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4 March 2019

ELEC-E5630 - Acoustics and Audio Technology Seminar

Almost
All About Audio Equalizers

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Teacher: Vesa Välimäki

- Professor of audio signal processing, 2002-07, 07-
  - Aalto Department of Signal Processing and Acoustics
- Vice Dean for Research in electrical engineering, 2017-
- MSc 1992, DSc 1995, Docent 1999
  - All from Helsinki University of Technology (now part of Aalto Univ.)
- Postdoc at University of Westminster, London, UK 1996
- Professor of signal processing, Tampere Univ. Tech./Pori, 2001-02
- Visiting scholar at CCRMA, Stanford University, 2008-09
- General Chair of DAFX-08 and SMC-17 international conferences

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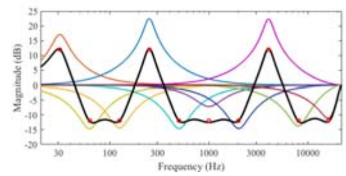
Audio Signal Processing Research at Aalto Acoustics Lab

- Headphone/headset audio
  - Virtual/augmented reality audio
  - Enhanced listening in noise
- Audio effects processing
  - Virtual analog modeling, now with deep learning
  - Artificial reverberation algorithms
- Digital filters for audio
  - Equalization, delay filters, filter design
- Sound synthesis
  - Modeling of sound/noise sources



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Review Article on Equalizers

applied sciences

MDPI

Review

All About Audio Equalization: Solutions and Frontiers

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Abstract: Audio equalization is a vast and active research area. The extent of research means that one often cannot identify the preferred technique for a particular problem. This review paper bridges those gaps, systemically providing a deep understanding of the problems and approaches in audio equalization, their relative merits and applications. Digital signal processing techniques for modifying the spectral balance in audio signals and applications of these techniques are reviewed, ranging from classic equalizers to emerging designs based on new advances in signal processing and machine learning. Emphasis is placed on putting the range of approaches within a common mathematical and conceptual framework. The application areas discussed herein are diverse, and include well-defined, solvable problems of filter design subject to constraints, as well as newly emerging challenges that touch on problems in semantics, perception and human computer interaction. Case studies are given in order to illustrate key concepts and how they are applied in practice. We also recommend preferred signal processing approaches for important audio equalization problems. Finally, we discuss current challenges and the uncharted frontiers in this field. The source code for methods discussed in this paper is made available at https://code.soundsoftware.ac.uk/projects/allaboutaudioeq.

- A review paper about audio equalizers published in *Applied Sciences* in 2016
- Joint work with Prof. Josh Reiss (Queen Mary Univ. London, UK)

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Outline

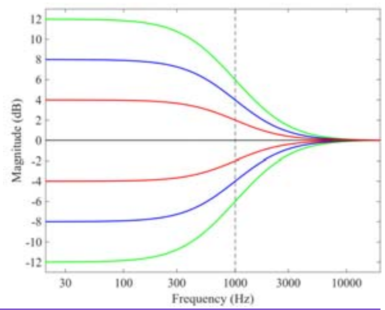
- Brief history
- Shelving filters
- Parametric equalizers
- Graphic equalizers
- Parallel graphic equalizers
- Cascade graphic equalizers

Brief History of Audio Equalization

- 1870s: telegraph line signals were selected with vibrating electromechanical reeds
- 1930s: telephone line equalizers boosting of high frequencies to compensate for distance loss
  - "Equalize" the frequency response
- 1940s: Equalizing filter for gramophone players
- 1950s: RIAA correction for LP records
- 1950s: First graphic equalizer, *Cinema Engineering 7080*
- 1970s: Parametric equalizer (Massenburg, 1972)
- 1980s: Digital equalizer, *Yamaha DEQ7* (1987)

Shelving Filter

- Shelving filter is a bass or treble tone control
- Amplifies or attenuates low (high) frequencies without affecting the high (low) ones



Derivation of 1st-Order Low Shelf

- First define the gain at 3 points: $H(0) = G$, $H(1) = 1$, and *geometric mean* gain (g/2 in dB) at crossover:

$$|H(e^{j\omega_c})| = \sqrt{G}$$

- When $\omega_c = \pi/2$, we get the prototype filter:

$$H_P(z) = \sqrt{G}\frac{z+p}{z-p} \quad \text{with the pole at} \quad p = \frac{G-\sqrt{G}}{G+\sqrt{G}}$$

- The crossover point is shifted using the lowpass-to-lowpass transformation (see, e.g. Reiss & McPherson, 2015):

$$z \rightarrow \frac{z-\beta}{1-\beta z} \qquad \beta = \frac{1-\tan(\omega_c/2)}{1+\tan(\omega_c/2)}$$

Derivation of 1st-Order Low Shelf (2)

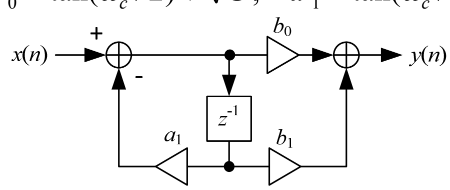
- The 1st-order shelf filter has the transfer function (Välimäki & Reiss, 2016):

All coefficients divided by a'_0

$$H_{LS}(z) = \frac{b'_0 + b'_1 z^{-1}}{a'_0 + a'_1 z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$

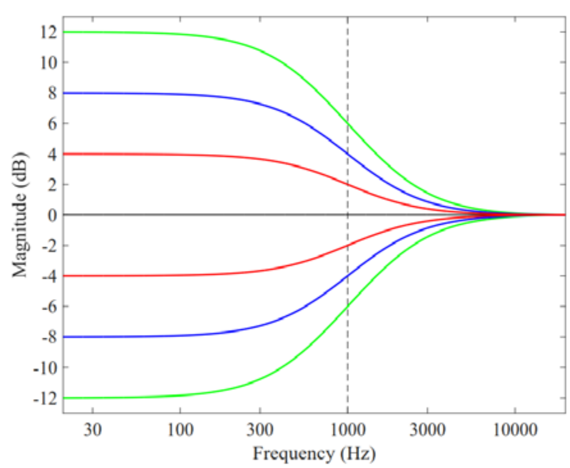
where $b'_0 = G\tan(\omega_c/2) + \sqrt{G},\quad b'_1 = G\tan(\omega_c/2) - \sqrt{G},$

