$

DYADIC : $\sum_{i=1}^{3} \bar{a}_i \bar{b}_i = \sum a_{ij}\bar{u}_i \bar{u}_j$

$$\bar{a}(\alpha\bar{b}) = \alpha(\bar{a}\bar{b})$$

$\bar{\bar{A}}$

EXAMPLE: $\bar{\bar{I}} = \bar{u}_x\bar{u}_x + \bar{u}_y\bar{u}_y + \bar{u}_z\bar{u}_z$

$$\bar{\bar{I}}\cdot\bar{a} = \bar{\bar{I}}\cdot(a_x\bar{u}_x + a_y\bar{u}_y + a_z\bar{u}_z)$$

$$= a_x\bar{u}_x\underbrace{\bar{u}_x\cdot\bar{u}_x}_{1} + \dots$$

$$= a_x\bar{u}_x + \dots = \bar{a}$$

$$\bar{\bar{P}} = \bar{u}_x\bar{u}_x$$

$$\bar{\bar{P}}\cdot\bar{b} = \bar{u}_x\bar{u}_x\cdot(b_x\bar{u}_x + b_y\bar{u}_y + b_z\bar{u}_z)$$

(PROJECTION!) $= b_x\bar{u}_x$

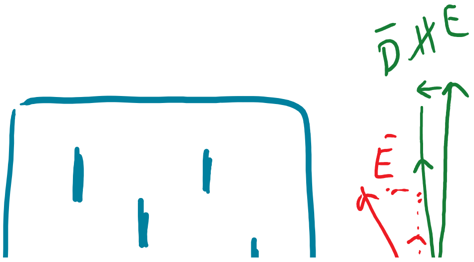
$\bar{\bar{A}}\cdot\bar{a} \rightarrow$ vector

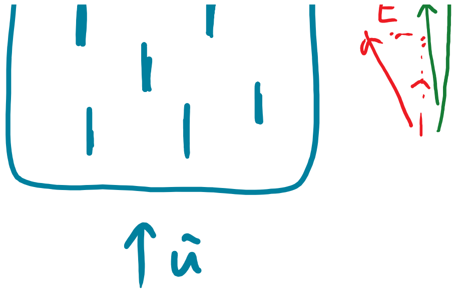
$\bar{\bar{A}}\times\bar{a} \rightarrow$ dyadic

$$\bar{D} = \varepsilon\bar{E}$$

$$= \bar{\bar{\varepsilon}}\cdot\bar{E}$$

$$\cancel{\varepsilon\bar{\bar{I}}}$$





$$\bar{\bar{\varepsilon}} = \varepsilon_{strong}\,\hat{u}\hat{u} + \varepsilon_{weak}\left(\bar{\bar{I}} - \hat{u}\hat{u}\right)$$