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Brief Recap of Chapter 8 of Brown et al. (2014)

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Contents of Chapter 8 ”Introductory Signal Acquisition Methods: Free Induction Decay, Spin Echoes, Inversion Recovery, and Spectroscopy”

Introduction

- **This chapter considers how to design rf pulse sequences:**
  1. Single pulse which induces collective precession
  2. Pair of rf pulses resulting in ‘echo’ which cancels out T_2’ relaxation
  3. Repeating various combinations of rf pulses
  4. ‘Inversion recovery’ pulses for determining T_1
- **Sequence diagram is introduced**
- **Spectroscopy is also introduced**

Free Induction Decay (FID)

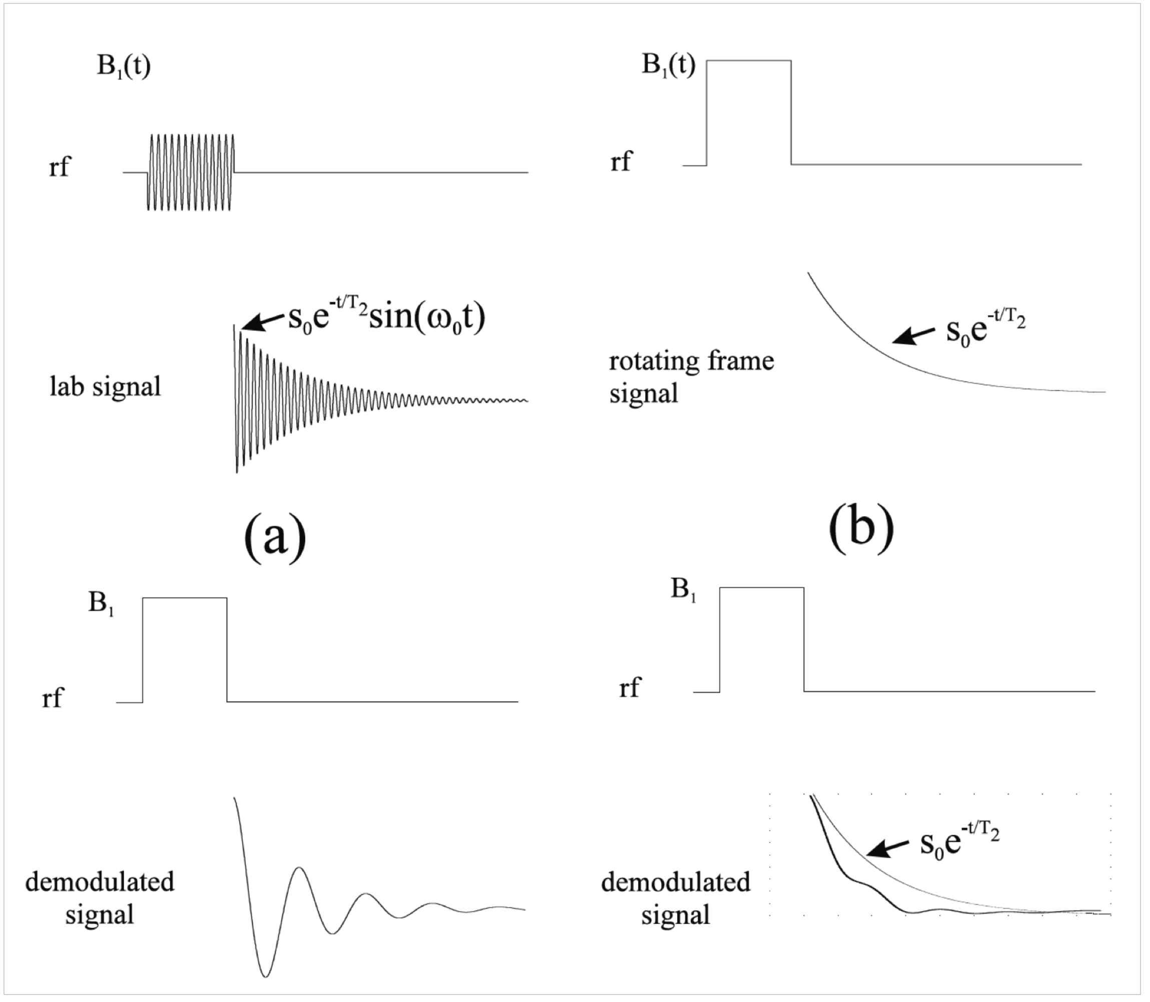
- **We can apply single π/2 pulse to the sample**
- **Rotates longitudinal magnetization to transverse plane**
- **Reveals the free induction decay (FID):**

$$s(t) \quad \propto \quad \omega_0 e^{-t/T_2} e^{i((\Omega-\omega_0)t+\phi_0-\theta_{\mathcal{B}})} \int d^3 r \mathcal{B}_\perp(\vec{r}) M_\perp(\vec{r},0)$$

$$\phi(\vec{r},t) = -\omega(\vec{r})t + \phi_0(\vec{r}) = -\gamma B_z(\vec{r})t + \phi_0(\vec{r})$$

- **In lab, we then see a FID signal of the form:**
  $s(t) = s_0 \exp(-t/T_2) \sin(\omega_0 t)$.
- **In rotating coordinate system this is just a signal**
  $s(t) = s_0 \exp(-t/T_2)$.

