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Brief Recap of Chapter 10 of Brown et al. (2014)

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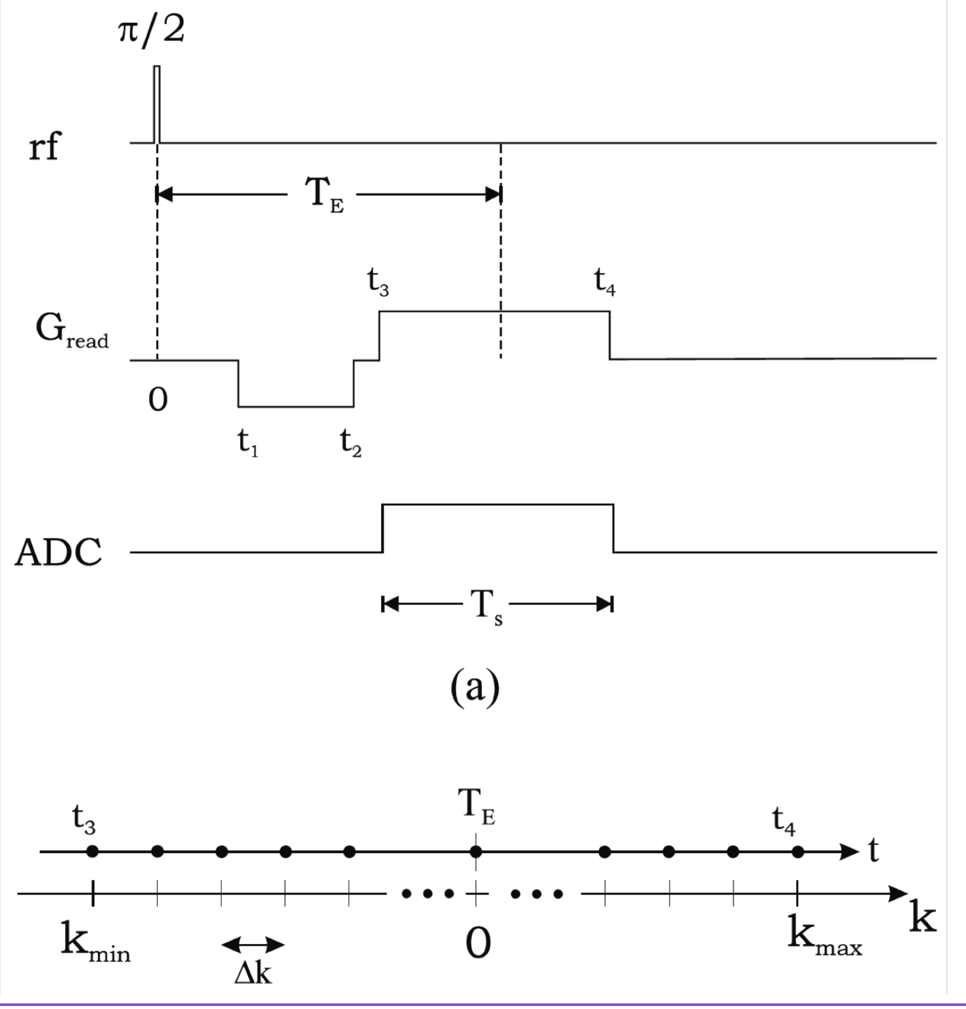
Contents of Chapter 10 ” Multi-Dimensional Fourier Imaging and Slice Excitation”

Introduction

- **The 1D Fourier encoding is here generalized to multiple dimensions**
- **The results are two- and three dimensional Fourier transforms**
- **Phase encoding by varying gradient amplitudes is introduced**
- **To avoid full 3D data collection, the sliced selection method is introduced**

Recap: Fourier Imaging in 1D



The 3D Imaging Equation

- **The signal from 3D excitation can be written as**

$$s(\vec{k}) = \int d^3 r \rho(\vec{r}) e^{-i2\pi \vec{k}\cdot\vec{r}}$$

- **Or**

$$\begin{aligned} s(k_x, k_y, k_z) &= \int\int\int dx\, dy\, dz\, \rho(x, y, z) e^{-i2\pi(k_x x + k_y y + k_z z)} \\ &= \mathcal{F}[\rho(x, y, z)] \end{aligned}$$

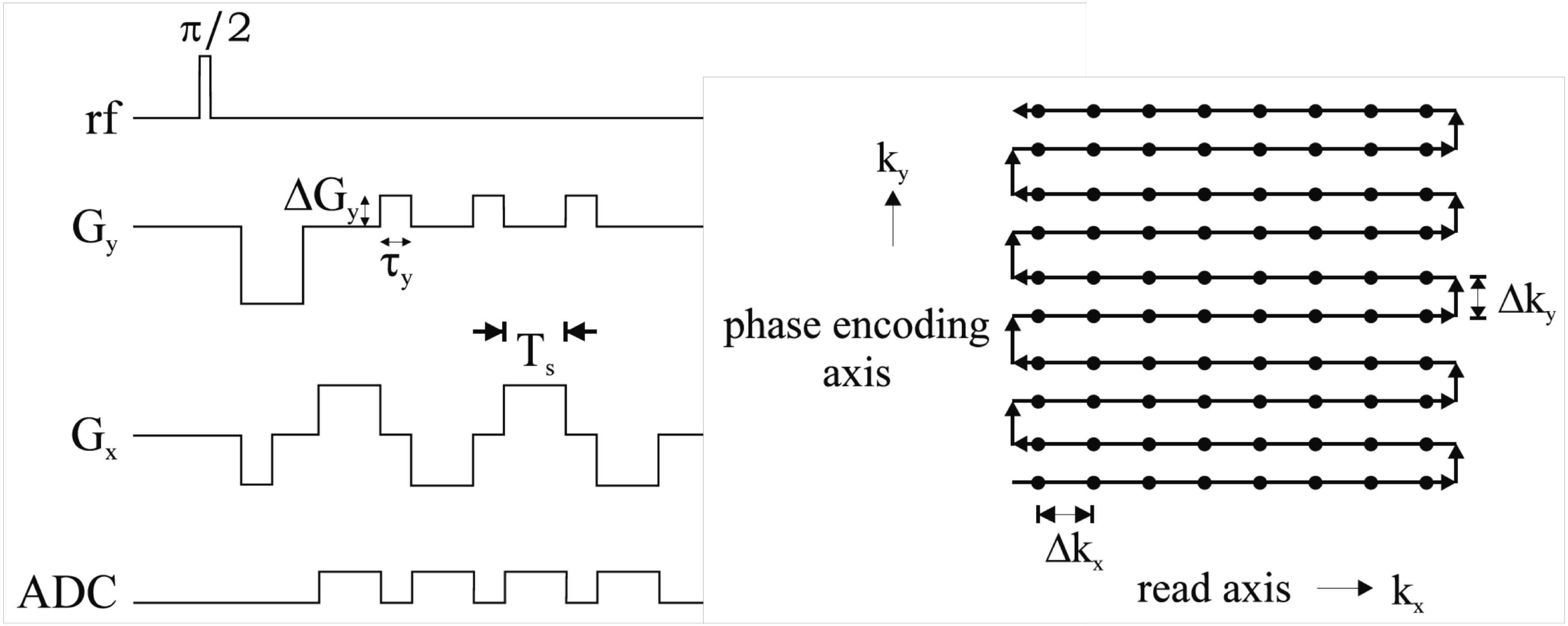
- **The time implicitly time dependent k-vector is given by**

$$k_x(t) = \gamma \int^t G_x(t')dt', \quad k_y(t) = \gamma \int^t G_y(t')dt', \quad k_z(t) = \gamma \int^t G_z(t')dt'$$