$$a'_0 = \tan(\omega_c/2) + \sqrt{G},\quad a'_1 = \tan(\omega_c/2) - \sqrt{G}$$



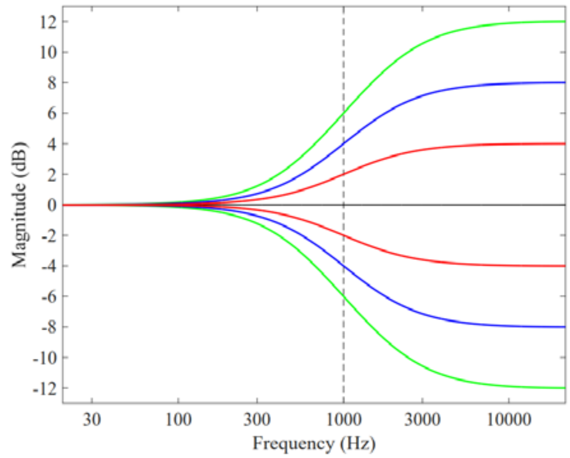
Magnitude Response of Low Shelf

- 1st-order shelf filter
- Notice that the gain at the crossover point is 0.5 times the max gain at dc



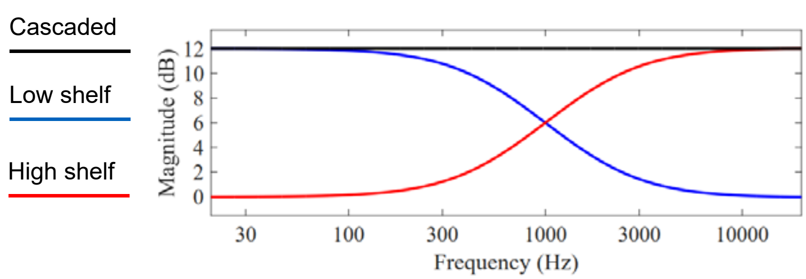
Magnitude Response of <u>High</u> Shelf

- 1^{st}-order shelf filter
- Notice that the gain at the crossover point is 0.5 times the max gain at dc



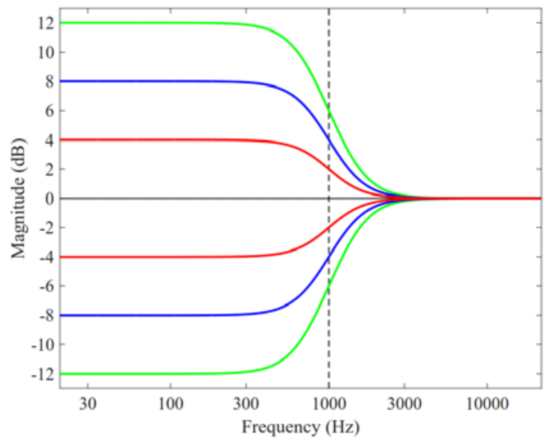
High Shelf Filter

- The 1^{st}-order high shelf filter can be derived by modifying the 1^{st}-order low shelf (Välimäki & Reiss, 2016)
- They can be made complementary (mirror images)



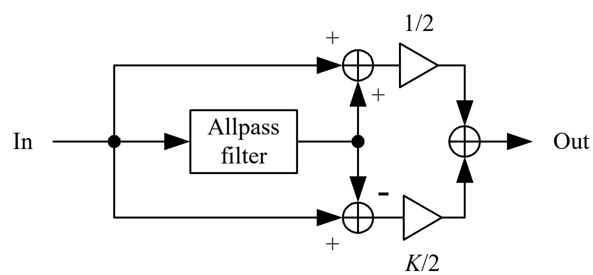
Higher-Order Shelf Filters

- Higher-order shelfs have a steeper transition
- The 2nd-order shelf can be derived by digitizing an analog shelf filter using the bilinear transform (Välimäki & Reiss, 2016)



Special Structure for Shelving and EQ Filters

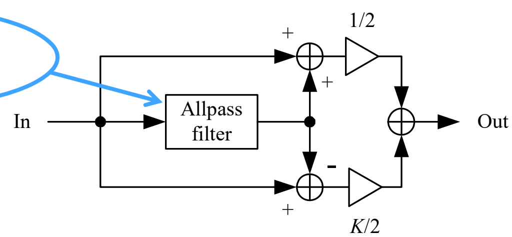
- A clever structure proposed by Regalia & Mitra (1987)
- Suitable for both shelving and equalizing filters
  - Probably common in commercial audio equipment
- Mainly useful in the time-varying case



Special Structure for Shelving and EQ Filters

- When $A(z)$ is a first-order allpass filter, a shelving filter is obtained
  - Transition frequency is determined by the phase response of the allpass filter
  - Gain of the shelf is controlled by parameter K

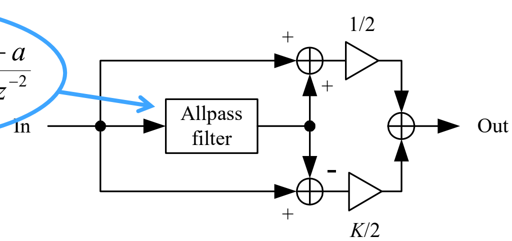
$$A(z) = -\frac{z^{-1} + a}{1 + az^{-1}}$$



Equalizing Filter Structure

- When $A(z)$ is a second-order allpass filter, a peak or notch filter is obtained
  - Parameter a controls bandwidth, b controls center frequency
  - Gain of the peak/notch is controlled by parameter K

$$A(z) = -\frac{z^{-2} + b(1+a)z^{-1} + a}{1 + b(1+a)z^{-1} + az^{-2}}$$



Bass Shelving Filter Design

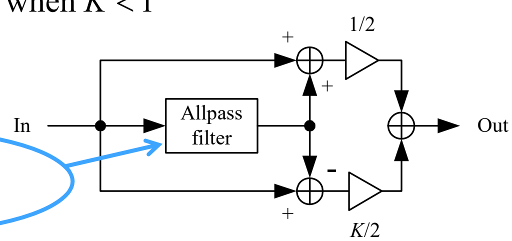
- Parameter *a* depends on cutoff frequency f_0 (in Hz) and gain *G* (in dB) (Regalia & Mitra 1987; Zölzer 1997)

$$a = \begin{cases} \dfrac{\tan(\Omega/2)-1}{\tan(\Omega/2)+1}, \text{ when } K \geq 1 \\ \dfrac{\tan(\Omega/2)-K}{\tan(\Omega/2)+K}, \text{ when } K < 1 \end{cases}$$