Free Induction Decay (FID)



T_2^* Decay

- **Due to magnetic field inhomogeneities we actually see**
  $s(t) = s_0 \exp(-t/T_2^*)$.
- **Which results from the magnetization**

$$M_\perp(\vec{r},t) = M_\perp(\vec{r},0)e^{-t/T_2^*}$$

where

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

- **The T_2' is a result of collective effect of dephasing:**

$$\sum_{\text{sample}} e^{i\phi(\vec{r},t)} \underset{\text{dephasing}}{\longrightarrow} 0$$

- **The phase accumulation at each position has the form**

$$\phi(\vec{r},t) = -\gamma\left(B_0 + \Delta B(\vec{r})\right)t$$

Sequence of FID experiments

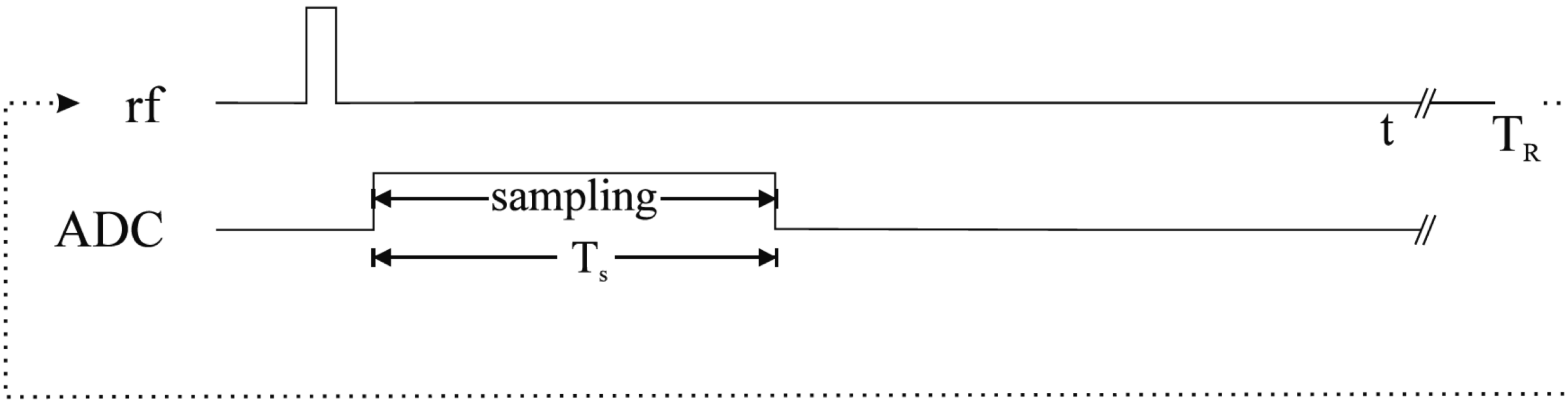
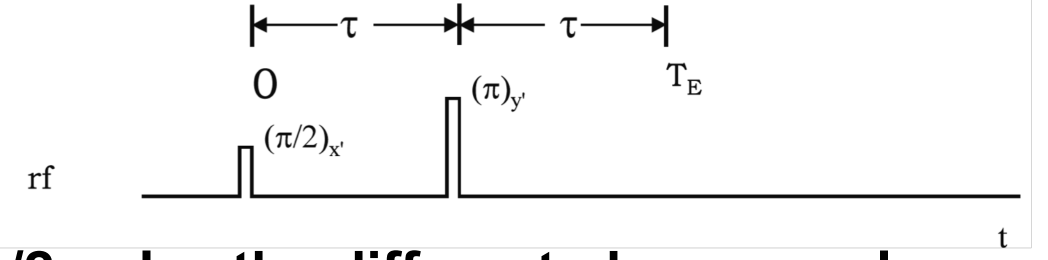


Fig. 8.2: Sequence diagram for a repeated FID experiment. Repetition of the rf pulse and sampling, with repeat time T_R, is indicated by the dotted line and arrow in this figure, but the fact that the process, or 'cycle,' is usually repeated is left understood in most diagrams. The ADC line represents the activity of the analog-to-digital converter, which is the device used to sample the signal over time T_s.

