DYAD $\quad \bar{a}\bar{b}$

$$(\bar{a}\bar{b})\cdot\bar{c} = \bar{a}(\bar{b}\cdot\bar{c}) \qquad \bar{c}\cdot(\bar{a}\bar{b}) = \bar{c}\cdot\bar{a}\,\bar{b}$$

$$\bar{a}(\bar{b}+\bar{c}) = \bar{a}\bar{b} + \bar{a}\bar{c}$$

$$\bar{\bar{I}}\cdot\bar{f} = \bar{f}$$

$$(\bar{a}+\beta\bar{b})\bar{c} = \bar{a}\bar{c} + \beta\bar{b}\bar{c}$$

$$\bar{\bar{I}} = \bar{a}\bar{a}' + \bar{b}\bar{b}' + \bar{c}\bar{c}'$$

$$\sum_{i=1}^{100} \bar{r}_i\bar{s}_i = \sum \bar{\bar{I}}\cdot\bar{r}_i\bar{s}_i$$

$$\bar{a}' = \frac{\bar{b}\times\bar{c}}{\bar{a}\times\bar{b}\cdot\bar{c}}$$

$$= \sum^{100}(\bar{a}\bar{a}' + \bar{b}\bar{b}' + \bar{c}\bar{c}')\cdot\bar{r}_i\bar{s}_i$$

$$= \bar{a}\underbrace{\sum(\bar{a}'\cdot\bar{r}_i)\bar{s}_i}_{\bar{d}} + \bar{b}\underbrace{\sum(\bar{b}'\cdot\bar{r}_i)\bar{s}_i}_{\bar{e}} + \bar{c}\underbrace{\sum(\bar{c}'\cdot\bar{r}_i)\bar{s}_i}_{\bar{f}}$$

$$= \bar{a}\bar{d} + \bar{b}\bar{e} + \bar{c}\bar{f}$$

$$\underbrace{(\bar{a}+\bar{b})}_{\bar{e}}\underbrace{(\bar{c}+\bar{d})}_{\bar{f}} = \bar{a}\bar{c} + \bar{a}\bar{d} + \bar{b}\bar{c} + \bar{b}\bar{d} = \bar{e}\bar{f}$$

SYMMETRIC DYADIC

$$\bar{\bar{A}}^T = \bar{\bar{A}}$$

TRANSPOSE OPERATION

$$(\bar{a}\bar{b})^T = \bar{b}\bar{a}$$

ANTISYMMETRIC DYADIC

$$\bar{\bar{A}}^T = -\bar{\bar{A}}$$

$$\begin{aligned}
\bar{a} \times (\bar{b} \times \bar{c}) &= \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b}) \\
&= \bar{b}\bar{a}\cdot\bar{c} - \bar{c}\bar{a}\cdot\bar{b} \\
&= \bar{b}\bar{c}\cdot\bar{a} - \bar{c}\bar{b}\cdot\bar{a} \\
&= (\bar{b}\bar{c} - \bar{c}\bar{b})\cdot\bar{a} \\
&= (\bar{c}\times\bar{b})\times\bar{\bar{I}}\cdot\bar{a}
\end{aligned}$$

$$\bar{h} \times \bar{\bar{I}}$$

$$\begin{aligned}
\bar{h} = \bar{u}_z \Rightarrow \bar{h}\times\bar{\bar{I}} &= \bar{u}_z \times (\bar{u}_x\bar{u}_x + \bar{u}_y\bar{u}_y + \bar{u}_z\bar{u}_z) \\
&= \bar{u}_y\bar{u}_x - \bar{u}_x\bar{u}_y
\end{aligned}$$

$$\bar{h}\times\bar{\bar{I}}\cdot\bar{s} = \bar{h}\times\bar{s}$$

$$\bar{s}\cdot\bar{h}\times\bar{\bar{I}} = \bar{s}\times\bar{h}$$

$$\cancel{\bar{h}\times\bar{s}}$$

$$\begin{aligned}
\bar{s}\cdot\bar{h}\times\bar{\bar{I}} &= (\bar{h}\times\bar{\bar{I}})^T\cdot\bar{s} \\
&= -\bar{h}\times\bar{\bar{I}}\cdot\bar{s} = -\bar{h}\times\bar{s} = \bar{s}\times\bar{h}
\end{aligned}$$

$$\begin{aligned}
&\bar{\bar{A}}\cdot\bar{a} \\
&= \bar{a}\cdot\bar{\bar{A}}^T
\end{aligned}$$

$$\bar{h}\times\bar{\bar{I}} = \bar{\bar{I}}\times\bar{h}$$

$$\begin{aligned}
\bar{u}_z\times\bar{\bar{I}} &= \bar{u}_y\bar{u}_x - \bar{u}_x\bar{u}_y \\
\bar{\bar{I}}\times\bar{u}_z &= (\bar{u}_x\bar{u}_x + \bar{u}_y\bar{u}_y + \bar{u}_z\bar{u}_z)\times\bar{u}_z \\
&= -\bar{u}_x\bar{u}_y + \bar{u}_y\bar{u}_x
\end{aligned}$$

$$(\bar{a}\bar{b})^T = \bar{a}\bar{b} = \bar{b}\bar{a} \qquad \bar{a} = \alpha\bar{b}$$