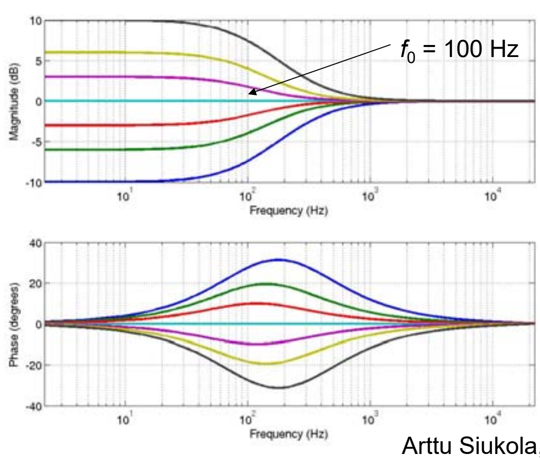
where $\Omega = \dfrac{2\pi f_0}{f_s}$

and $K = 10^{G/20}$

$$A(z) = -\frac{z^{-1}+a}{1+az^{-1}}$$



Bass Control With Shelving Filter



Original

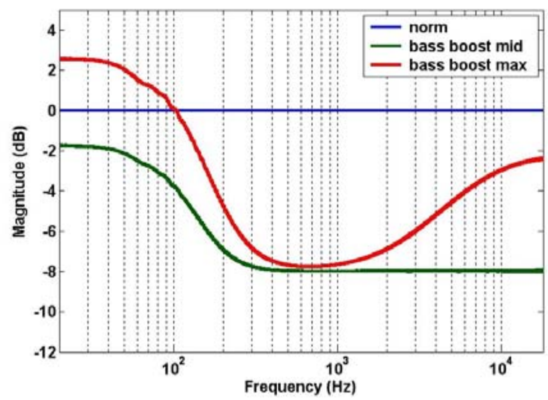
+10 dB

-10 dB

Arttu Siukola, Antti Kelloniemi, TKK, 2004

Practical Example of Tone Control

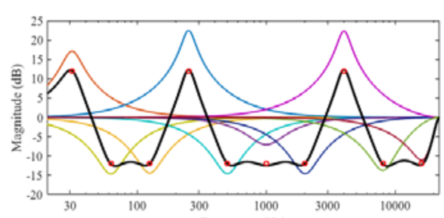
- Sony D-345 portable CD player has 3 options



Arttu Siukola, Antti Kelloniemi, TKK, 2004

Equalizers

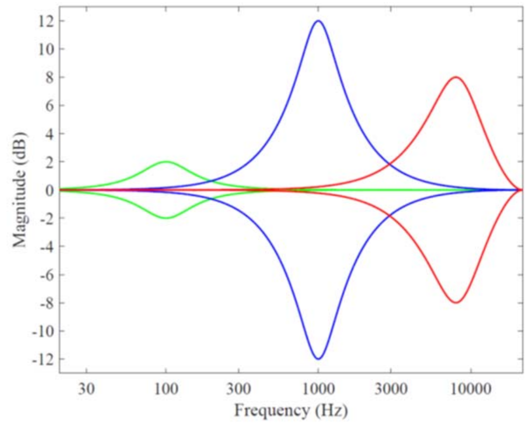
- An equalizer modifies the magnitude response
- Originally used for flattening the response (in telephone systems)
- Often used for flattening the magnitude response of loudspeakers (in an anechoic or listening room)
- Now widely used in audio production for spectral coloration (bumpy – not flat)



(www.universal-radio.com/
used/u023knob.jpg)

Magnitude Response of Parametric EQ

- A peak (boost) or a valley (cut)
  - Adjust gain, center frequency, and bandwidth
  - Elsewhere the gain is about 0 dB



Digital Equalizing Filter

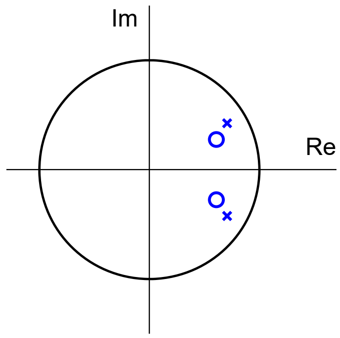
- EQ filter can be designed from gain constraints at 5 points: 0, Nyquist, peak, left and right bandwidth points
- One way is to use the prototype shelf and the lowpass-to-bandpass transformation (Välimäki & Reiss 2016)
- This leads the following 2nd-order transfer function:

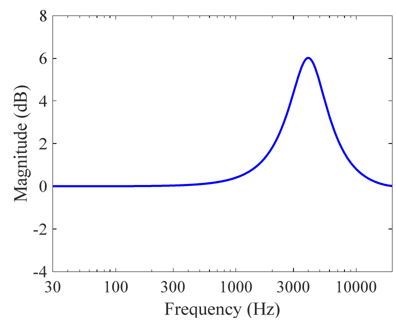
$$H_{\mathrm{PN}}(z) = \frac{\sqrt{G} + G\tan(B/2) - [2\sqrt{G}\cos(\omega_c)]z^{-1} + [\sqrt{(G)} - G\tan(B/2)]z^{-2}}{\sqrt{G} + \tan(B/2) - [2\sqrt{G}\cos(\omega_c)]z^{-1} + [\sqrt{G} - \tan(B/2)]z^{-2}}$$

- This filter is symmetric for G and $1/G$.