Spin-echo sequence

- **The spin-echo sequence uses two pulses:**
  1. $\pi/2$ pulse which flips the magnetization around x’ axis
  2. π pulse around y’ axis which refocuses the spins



- **After the $\pi/2$ pulse the different phases evolve as:**

$$\phi(\vec{r},t) = -\gamma \Delta B(\vec{r})\, t \qquad \text{for } 0 < t < \tau$$

- **The π pulse negates the phases and they become:**

$$\begin{aligned} \phi(\vec{r},\tau^{+}) &= -\phi(\vec{r},\tau^{-}) \\ &= \gamma \Delta B(\vec{r})\, \tau \end{aligned}$$

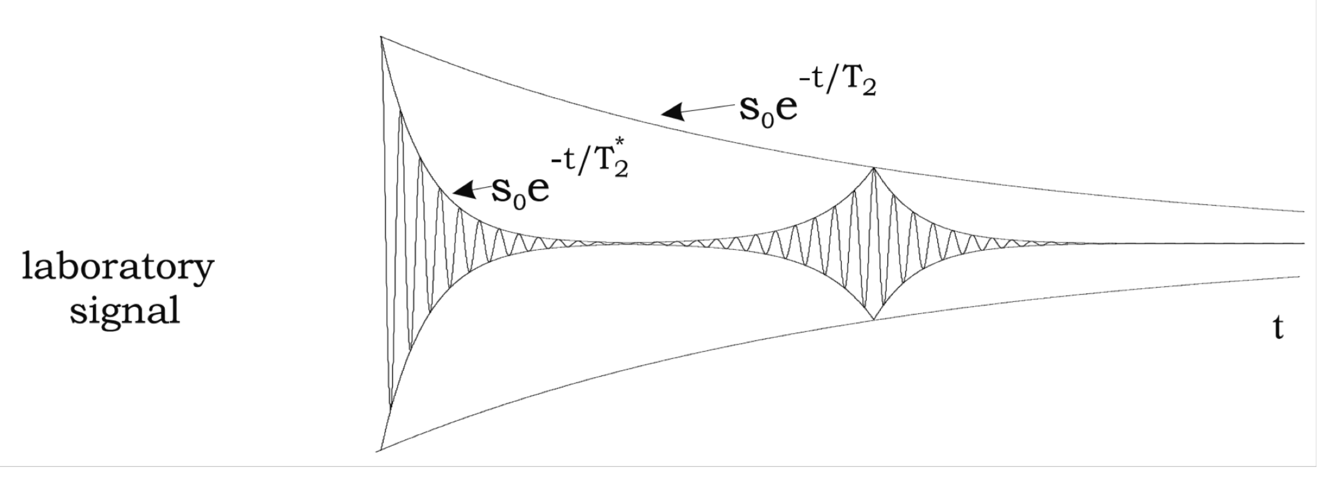
Spin-echo sequence

- **The phases then continue evolving as:**

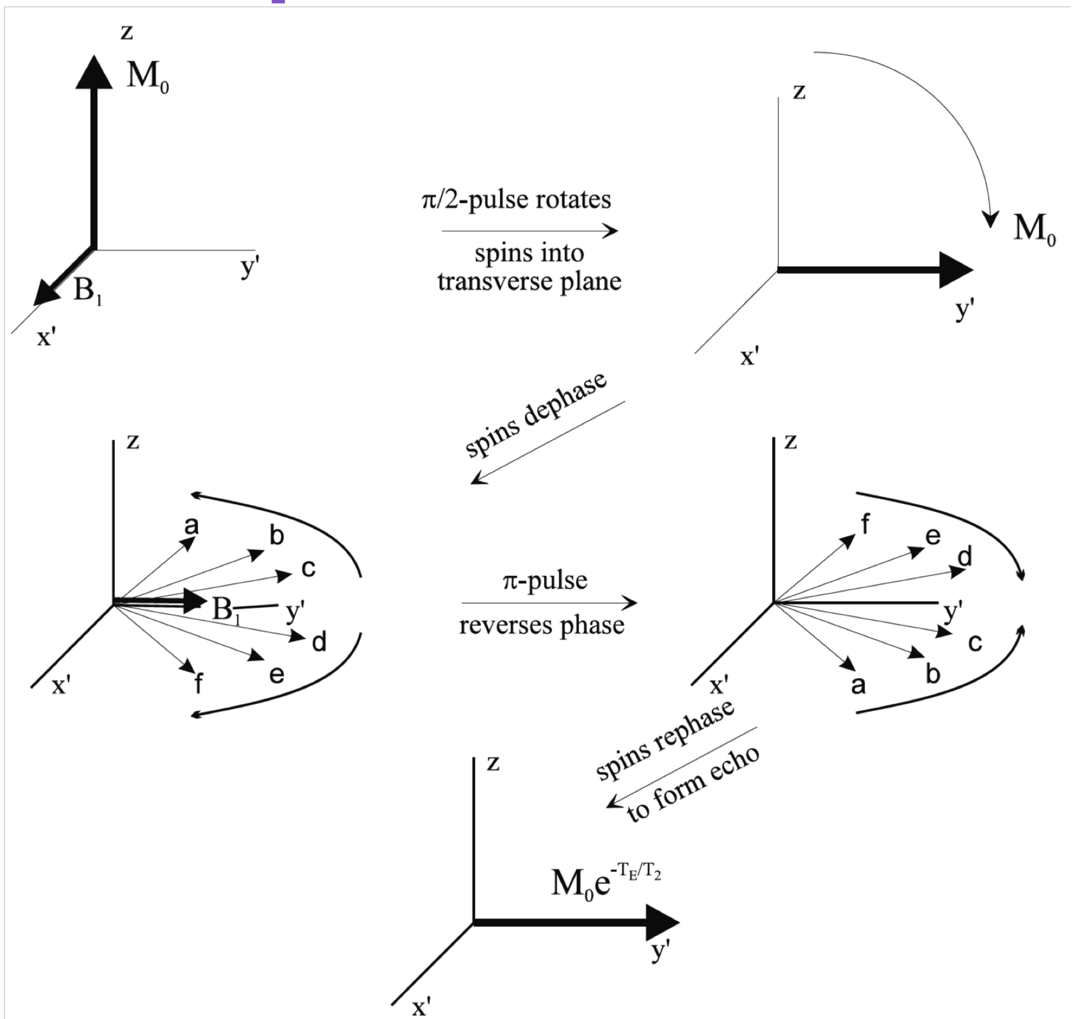
$$\begin{aligned}\phi(\vec{r},t) &= \phi(\vec{r},\tau^{+}) - \gamma\Delta B(\vec{r})(t-\tau)\\ &= -\gamma\Delta B(\vec{r})(t-2\tau)\\ &= -\gamma\Delta B(\vec{r})(t-T_E)\,, \qquad t>\tau\end{aligned}$$