- **The image itself can be recovered by**

$$\hat{\rho}(\vec{r}) = \int d^3 k s_m(\vec{k}) e^{i2\pi \vec{k}\cdot\vec{r}}$$

2D Imaging and Phase Encoding



$$s(k_x, k_y) \quad = \quad \int\int\int dx\, dy\, dz\, \rho(x, y, z) e^{-i2\pi(k_x x + k_y y)}$$

$$\hat{\rho}(x, y) = \int dk_x s(k_x, k_y) e^{i2\pi(k_x x + k_y y)} = \int dz\, \rho(x, y, z)$$

Full 3D Imaging with Phase Encoding

- **We could also traverse the whole k_y,k_z plane with phase encoding**
- **This is called 3D imaging**
- **Each x-line is sampled during application of gradient Gx:**

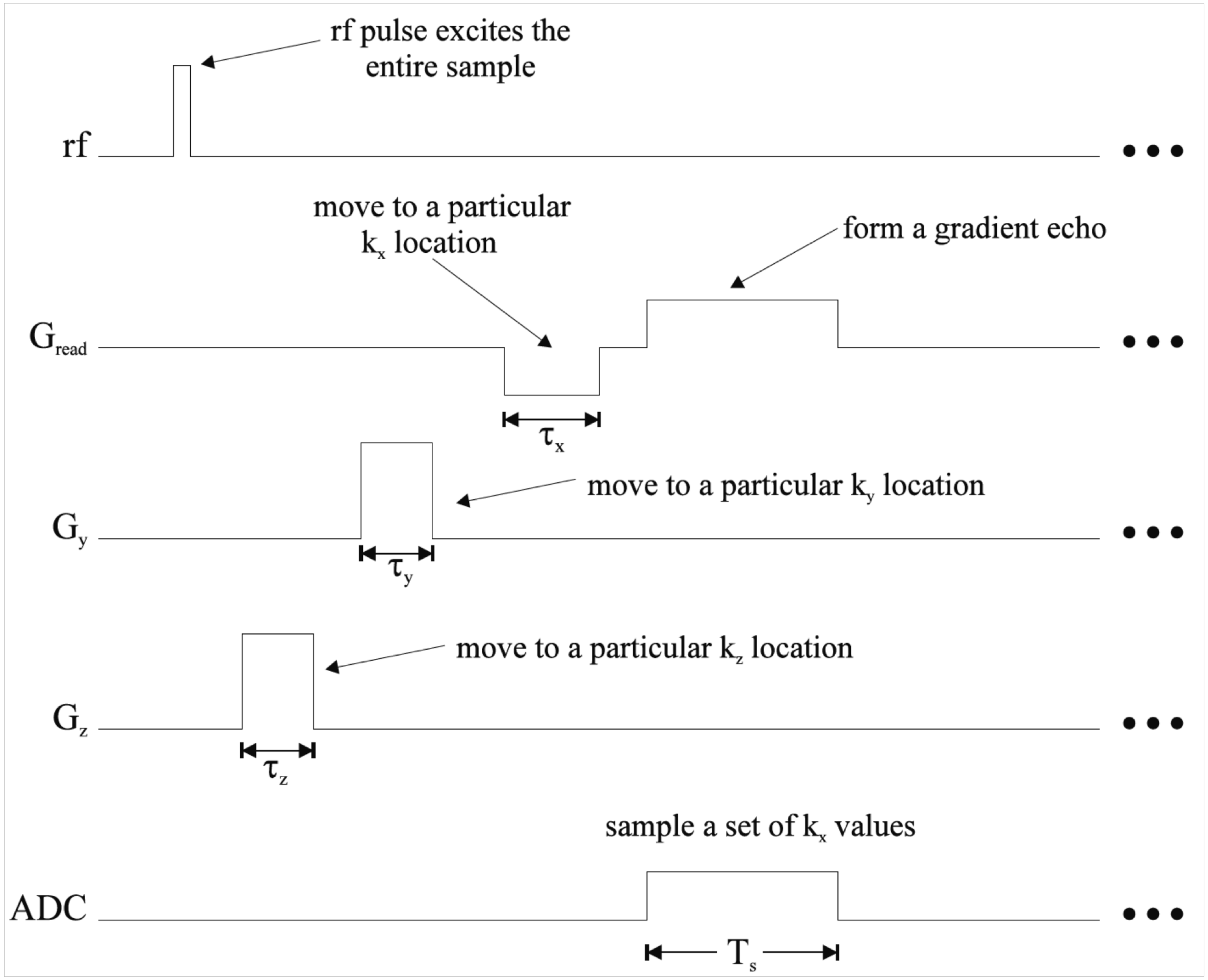
$$\Delta k_x = \gamma\, G_x \Delta t$$

- **The y and z directions are 'looped over' by taking steps**

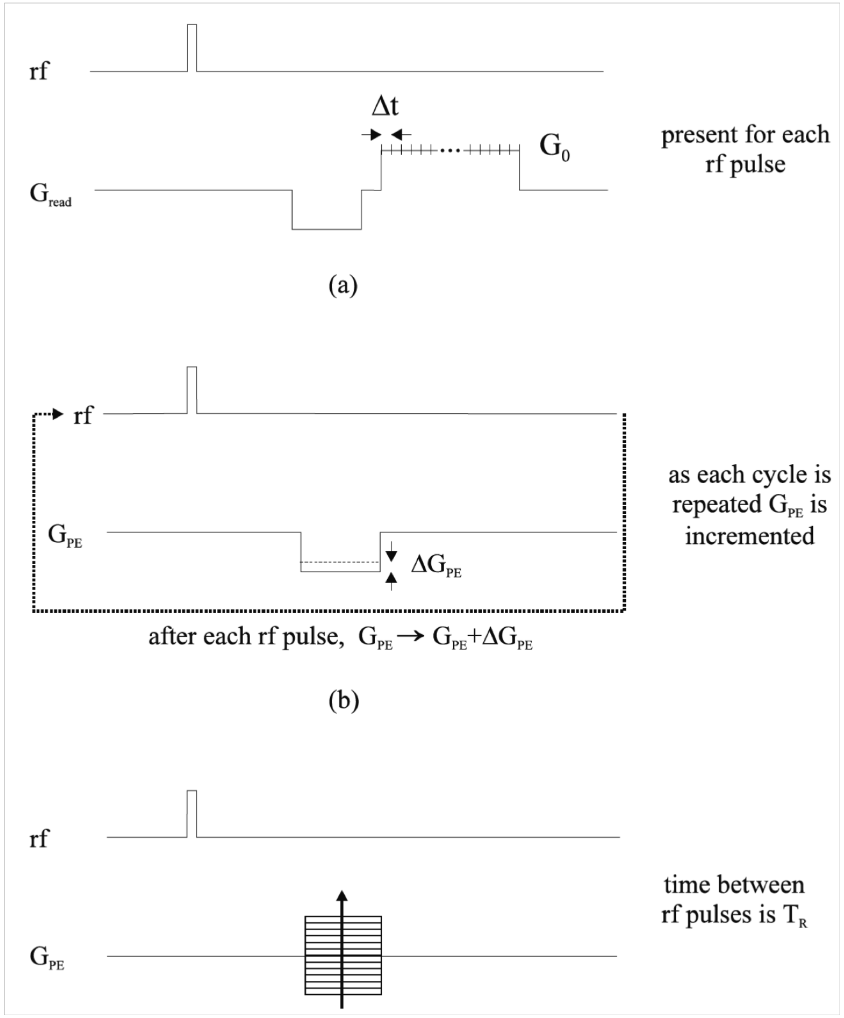
$$\begin{aligned} \Delta k_y &= \gamma\, \Delta G_y \tau_y \\ \Delta k_z &= \gamma\, \Delta G_z \tau_z \end{aligned}$$