Poles & Zeros of an EQ Filter

- The pole and the zero are close to each other to cancel the gain far away from the pole





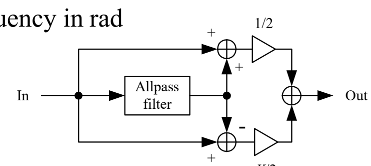
Special Structure for Shelving and EQ Filters

- Closed form formulas are available for *a* and *b* (Regalia & Mitra 1987; Zölzer 1997)

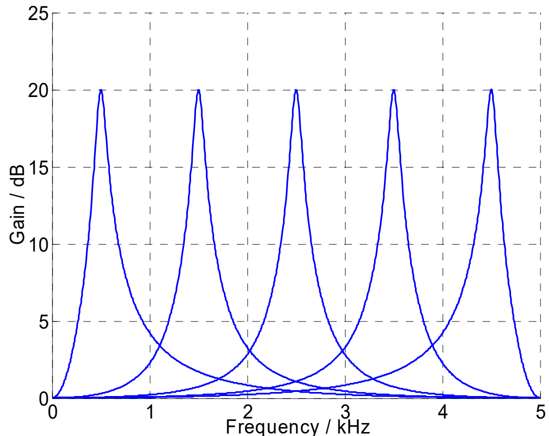
$$a = \begin{cases} \dfrac{1-\tan(\Omega/2)}{1+\tan(\Omega/2)}, \text{when } K \geq 1 \\ \dfrac{K-\tan(\Omega/2)}{K+\tan(\Omega/2)}, \text{when } K < 1 \end{cases} \text{, where } \Omega \text{ is normalized} -3 \text{ dB bandwidth and } K \text{ is gain}$$

$$b = -\cos(\omega_0), \text{ where } \omega_0 \text{ is the center frequency in rad}$$

$$A(z) = -\frac{z^{-2} + b(1+a)z^{-1} + a}{1 + b(1+a)z^{-1} + az^{-2}}$$



Continuous Control of Center Frequency



K = 7, BW = 100 Hz

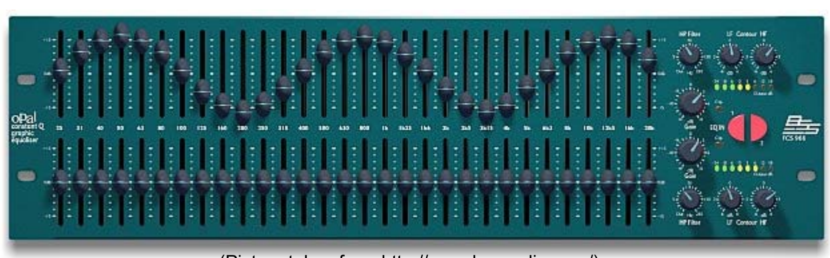
White noise:

f_0 = 300 Hz…6.5 kHz …300 Hz:

Markus Lehtonen, Martti Viljainen, TKK, 2003

Graphic Equalizer

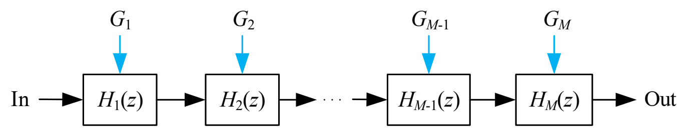
- A set of equalizers with fixed center freq. and Q value
  - One equalizing filter per band for each channel
  - Octave graphic EQ: 10 bands per channel
  - 1/3-oct graphic EQ: ~30 bands per channel



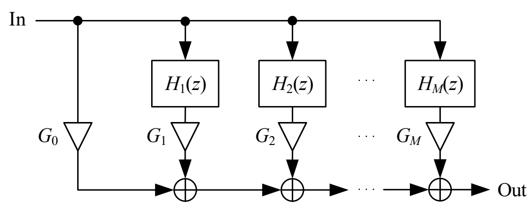
(Picture taken from http://www.bssaudio.com/)

Graphic Equalizer Types

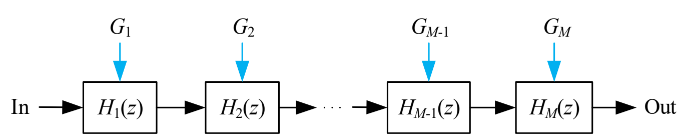
- Cascade structure



- Parallel structure



Cascade Graphic Equalizer

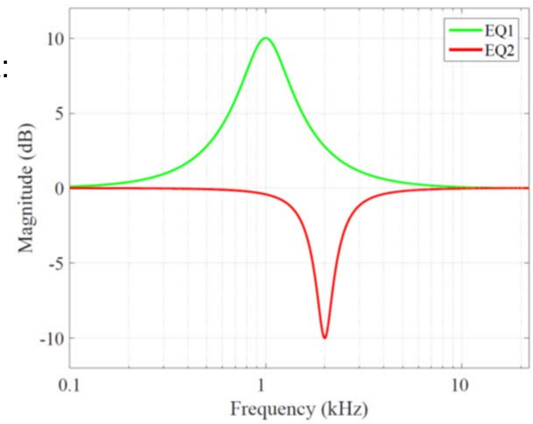


- Each subfilter is an equalizing filter with fixed f_c and Q: Gain *G* at peak, 0 dB elsewhere
- Problem: **interaction** between bands…

Cascade Graphic Equalizer, Ex. #1

- Two EQ filters with at 1 kHz and 2 kHz:

  +10dB and -10dB



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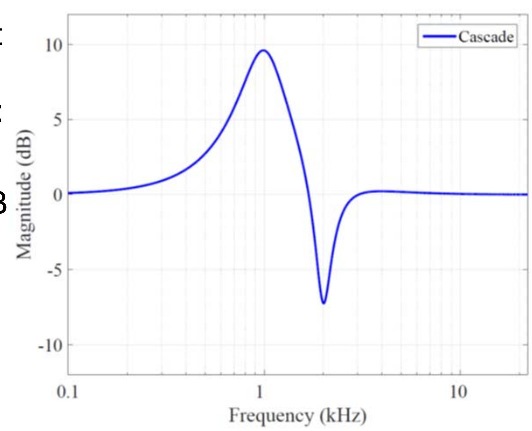
Cascade Graphic Equalizer, Ex. #1

- Responses interact so the overall response is biased:

  +9.6dB and -7.1dB



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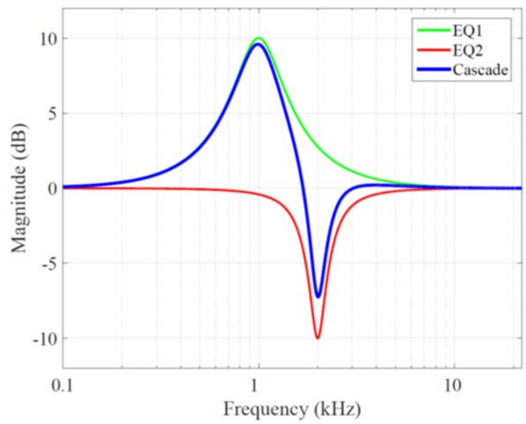
Cascade Graphic Equalizer, Ex. #1