- **At the echo time the phases again coincide and we get echo:**

$$T_E \equiv 2\tau$$



Spin-echo sequence



Problem

Problem 6.1

The parameter T_2' is associated with the (relatively smooth) variation in the z-component of the external field. An estimate of the average gradient in this component can be found from a given phase variation. If the z-component changes from $B_0 + \Delta B(\vec{r}_1)$ to $B_0 + \Delta B(\vec{r}_2)$, then the average gradient of that component between the two points $\vec{r}_1$ and $\vec{r}_2$ can be defined as

$$\overline{G} = \frac{|\Delta B(\vec{r}_2) - \Delta B(\vec{r}_1)|}{|\vec{r}_2 - \vec{r}_1|} \tag{8.8}$$

Suppose two protons are situated at these points. If $|\vec{r}_2 - \vec{r}_1|$ is 2 mm, and if there is no initial phase difference between their spins, find the value of $\overline{G}$ leading to a 2π difference in phase, after a time of 5 ms, for the two proton spins.

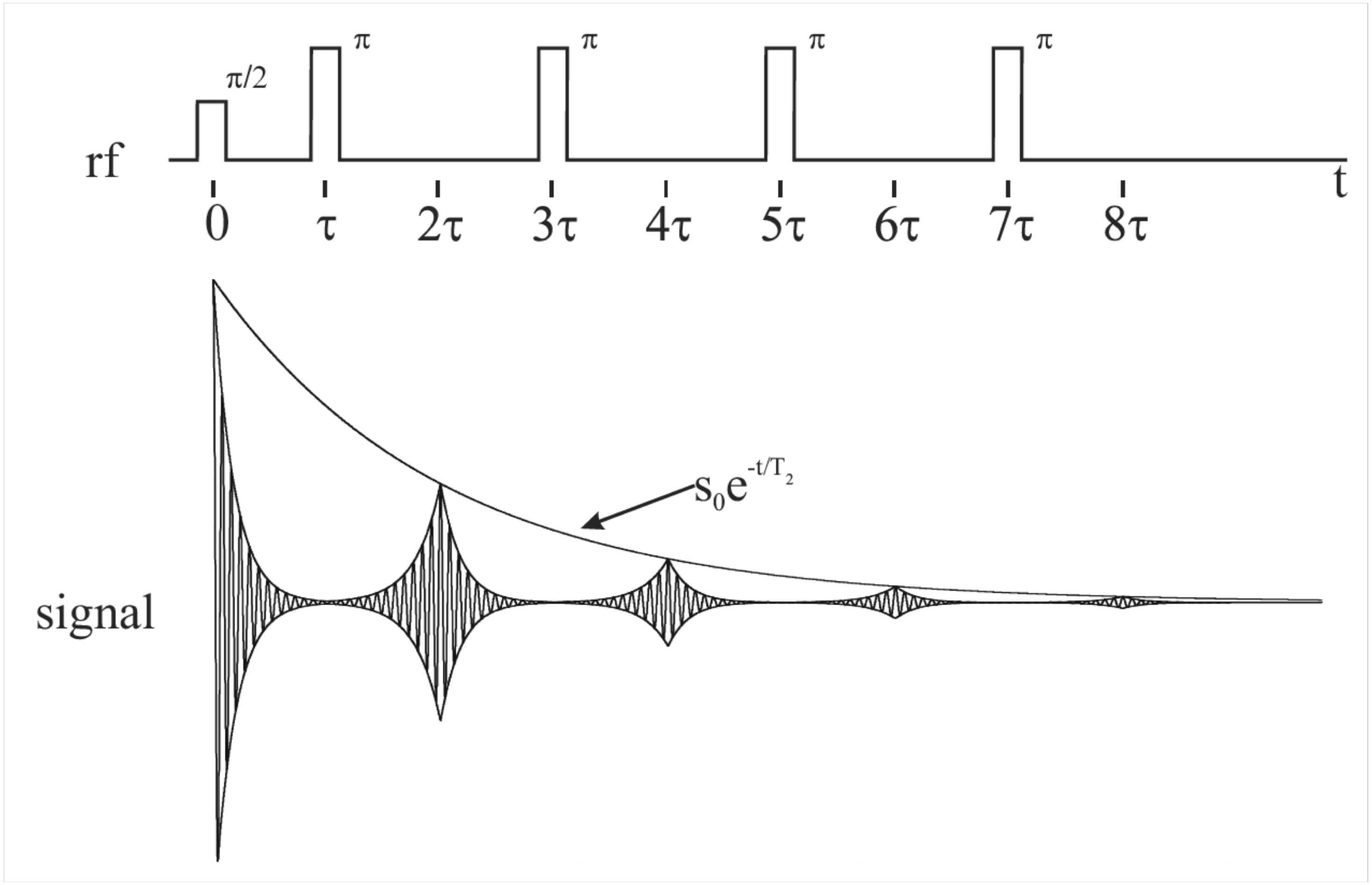
More on spin-echo

- **Spin-echo envelopes at each interval:**

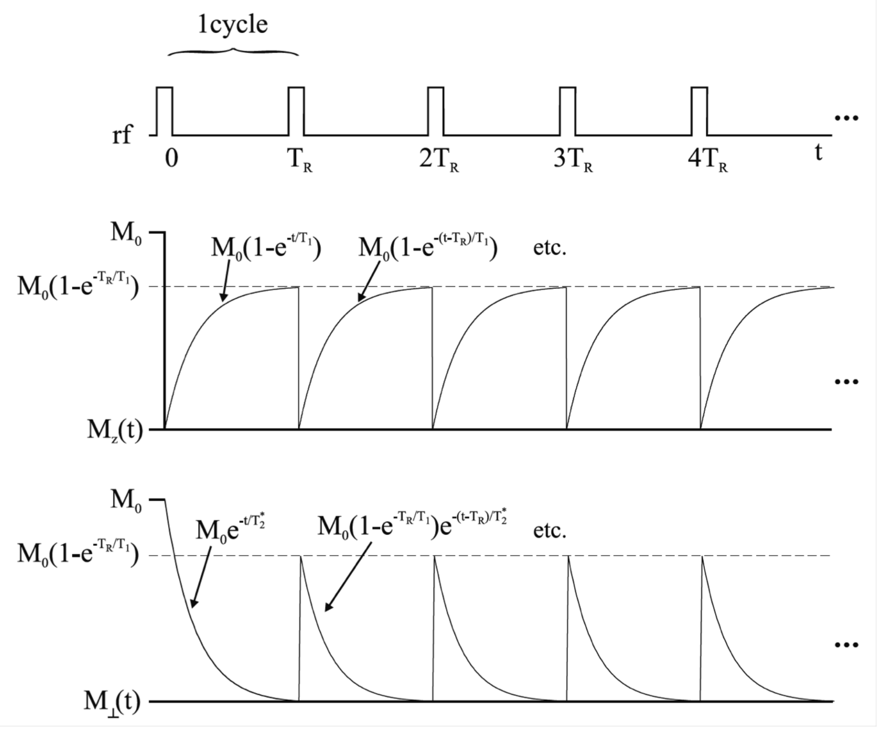
$$M_\perp(t) = M_\perp(0) \begin{cases} e^{-t/T_2^*} & 0 < t < \tau \\ e^{-t/T_2} e^{-(T_E - t)/T_2'} & \tau < t < 2\tau = T_E \\ e^{-t/T_2} e^{-(t - T_E)/T_2'} = e^{-t/T_2^*} e^{T_E/T_2'} & t > 2\tau = T_E \end{cases}$$

- **Spin-echo eliminates the effect of T2' relaxation**
- **It does not reduce the effect of T2**