- **One rarely works this way, because T2 and T2* decays cause the signal to attenuate**

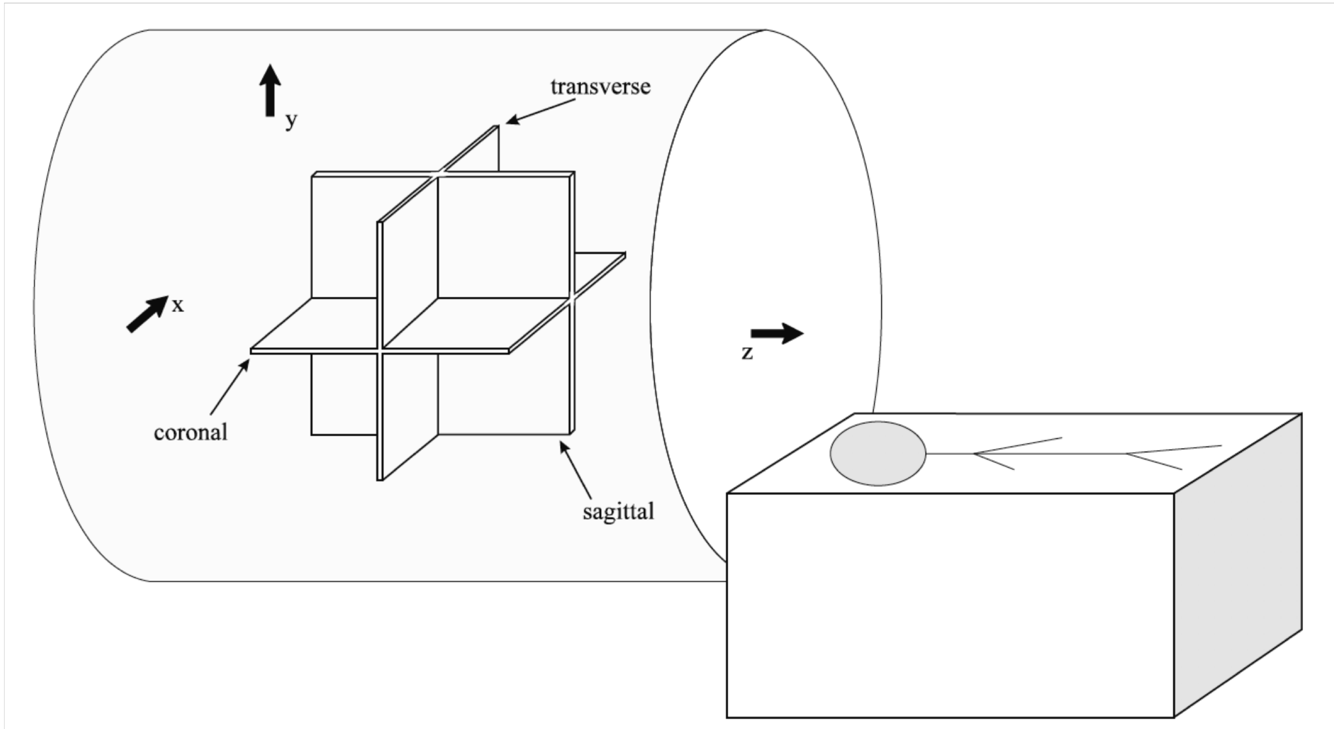
3D Imaging with Repeated rf-Pulses



2D/3D Imaging with Repeated rf-Pulses



Slice Selection



Applied slice select gradient	Name	Slice plane orientation
G_x	sagittal	parallel to y-z plane
G_y	coronal	parallel to x-z plane
G_z	transverse	parallel to x-y plane

Exciting a Slice

- **We turn on a 'slice selection' gradient, which causes Larmor frequencies to depend on position:**

$$f(z) = f_0 + \gamma G_z z$$

- **We then apply a band-limited pulse which excites only a slice of thickness *Δz:***

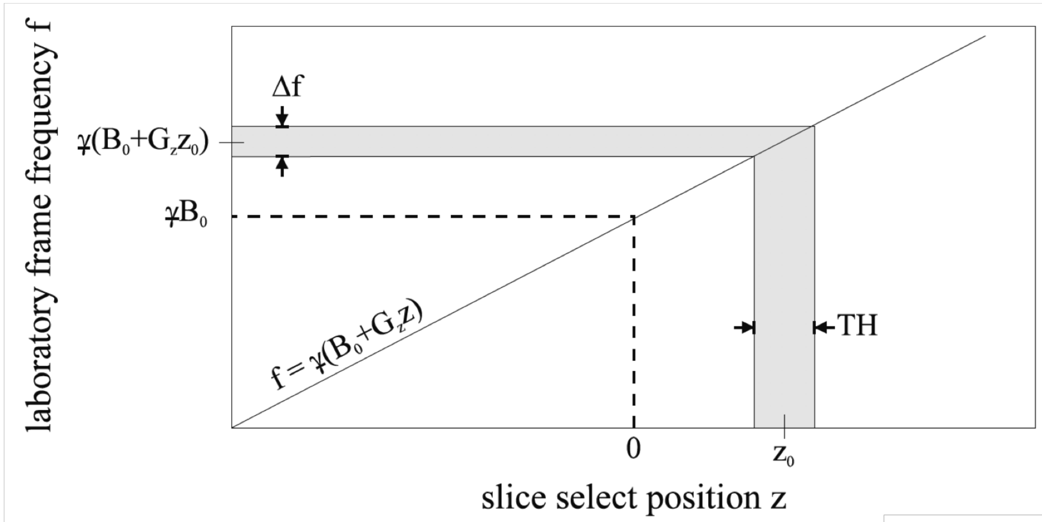
$$\begin{aligned} BW_{rf} &\equiv \Delta f \\ &= (\gamma G_z z_0 + \gamma G_z \Delta z/2) - (\gamma G_z z_0 - \gamma G_z \Delta z/2) \\ &= \gamma G_z \Delta z \end{aligned}$$