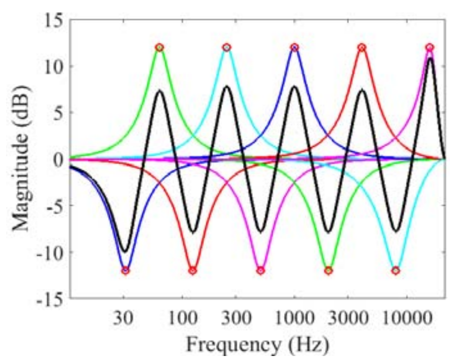
- Interaction problem:

1. Gain of EQ1 at center freq. of EQ2 > 0 dB
2. Gain of EQ2 at center freq. of EQ1 < 0 dB



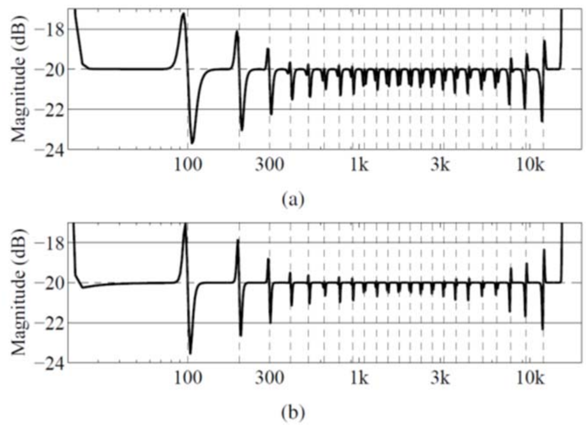
Cascade Graphic EQ for Octave Bands

- Naïve design leads to severe approximation problems
  - Filter gains = command gains



High-Order Cascade Graphic EQ

- Bark-band graphic EQ
  - (a)16th-order EQ filters (total order: 384)
  - (b) 28th-order EQ filters (672)
- **No interaction** between band filters!
- A new problem: **Ripple** at band edges



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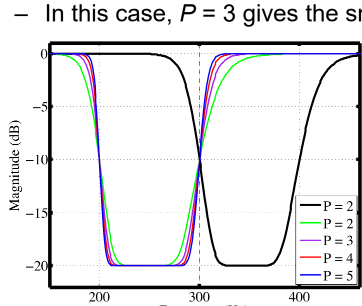
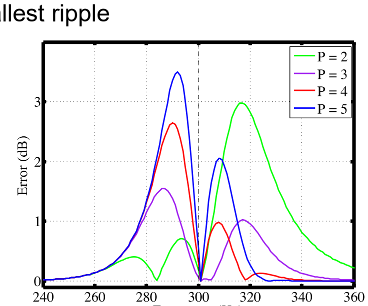
Optimizing High-Order Graphic EQ

- Ripple between bands is reduced, if the neighboring filters are symmetric (Rämö & Välimäki 2014)
- Iteratively choose the best orders P, e.g. $P < 5$
  - In this case, $P = 3$ gives the smallest ripple





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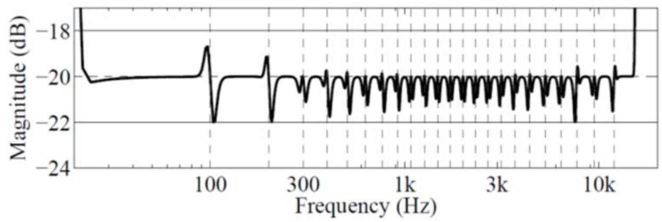
Optimized High-Order Cascade GEQ

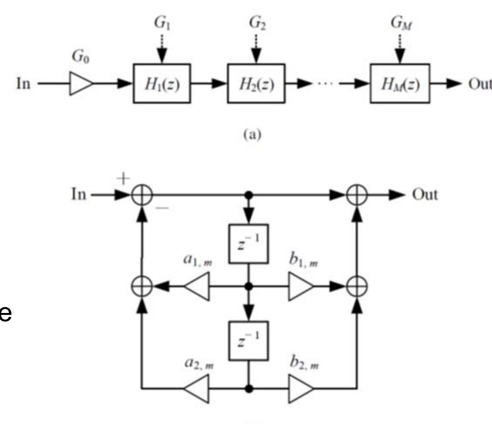
- Different filter order *P* selected for each band:
  28, 20, 16, 12, 12, 12, …, 12, 16, 20
  (24 bands, total order 328)
- **Ripple** reduced to < 2 dB



(Rämö &Välimäki 2014)

Novel Cascade Graphic EQ Design

- It is desirable to use 2nd order band filters
- The most accurate cascade design uses different peak filter shapes than previously (Välimäki & Liski 2017)
- The magnitude response is optimized at center frequencies and intermediate frequencies



Ref: Välimäki & Liski, IEEE SPL, 2017

Second-order cell $H_k(z)$

Cascade GEQ Based on 2nd-Order Filters

- It is possible to measure the interaction and form an interaction matrix (Abel & Berners 2004; Oliver & Jot 2015)
- Interaction matrix **B** shows how much each band filter leaks to neighboring bands:
  - Normalized so it shows how much 1 dB of gain affects other bands

Filter ↓

$$\mathbf{B} = \begin{bmatrix} 0.80 & 0.23 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.19 & 1 & 0.21 & 0.04 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0.21 & 1 & 0.20 & 0.04 & 0.01 & 0 & 0 & 0 & 0 \\ 0.01 & 0.04 & 0.20 & 1 & 0.20 & 0.04 & 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0.04 & 0.20 & 1 & 0.20 & 0.04 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0.04 & 0.20 & 1 & 0.20 & 0.04 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 & 0.04 & 0.20 & 1 & 0.20 & 0.03 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0.04 & 0.21 & 1 & 0.18 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0.01 & 0.06 & 0.25 & 1 & 0.10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0.14 & 0.94 \end{bmatrix}$$

