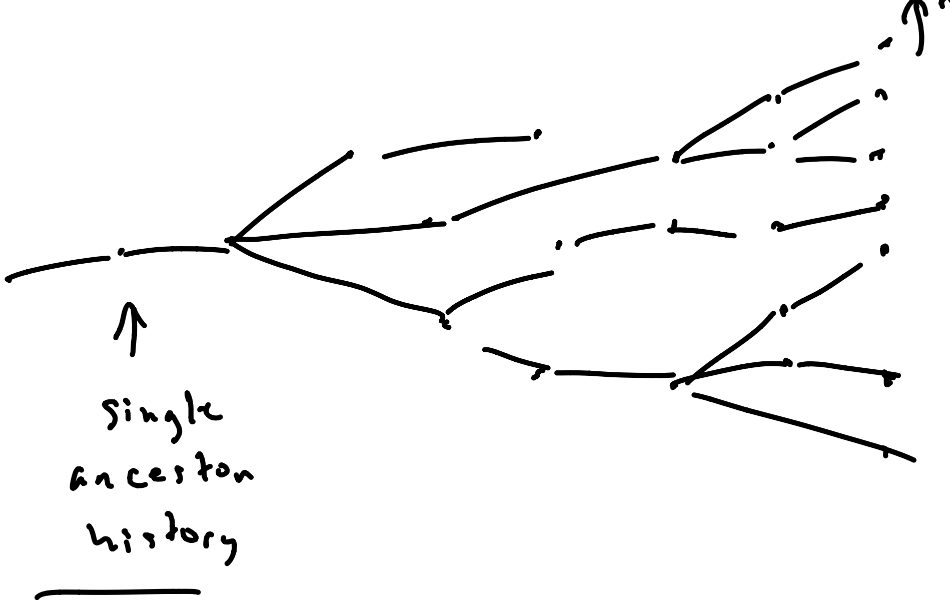
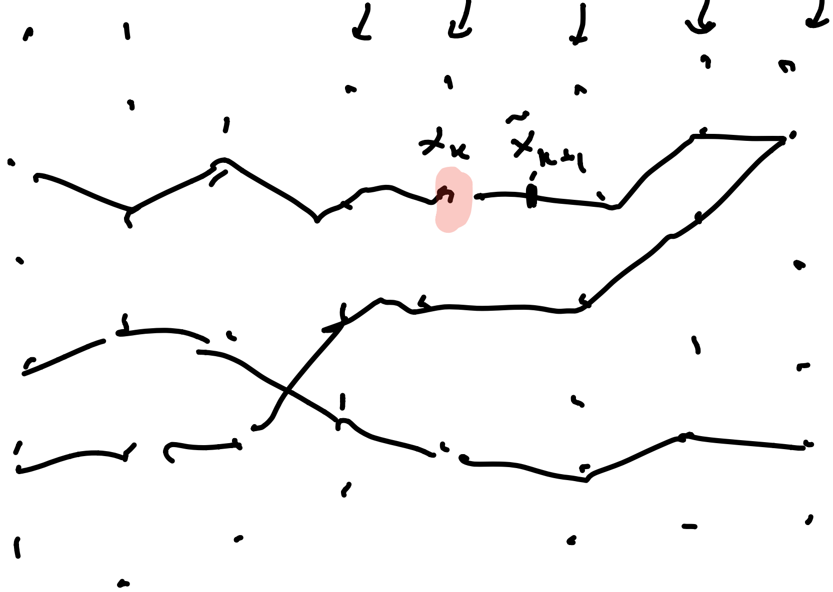
Direct SMC:



Backward Sim. filtering approximations



$$
\begin{aligned}
& p(x_k \mid \tilde{x}_{k+1}, y_{1:T}) \\
&= \frac{p(\tilde{x}_{k+1} \mid x_k)\, p(x_k \mid y_{1:k})}{p(\tilde{x}_{k+1} \mid y_{1:k})} \\
&\propto p(\tilde{x}_{k+1} \mid x_k)\, p(x_k \mid y_{1:k})
\end{aligned}
$$

Sample with weights $\underline{\underline{w_k^{(i)}}}$

Sample with weights

$$p(\tilde{x}_{k+1} | x_k^{(i)}) \cdot w_k^{(i)}$$

Reweighting smoother:

$$p(x_k | y_{1:T}) \approx \sum_i w_{k|T} \, \delta(x_k - x_k^{(i)})$$

new weights — filter particles

assume:

$$p(x_{k+1} | y_{1:T}) \approx \sum_i w_{k+1|T} \, \delta(x_{k+1} - x_{k+1}^{(i)})$$

known

$$p(x_k | y_{1:k}) \approx \sum_i w_k^{(i)} \delta(x_k - x_k^{(i)})$$

$$p(x_k | y_{1:T}) = p(x_k | y_{1:k}) \int \frac{p(x_{k+1} | x_k) \, p(x_{k+1} | y_{1:T})}{p(x_{k+1} | y_{1:k})} \, dx_{k+1}$$

$$\int \underbrace{\frac{p(x_{k+1} | x_k)}{p(x_{k+1} | y_{1:k})}}_{g(x_{k+1})} \cdot p(x_{k+1} | y_{1:T}) \, dx_{k+1}$$

$$\approx \sum_i w_{k+1|T} \underbrace{\frac{p(x_{k+1}^{(i)} | x_k)}{p(x_{k+1}^{(i)} | y_{1:k})}}_{g(x_{k+1})}$$

recall:

$$E[g(x_{k+1}) | y_{1:T}] \approx \sum_i w_{k+1|T} \, g(x_{k+1}^{(i)})$$

$$p(x^{(i)}_{k+1} \mid y_{1:k}) = \int p(x^{(i)}_{k+1} \mid x_k) \cdot \underline{p(x_k \mid y_{1:k})\, dx_k}$$

$$\approx \sum_i w_k^{(i)}\, p(x^{(i)}_{k+1} \mid x_k^{(i)})$$

we have

$$p(x_k \mid y_{1:T}) = p(x_k \mid y_{1:k}) \cdot \int \frac{p(x_{k+1} \mid x_k)\, p(x_{k+1} \mid y_{1:T})}{p(x_{k+1} \mid y_{1:k})}\, dx_{k+1}$$

$$\approx \sum_i w_k^{(i)} \delta(x_k - x_k^{(i)})$$

$$\sum_j w^{(j)}_{k+1|T} \frac{p(x^{(j)}_{k+1} \mid x_k)}{\sum_l w_k^{(l)}\, p(x^{(j)}_{k+1} \mid x_k^{(l)})}$$

$$= \sum_i \left[w_k^{(i)} \sum_j \frac{w^{(j)}_{k+1|T}\, p(x^{(j)}_{k+1} \mid x_k^{(i)})}{\sum_l w_k^{(l)}\, p(x^{(j)}_{k+1} \mid x_k^{(l)})} \right] \delta(x_k - x_k^{(i)})$$