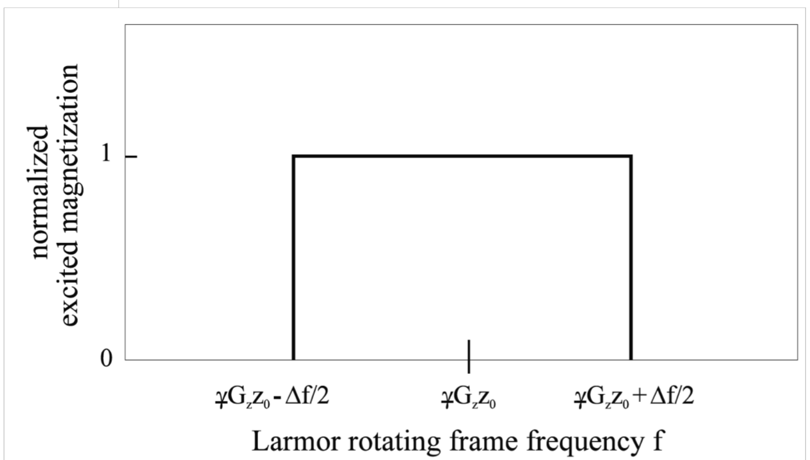
- **The slice thickness *TH* = *Δz* is then given as**

$$TH = \frac{BW_{rf}}{\gamma G_z}$$

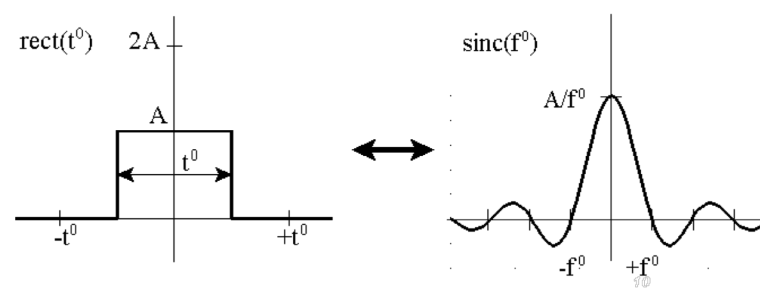
- **Band-limited boxcar pulse in time-domain is sinc(.)**

Slice Selection with Boxcar Excitations



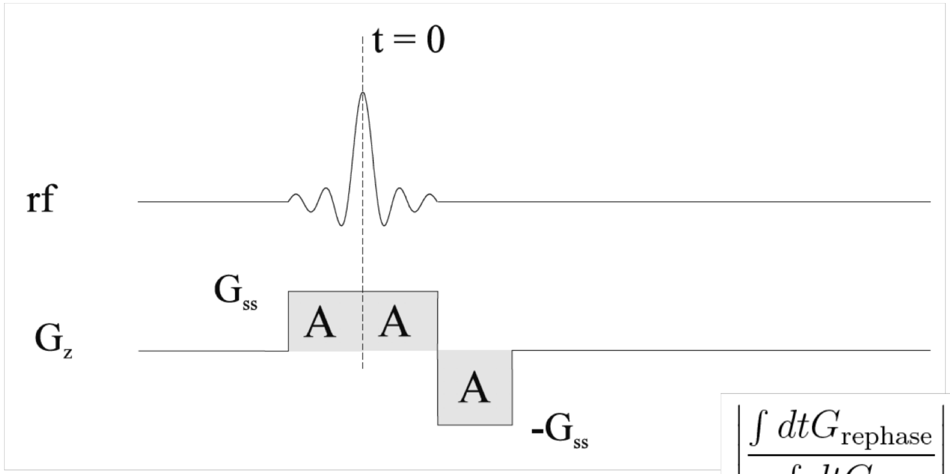


$$B_1(t) \propto \mathrm{sinc}(\pi \Delta f\ t)$$



Gradient Rephasing After Slice Selection

- **The slice selection gradient causes spins to dephase at different sides of the slice**
- **This effect can be eliminated by applying a reverse gradient for half of the time after the slice selection:**

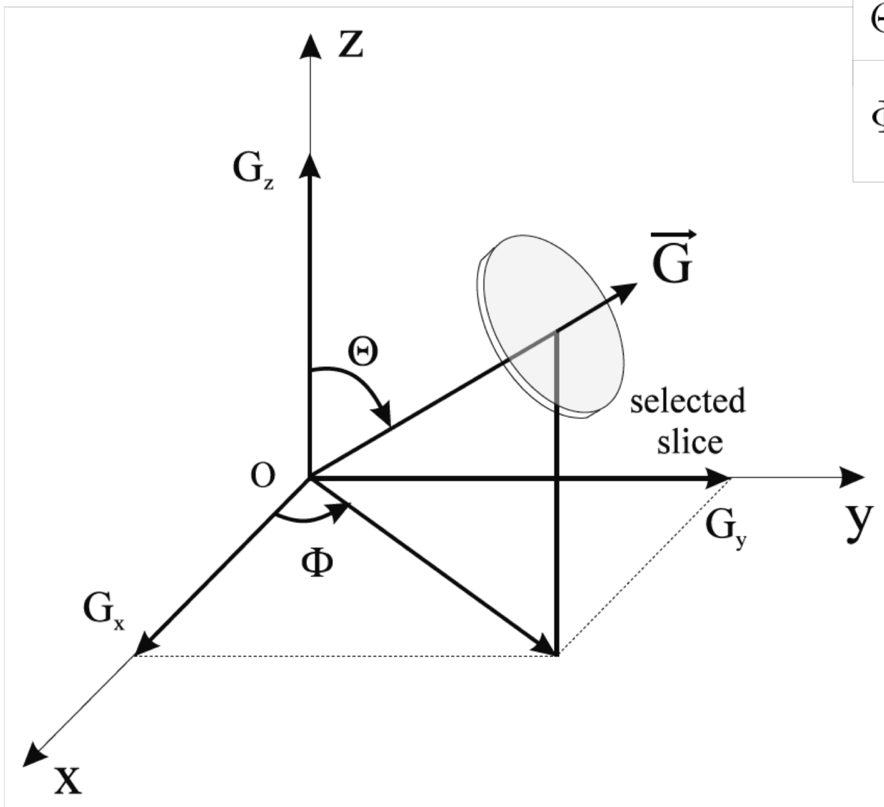


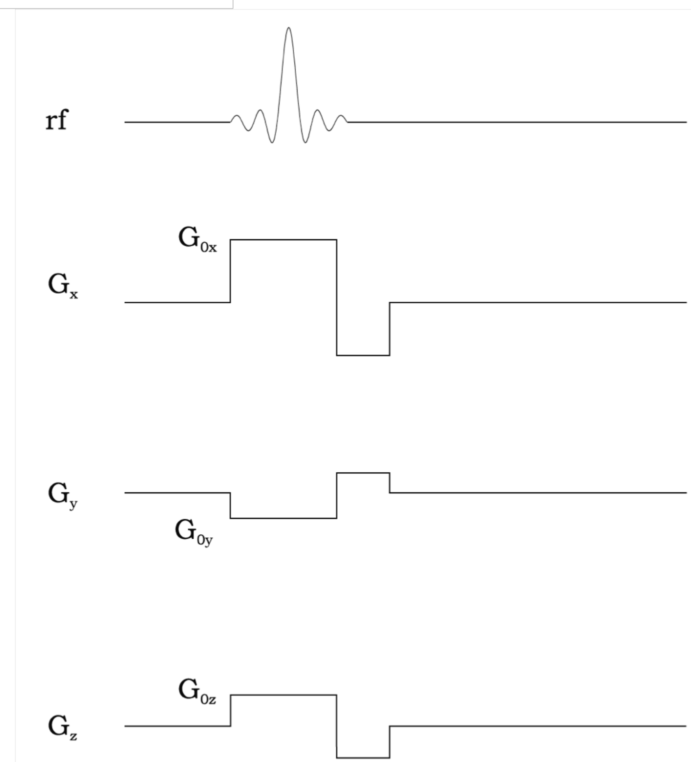
$$\left|\frac{\int dt G_{\mathrm{rephase}}}{\int dt G_{ss}}\right| = \left|\frac{\text{total area under } G_{\mathrm{rephase}}}{\text{total area under } G_{ss}}\right| = 50\%$$