ANTISYMMETRIC DYAD

$$(\bar{a}\bar{b})^T = \bar{b}\bar{a} = -\bar{a}\bar{b} \qquad \bar{a} = \bar{0} \text{ OR } \bar{b} = \bar{0}$$

$$\bar{\bar{A}} = \underbrace{\frac{\bar{\bar{A}} + \bar{\bar{A}}^T}{2}}_{\text{SYMMETRIC}} + \underbrace{\frac{\bar{\bar{A}} - \bar{\bar{A}}^T}{2}}_{\text{ANTISYMMETRIC}} \qquad (\bar{\bar{A}}^{TT} = \bar{\bar{A}})$$

$$(\bar{\bar{A}}\cdot\bar{\bar{B}})^T = \bar{\bar{B}}^T\cdot\bar{\bar{A}}^T$$

$$\bar{\bar{A}} \downarrow \bar{a}\bar{b} \qquad \bar{\bar{B}} \downarrow \bar{c}\bar{d}$$

$$(\bar{a}\bar{b}\cdot\bar{c}\bar{d})^T = \bar{b}\cdot\bar{c}\,\bar{d}\bar{a} = \bar{d}\,\bar{b}\cdot\bar{c}\,\bar{a}$$
$$= \bar{d}\bar{c}\cdot\bar{b}\bar{a}$$

$$\bar{\bar{A}}\cdot\bar{\bar{A}}^{-1} = \bar{\bar{A}}^{-1}\cdot\bar{\bar{A}} = \bar{\bar{I}}$$

$$(\bar{\bar{A}}\cdot\bar{\bar{B}})^{-1} = \bar{\bar{B}}^{-1}\cdot\bar{\bar{A}}^{-1}$$

$$(\bar{\bar{A}}\cdot\bar{\bar{B}})^{-1}\cdot\bar{\bar{A}}\cdot\bar{\bar{B}} = \bar{\bar{I}}$$

$$\bar{\bar{B}}^{-1}\cdot\bar{\bar{A}}^{-1}\cdot\bar{\bar{A}}\cdot\bar{\bar{B}} = \bar{\bar{B}}^{-1}\cdot\bar{\bar{I}}\cdot\bar{\bar{B}} = \bar{\bar{I}}$$

$$\bar{\bar{A}} \cdot (\bar{\bar{B}} \cdot \bar{C}) = (\bar{\bar{A}} \cdot \bar{\bar{B}}) \cdot \bar{C} \qquad \text{(ASSOC.)}$$

NON - COMMUTATIVE : $\bar{\bar{A}} \cdot \bar{\bar{B}} \neq \bar{\bar{B}} \cdot \bar{\bar{A}}$

$$\bar{\bar{A}} \cdot \bar{\bar{B}} = (\bar{\bar{B}}^T \cdot \bar{\bar{A}}^T)^T$$

$$(\bar{u}_x + \bar{u}_y)(\bar{u}_x + \bar{u}_y) \cdot (\bar{u}_y + \bar{u}_z)(\bar{u}_y + \bar{u}_z)$$

$$= (\bar{u}_x + \bar{u}_y)(\bar{u}_y + \bar{u}_z)$$

$$(\bar{u}_y + \bar{u}_z)(\bar{u}_y + \bar{u}_z) \cdot (\bar{u}_x + \bar{u}_y)(\bar{u}_x + \bar{u}_y)$$

$$= (\bar{u}_y + \bar{u}_z)(\bar{u}_x + \bar{u}_y)$$

$$\bar a \bar b : \bar c \bar d = (\bar a \cdot \bar c)(\bar b \cdot \bar d) \qquad \bar{\bar A} : \bar{\bar B}$$

$$\bar a \bar b \mathbin{\overset{\times}{\times}} \bar c \bar d = (\bar a \times \bar c)(\bar b \times \bar d) \qquad \bar{\bar A} \mathbin{\overset{\times}{\times}} \bar{\bar B}$$

$$\bar a \bar b \mathbin{\overset{\times}{\cdot}} \bar c \bar d = (\bar a \times \bar c)(\bar b \cdot \bar d)$$

$$\bar a \bar b : \bar{\bar I} = \bar a \cdot \bar b \qquad \bar{\bar A} : \bar{\bar I} = \mathrm{tr}\, \bar{\bar A}$$

$$\bar{\bar A} : \bar{\bar B} = \mathrm{tr}(\bar{\bar A} \cdot \bar{\bar B}^T) = \bar{\bar A} \cdot \bar{\bar B}^T : \bar{\bar I}$$