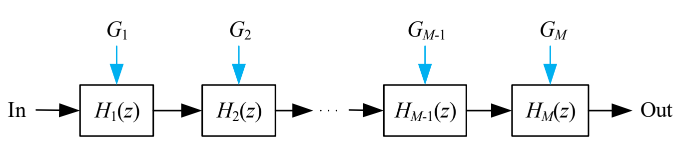
Frequency band →

Cascade GEQ Based on 2nd-Order Filters

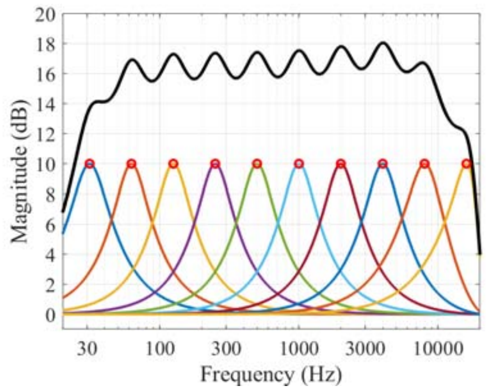
- Use inverse matrix of **B** to solve the optimal dB gains in the least squares (LS) sense:

$$\mathbf{g}_{\mathrm{opt}} = \mathbf{B}^{-1}\mathbf{g}$$

- After optimization: filter gains ≠ command gains



Octave Equalizer: Trivial Design



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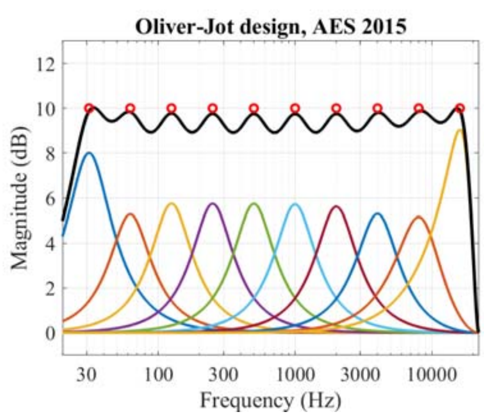
Octave Equalizer: Oliver & Jot, 2015



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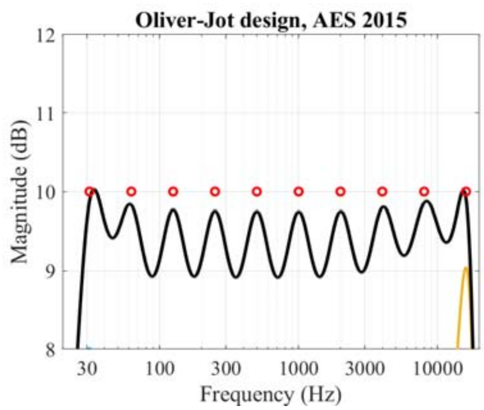
Octave Equalizer: Oliver & Jot, 2015



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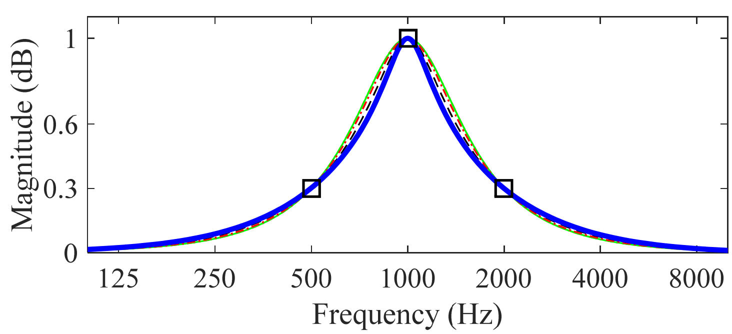
Cascade GEQ with Variable-Q Band Filters

- It is possible to force the response ”proportional” at its f_c and the 2 neighboring f_c’s (Välimäki & Liski 2017)



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Orfanidis Peak Filter

- We use the following peak filter

$$H(z) = \frac{1 + G\beta - 2\cos(\omega_c)z^{-1} + (1 - G\beta)z^{-2}}{1 + \beta - 2\cos(\omega_c)z^{-1} + (1 - \beta)z^{-2}},$$

where G is the peak gain, ω_c is the center frequency,

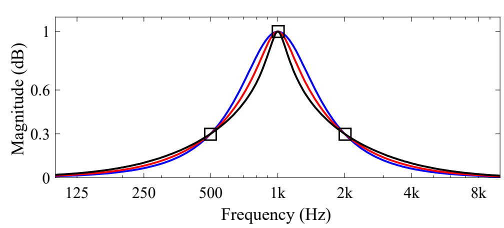
$$\beta = \begin{cases} \tan(B/2), & \text{when } G = 1 \\ \sqrt{\frac{|G_B^2 - 1|}{|G^2 - G_B^2|}} \tan\left(\frac{B}{2}\right), & \text{otherwise} \end{cases}$$

- You can choose the bandwidth definition: G_B is the gain at bandwidth B
- Original design by Orfanidis (2010)

Parametric Filters for Octave Design

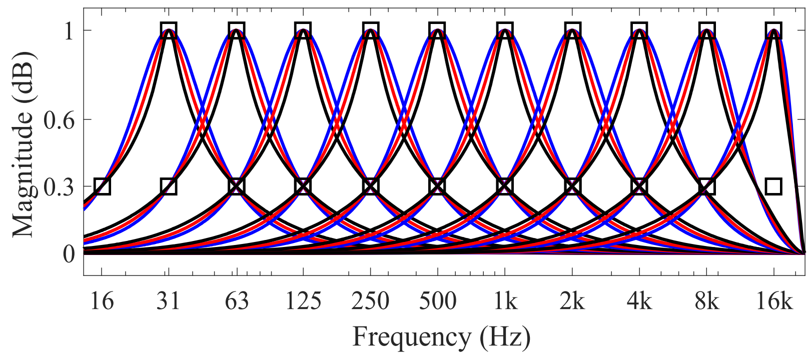
(Välimäki and Liski, IEEE SPL 2017)

- Inspired by designs by Abel and Berners (ICMC 2004) and by Oliver and Jot (AES 2015)
- We set $G_B = cG$, where $c = 0.3$ and B = difference of center freqs. on each side (e.g., 500 Hz and 2 kHz)



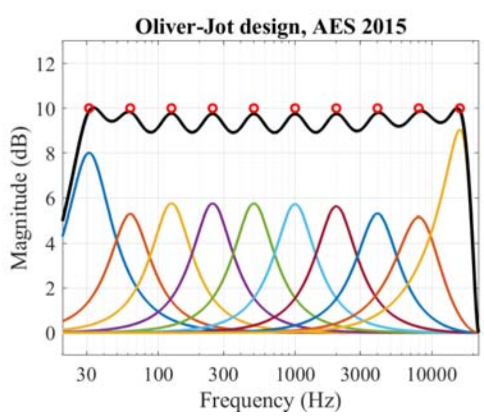
Orfanidis, *Introduction to Signal Processing*, 2010.