Arbitrary Slice Orientation

$$\Theta = \tan^{-1} \frac{\sqrt{G_{0x}^2 + G_{0y}^2}}{G_{0z}}$$

$$\Phi = \tan^{-1} \frac{G_{0y}}{G_{0x}}$$



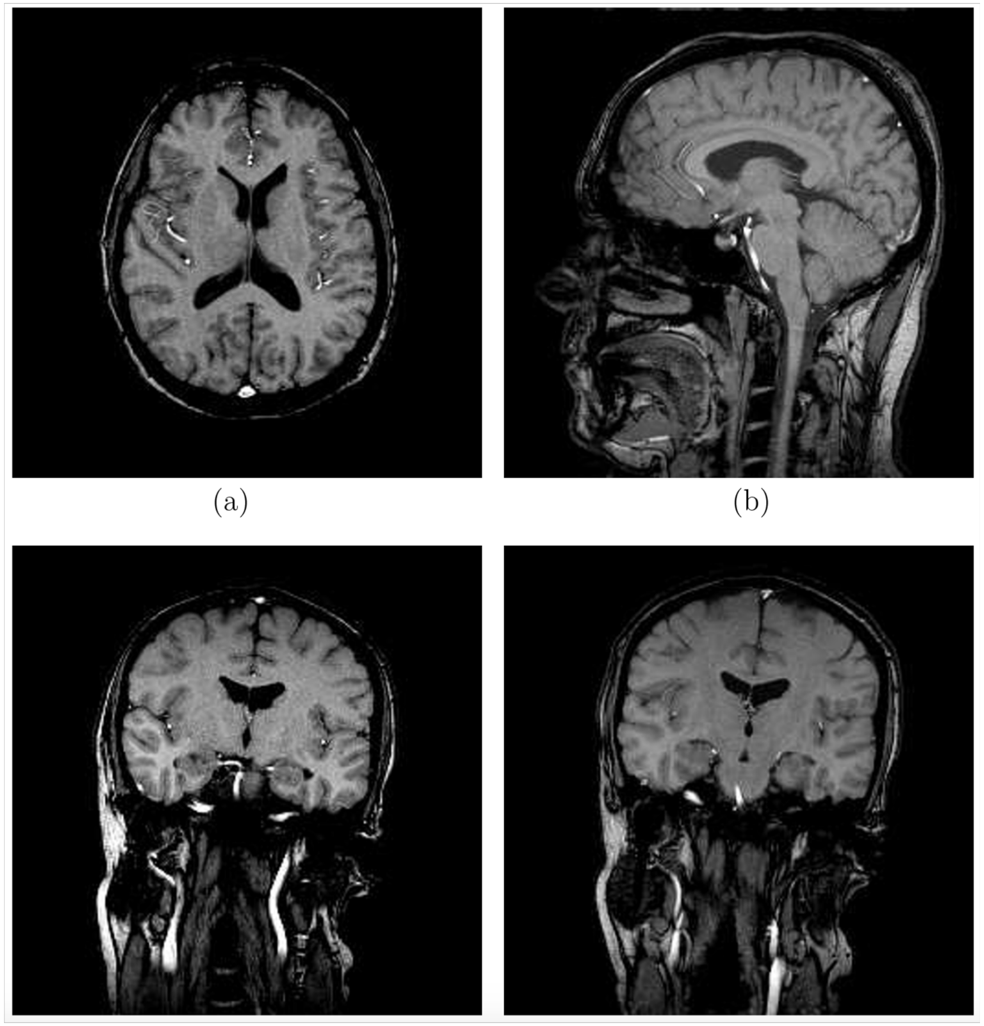


Problem

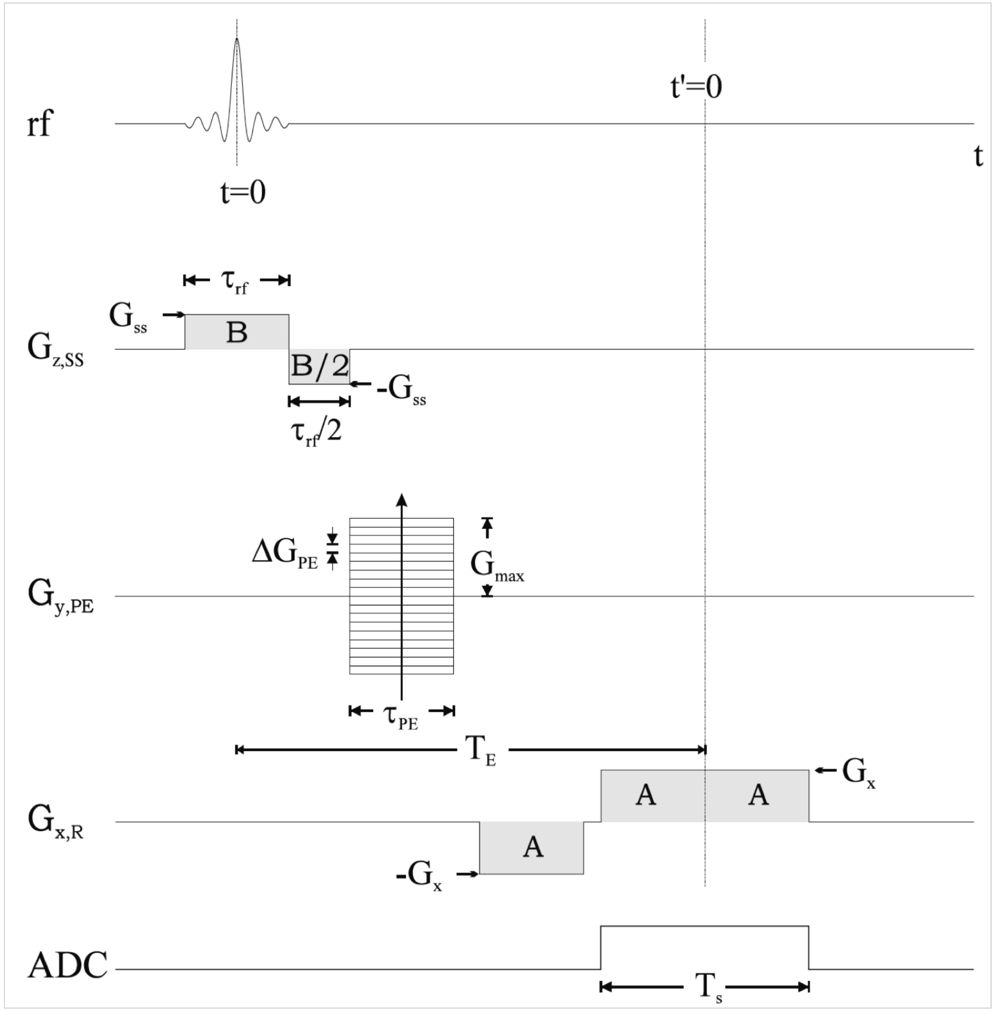
Problem 8.1

Assume that the slice select gradient amplitudes are changed from those shown in Fig. 10.11 to be $G_{0x} = -G_{0y} = 2G_{0z}$. What are the angles Φ and Θ of the resulting slice? Draw the excited plane (use Fig. 10.10 as a guide).