$$\bar a \bar b : \bar c \bar d = \bar a \cdot \bar c \; \bar b \cdot \bar d$$

$$\bar a \bar b \mathbin{\overset{\times}{\times}} \bar c \bar d = \bar a \times \bar c \; \bar b \times \bar d$$

$$= \bar c \bar d \mathbin{\overset{\times}{\times}} \bar a \bar b$$

$$\bar{\bar A} \mathbin{\overset{\times}{\times}} \bar{\bar B} = \bar{\bar B} \mathbin{\overset{\times}{\times}} \bar{\bar A}$$

$$(\bar{\bar A} \mathbin{\overset{\times}{\times}} \bar{\bar B}) \mathbin{\overset{\times}{\times}} \bar{\bar C} \neq \bar{\bar A} \mathbin{\overset{\times}{\times}} (\bar{\bar B} \mathbin{\overset{\times}{\times}} \bar{\bar C})$$

$$\bar{\bar{A}} \overset{\times}{\times} \left(\bar{\bar{B}} \overset{\times}{\times} \bar{\bar{C}}\right)$$

$$\bar a \bar b \overset{\times}{\times} \left(\bar c \bar d \overset{\times}{\times} \bar e \bar f\right) = \bar a \bar b \overset{\times}{\times} (\bar c \times \bar e)(\bar d \times \bar f)$$

$$= \bar a \times (\bar c \times \bar e)\;\; \bar b \times (\bar d \times \bar f)$$

$$= (\bar c\; \bar a\cdot\bar e - \bar e\; \bar a\cdot\bar c)(\bar d\; \bar b\cdot\bar f - \bar f\; \bar b\cdot\bar d)$$

$\bar a\bar b : \bar c\bar d$

$$= \bar c\bar d\, \underbrace{(\bar a\cdot\bar e)(\bar b\cdot\bar f)}_{\bar a\bar b\,:\,\bar e\bar f} + \bar e\bar f\,(\bar a\cdot\bar c)(\bar b\cdot\bar d)$$

$$- \underbrace{\bar e\bar d\,(\bar a\cdot\bar c)(\bar b\cdot\bar f)}_{\bar e\bar f\cdot\bar b\bar a\cdot\bar c\bar d} - \bar c\bar f\,(\bar a\cdot\bar e)(\bar b\cdot\bar d)$$

$\neq$

$\bar c\bar d\cdot\bar b\bar a\cdot\bar e\bar f$

$$\bar{\bar{A}} \overset{\times}{\times} \left(\bar{\bar{B}} \overset{\times}{\times} \bar{\bar{C}}\right) = \left(\bar{\bar{A}} : \bar{\bar{C}}\right)\bar{\bar{B}} + \left(\bar{\bar{A}} : \bar{\bar{B}}\right)\bar{\bar{C}}$$

$$- \bar{\bar{C}}\cdot\bar{\bar{A}}^{T}\cdot\bar{\bar{B}} - \bar{\bar{B}}\cdot\bar{\bar{A}}^{T}\cdot\bar{\bar{C}}$$

$$\mathrm{tr}\,\bar{\bar{A}} = \bar{\bar{A}} : \bar{\bar{I}}$$

$$\bar{\bar{A}} \cdot \bar{\bar{A}} = \bar{\bar{A}}^{2}$$

$$\bar{\bar{A}}^{(2)} = \frac{1}{2}\, \bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}}$$

$$\mathrm{spm}\,\bar{\bar{A}} = \mathrm{tr}\,\bar{\bar{A}}^{(2)} = \frac{1}{2}\, \bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{I}}$$

$$\det \bar{\bar{A}} = \frac{1}{6}\, \bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{A}}$$

$$\bar{\bar{A}}^{-1} = \frac{\bar{\bar{A}}^{(2)T}}{\det \bar{\bar{A}}} = 3\, \frac{\left(\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}}\right)^{T}}{\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{A}}}$$

EXAMPLE: ANTISYMMETRIC

$$\bar{u}_z \times \bar{\bar{I}} = \bar{u}_y \bar{u}_x - \bar{u}_x \bar{u}_y$$

$$\mathrm{tr}\, \bar{u}_z \times \bar{\bar{I}} = 0$$

$$\left(\bar{u}_y \bar{u}_x - \bar{u}_x \bar{u}_y\right) \overset{\times}{\times} \left(\bar{u}_y \bar{u}_x - \bar{u}_x \bar{u}_y\right)$$

$$= 2\, \bar{u}_z \bar{u}_z \qquad \Rightarrow \mathrm{spm}\left(\bar{u}_z \times \bar{\bar{I}}\right) = 1$$

$$2\, \bar{u}_z \bar{u}_z : \left(\bar{u}_z \times \bar{\bar{I}}\right) = 0 \qquad \Rightarrow \det\left(\bar{u}_z \times \bar{\bar{I}}\right) = 0$$