Octave-Band Filters with Gain Normalized to 1 dB

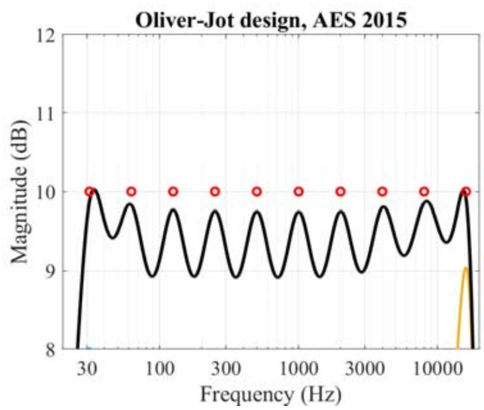


- Filters with different gains (3 dB, 17 dB, 24 dB) are self-similar meeting at 3 points: own center freq & 2 adjacent center freqs

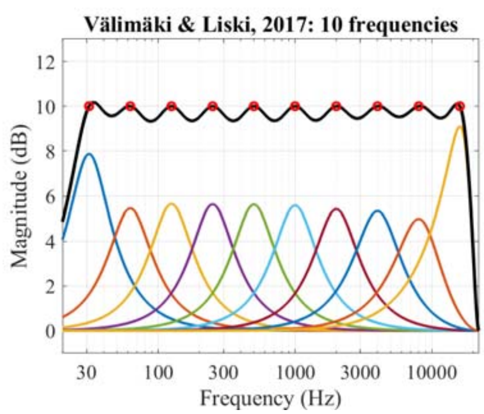
Octave Equalizer: Oliver & Jot, 2015



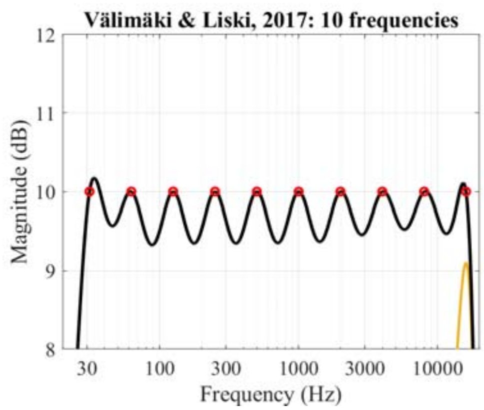
Octave Equalizer: Oliver & Jot, 2015



Octave Equalizer: Välimäki & Liski, 2017



Octave Equalizer: Välimäki & Liski, 2017



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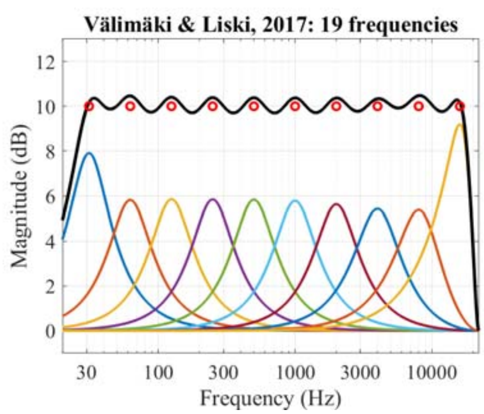
Octave Equalizer: Välimäki & Liski, 2017



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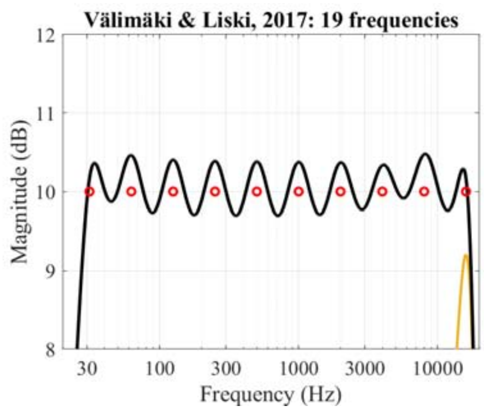
Octave Equalizer: Välimäki & Liski, 2017



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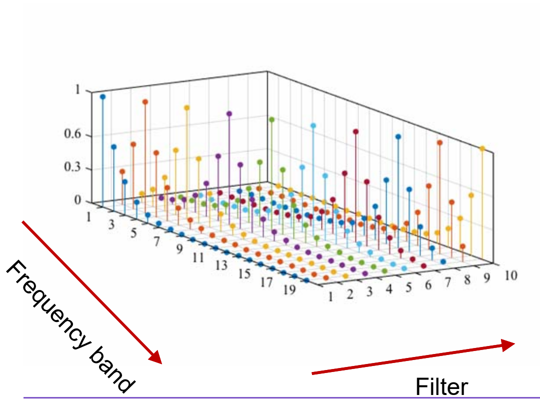
Graphic Equalizer Design Using Interaction Matrix

Interaction matrix **B**



Optimize dB gains in least-squares sense using pseudoinverse:

$$\mathbf{g} = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{t}_1$$

where

$\mathbf{t}_1$ = target gains (dB)

$\mathbf{B}$ = interaction matrix

$\mathbf{g}$ = filter gains (dB)