Multiple Spin Echo Experiments

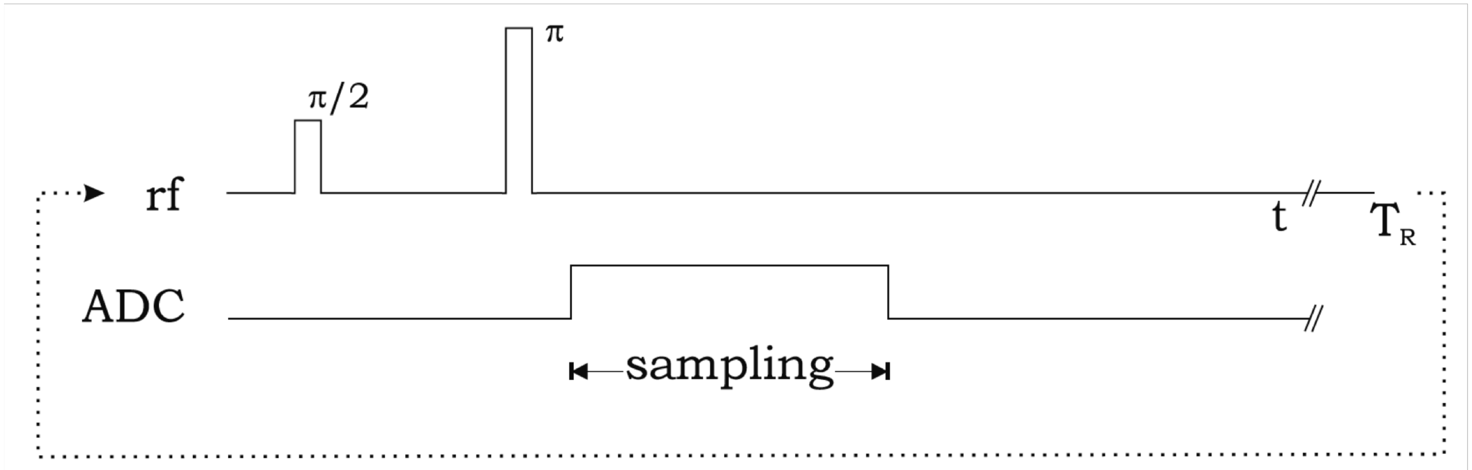


The FID Signal from Repeated RF Pulse Structures



$$M_\perp(t_n) = M_0(1 - e^{-T_R/T_1})e^{-t_n/T_2^*} \qquad (n \geq 2)$$

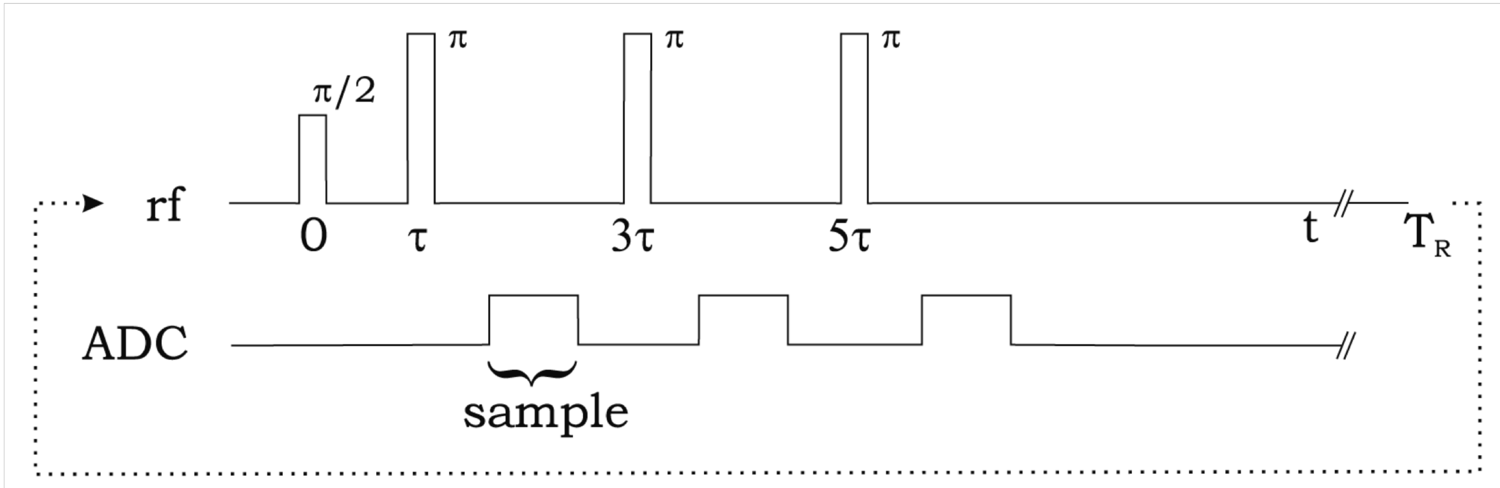
The Spin Echo Signal from Repeated RF Pulse Structures