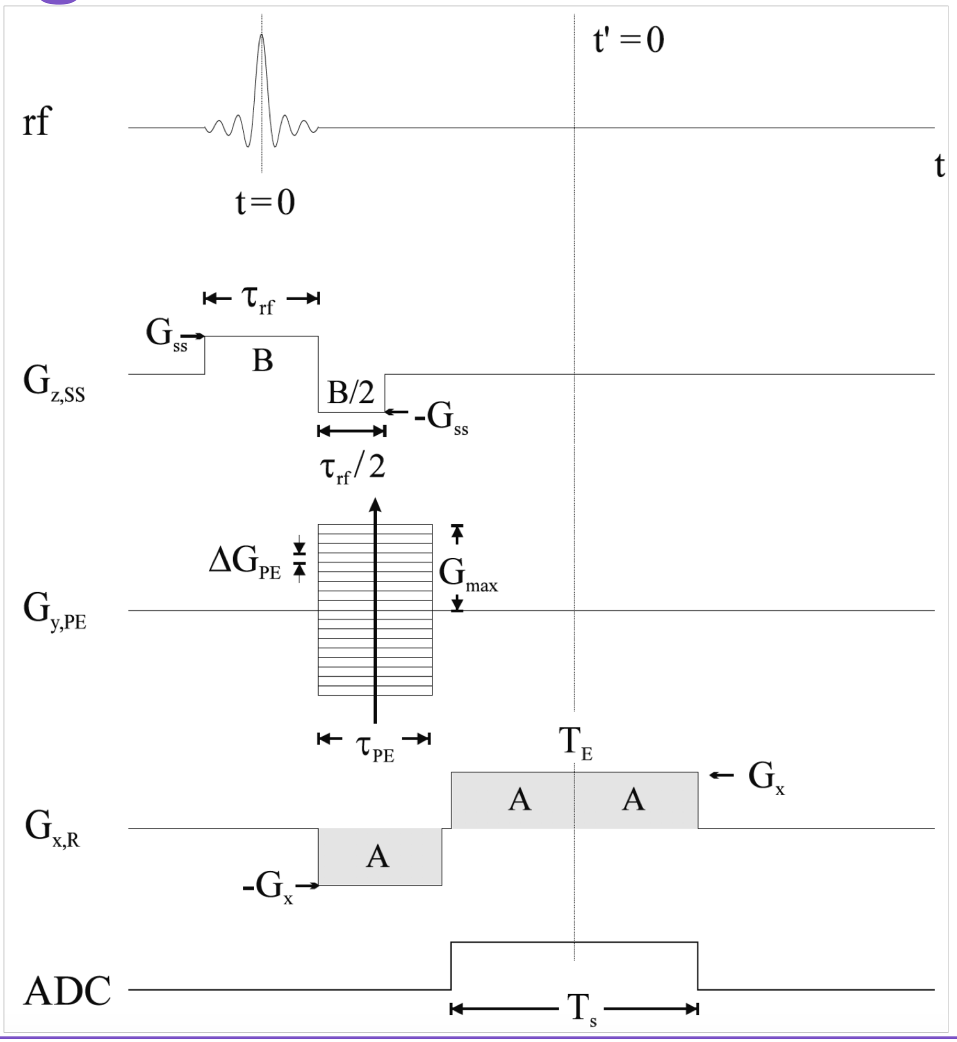
Examples 2D Gradient Echo Images



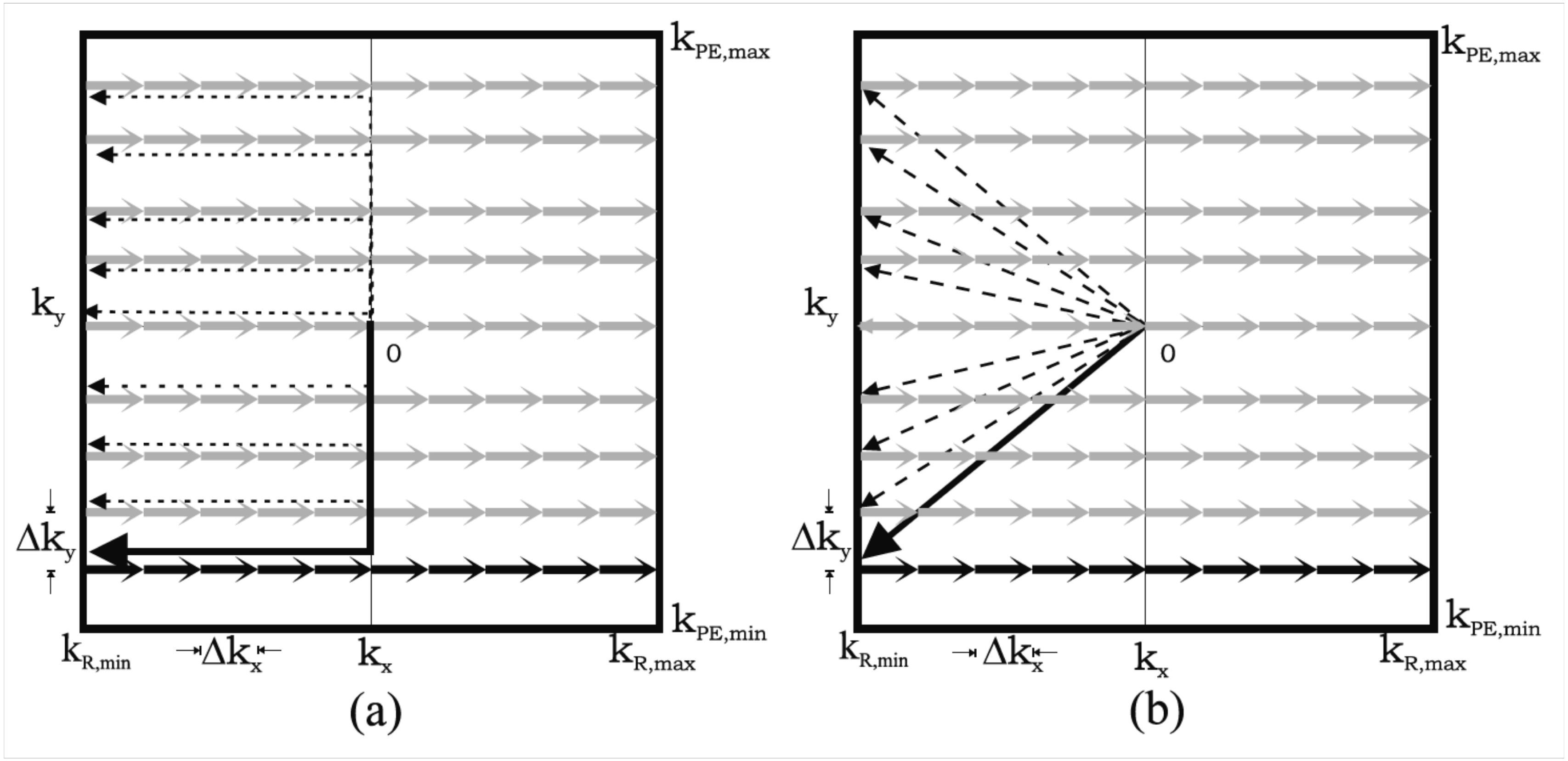
2D Gradient Echo Imaging Sequence