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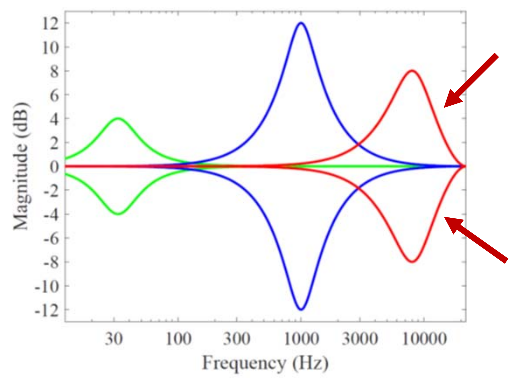
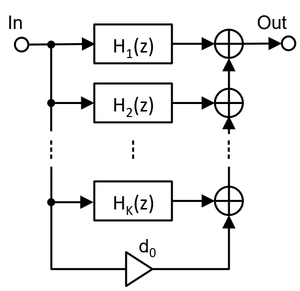
Asymmetry at High Center Frequency

- A remaining (minor) problem in graphic equalizers at high frequencies

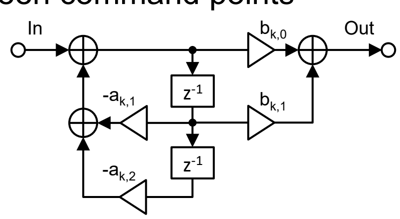


Parallel Graphic EQ Structure

- Use the parallel IIR filter structure, with fixed poles, proposed by B. Bank (2008)
- Use extra band filters between command points



Parallel Structure

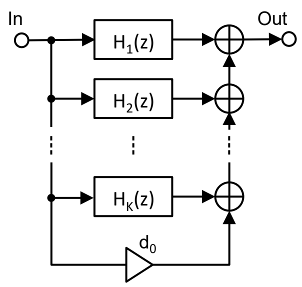


Second-order block $H_k(z)$

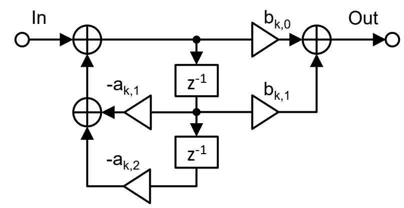
Ref: Rämö *et al.* IEEE Tr. ASL, 2014

Parallel Graphic EQ Design

- Poles can be set first (log freq. distribution)
- Least squares design of coefficients $b_{k,0}$ and $b_{k,1}$



Parallel Structure

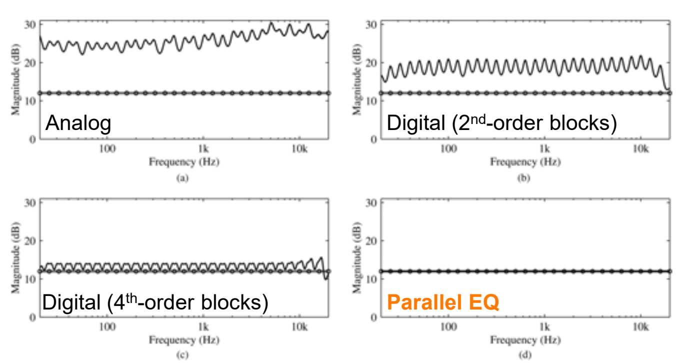


Second-order block $H_k(z)$

Ref: Rämö *et al*. IEEE Tr. ASL, 2014

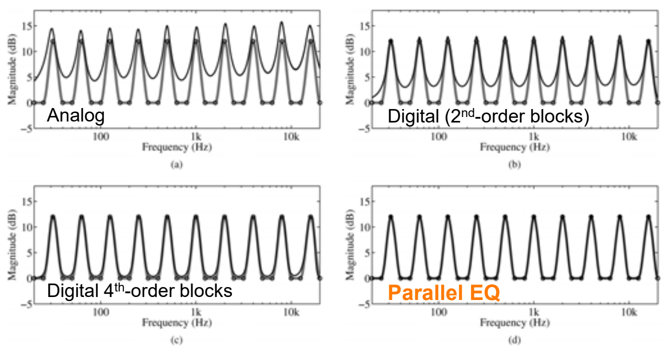
Graphic EQ Response Example #1

- Constant +12 dB command gain at all bands – surprisingly hard!



Graphic EQ Response Example #2

- Every 3rd gain at +12 dB, others at 0 dB – known to be difficult



Conclusion

- Basic building blocks of equalizers: shelving filters and parametric equalizing filters (peak/notch)
- Graphic equalizer design is more difficult than it seems
  - Bias caused by band interactions
  - Ripple in the magnitude response
- Several graphic equalizer designs are available
  - Optimized high-order cascade graphic equalizer (Rämö & Välimäki 2014)
  - New parallel graphic equalizer (Rämö, Bank, Välimäki 2014)
  - New cascade graphic equalizer (Välimäki & Liski 2017)

References

- J. S. Abel and D. P. Berners, “Filter design using second-order peaking and shelving sections,” in *Proc. Int. Computer Music Conf.*, Miami, FL, USA, Nov. 2004.
- B. Bank, "Perceptually motivated audio equalization using fixed-pole parallel second-order filters," *IEEE Signal Processing Letters*, vol. 15, pp. 477-480, 2008.
- R. J. Oliver and J.-M. Jot, “Efficient multi-band digital audio graphic equalizer with accurate frequency response control,” in *Audio Engineering Society 139th Convention*, New York, NY, USA, Oct. 2015.
- S. J. Orfanidis, *Introduction to Signal Processing*. Piscataway, NJ, USA: Rutgers Univ., 2010.
- P. A. Regalia and S. K. Mitra, “Tunable digital frequency response equalization filters,” *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 35, no. 1, pp. 118–120, Jan. 1987
- J. Rämö and V. Välimäki, “Optimizing a high-order graphic equalizer for audio processing,” *IEEE Signal Processing Letters*, vol. 21, no. 3, pp. 301–305, Mar. 2014.
- J. Rämö, V. Välimäki, B. Bank, “High-precision parallel graphic equalizer,” *IEEE/ACM Transactions on Audio, Speech and Language Process*ing, vol. 22, no. 12, pp. 1894–1904, Dec. 2014.
- V. Välimäki and J. D. Reiss, “All about audio equalization: Solutions and frontiers,” *Applied Sciences*, vol. 6, no. 5, p. 129, May 2016. URL: http://www.mdpi.com/2076-3417/6/5/129
- V. Välimäki and J. Liski, “Accurate cascade graphic equalizer,” *IEEE Signal Processing Letters*, vol. 24, no. 2, pp. 176–180, Feb. 2017.