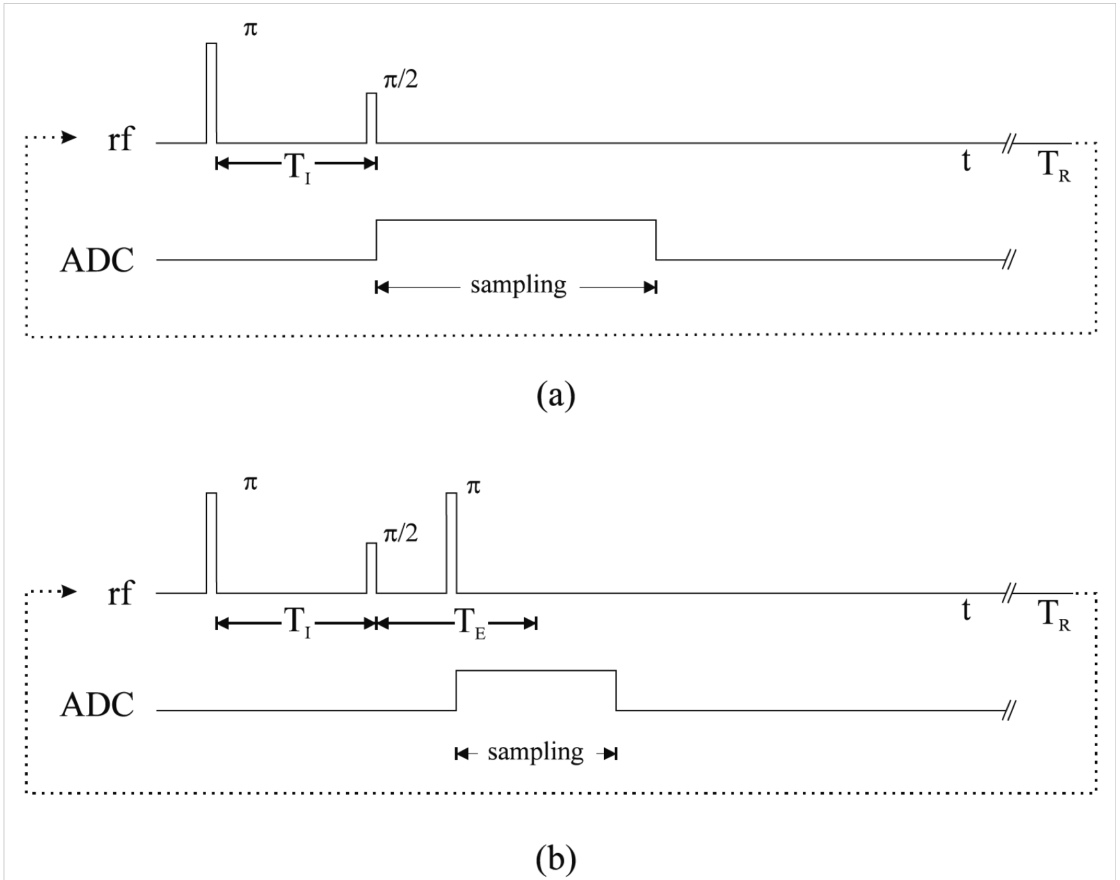
$$M_z(t) = M_0(1 - 2e^{-(t-\tau)/T_1} + e^{-t/T_1}) \qquad \tau < t < T_R$$

$$M_\perp(T_E) = M_0 e^{-T_E/T_2}$$

Multiple Spin Echo Sequence Diagram



Inversion Recovery and T1 Measurements



Inversion Recovery and T1 Measurements

- **After π pulse we have:**

$$M_z(0^+) = -M_0$$

- **The signal then evolves as:**

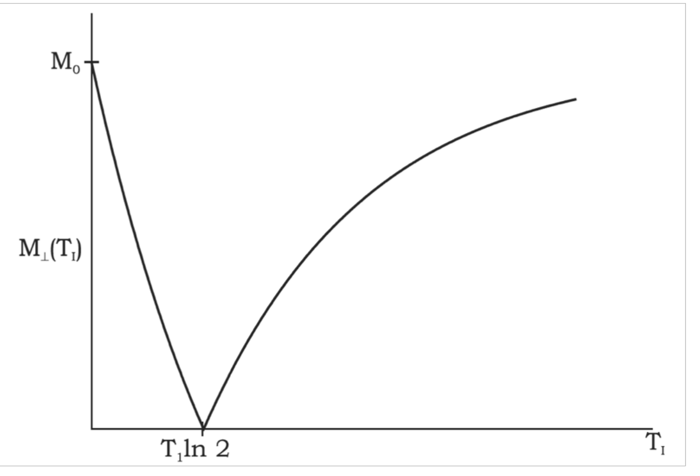
$$M_z(t) = -M_0 e^{-t/T_1} + M_0(1 - e^{-t/T_1}) = M_0(1 - 2e^{-t/T_1}), \quad 0 < t < T_I$$