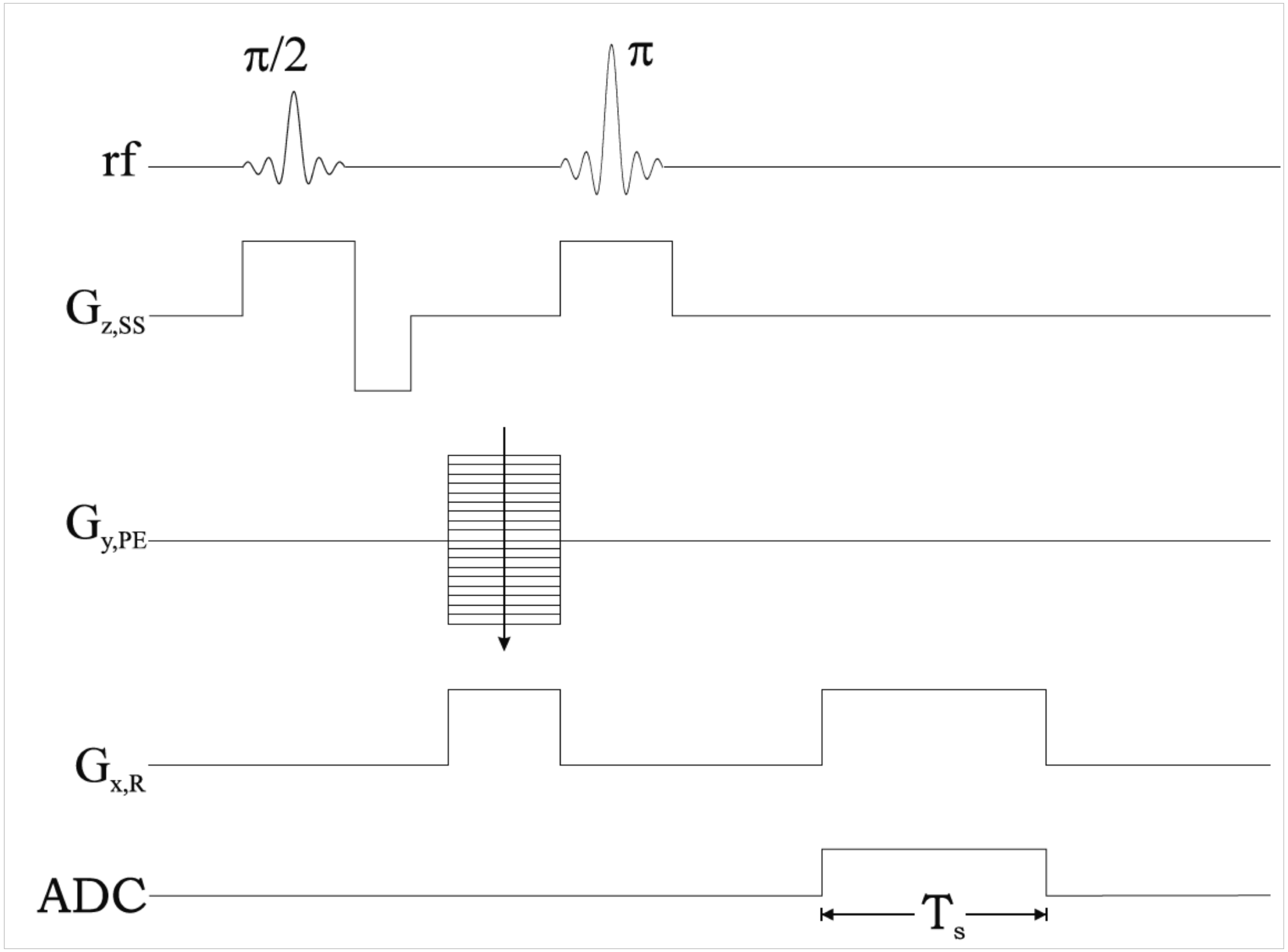
Applying Gradients at the Same Time



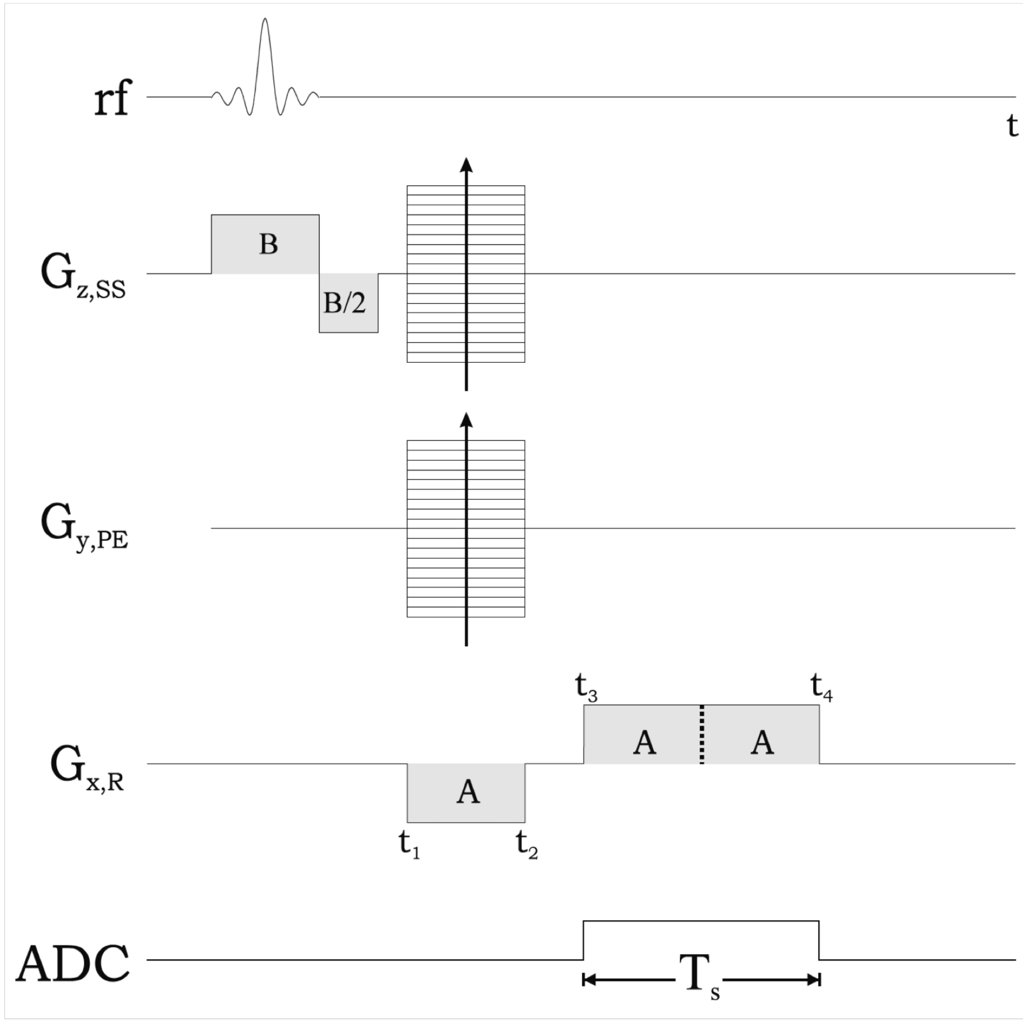
K-space Trajectory



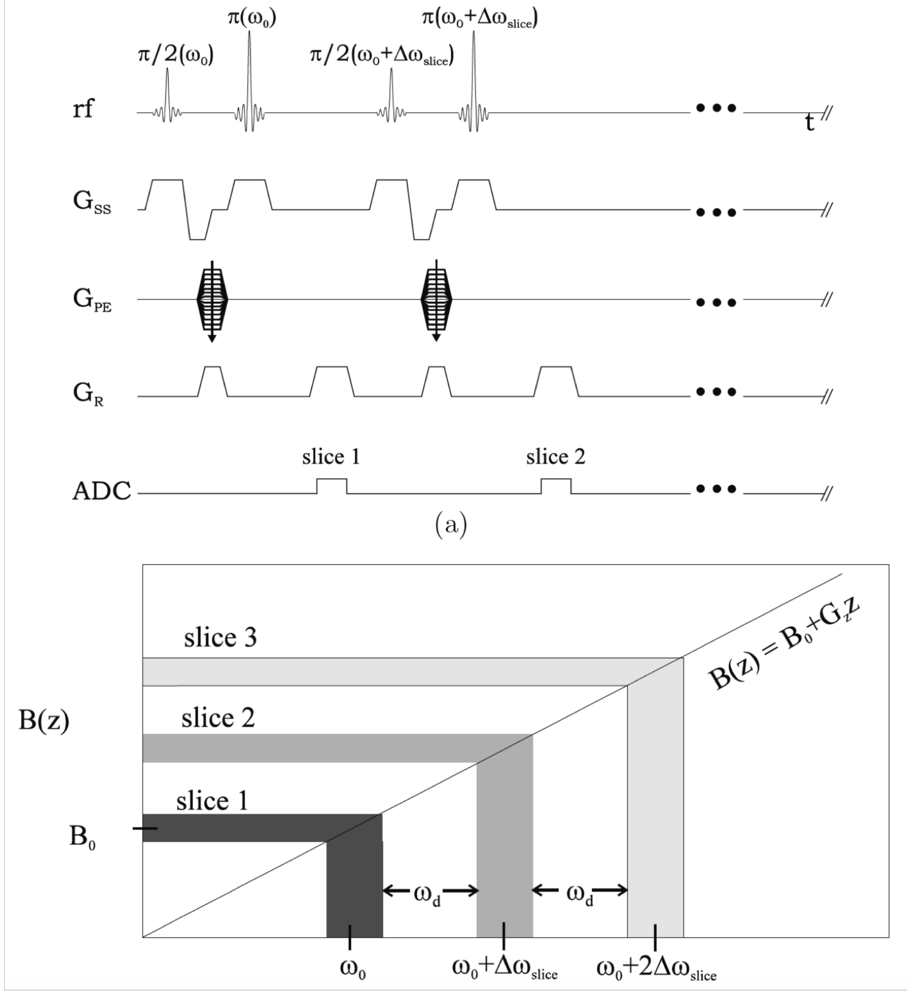