- **After $\pi/2$ pulse we have:**

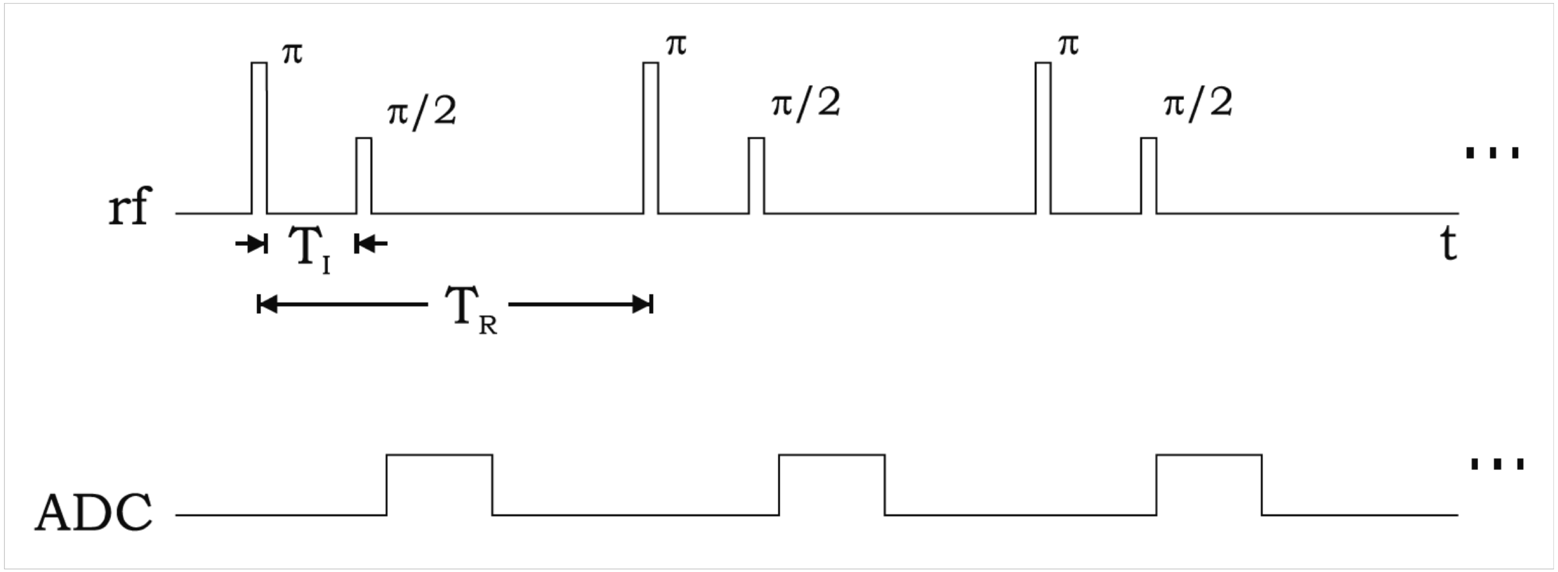
$$M_\perp(t) = \left| M_0(1 - 2e^{-T_I/T_1}) \right| e^{-(t-T_I)/T_2^*}, \quad t > T_I$$

- **This is zero when**

$$T_{I,null} = T_1 \ln 2$$



Repeated Inversion Recovery



$$
\begin{aligned}
M_\perp(t_n) &= \left| -M_0(1 - e^{-(T_R - T_I)/T_1})e^{-T_I/T_1} + M_0(1 - e^{-T_I/T_1}) \right| e^{-(t_n - T_I)/T_2^*} \\
&= M_0 \left| 1 + e^{-T_R/T_1} - 2e^{-T_I/T_1} \right| e^{-(t_n - T_I)/T_2^*} \\
& \qquad T_I < t_n < T_R \qquad (8.57)
\end{aligned}
$$

Problem

Problem 6.2

Experiments with $\pi/2$-pulses and short T_R can be expected to have reduced signal. That is, in the limit that T_R becomes much less than T_1 (but still much larger than T_2^*), show that $M_z(nT_R^-)$ is proportional to T_R/T_1.

Hint: Recall (8.31) and the Taylor expansion of the exponential function.

$$M_\perp(t_n) = M_0(1 - e^{-T_R/T_1})e^{-t_n/T_2^*} \qquad (n \geq 2) \tag{8.31}$$

Spectroscopy and Chemical Shift

- **Local magnetic 'shielding' causes local changes in B0**

$$B_{\text{shifted}}(j) = (1 - \sigma_j)B_0$$

- **The frequency shift is thus**

$$f_\sigma = -\sigma \gamma B_0$$

- **The phases evolve according to**

$$\phi_j(t) = -\gamma B_0(1 - \sigma_j)t$$

- **The resulting demodulated signal from continuous species has the form of a Fourier integral:**

$$s(t) \propto \int d\sigma \, \mathcal{N}(\sigma) e^{i\gamma\sigma B_0 t}$$