Spin Echo Imaging Sequence



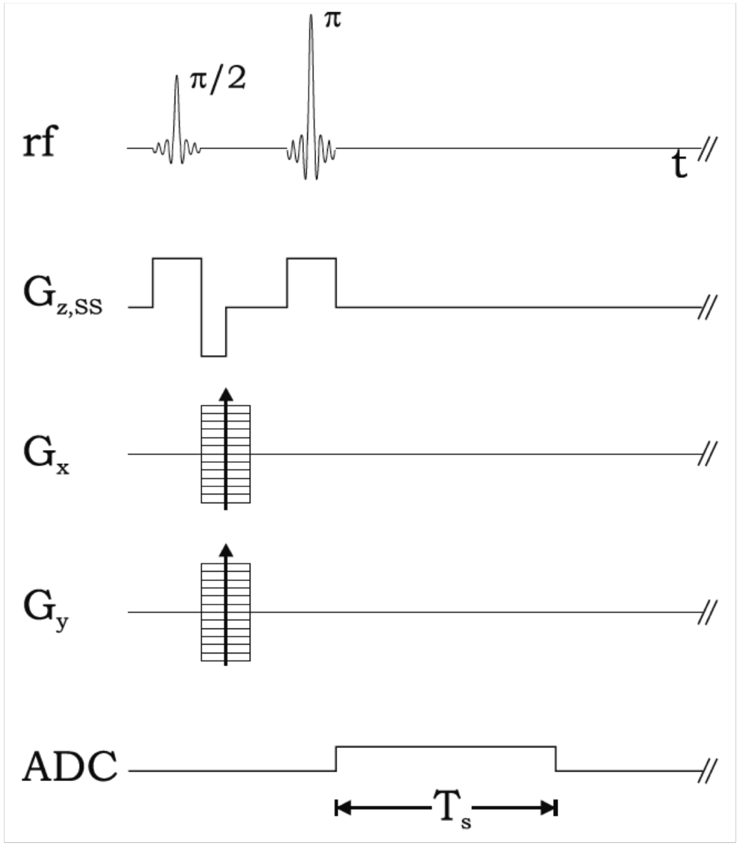
Short-TR 3D Gradient Echo Imaging

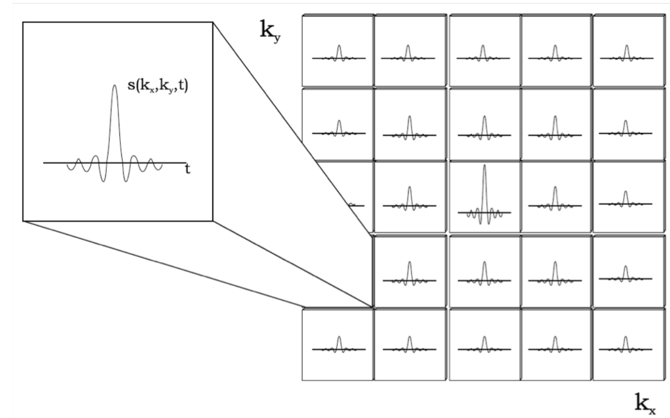


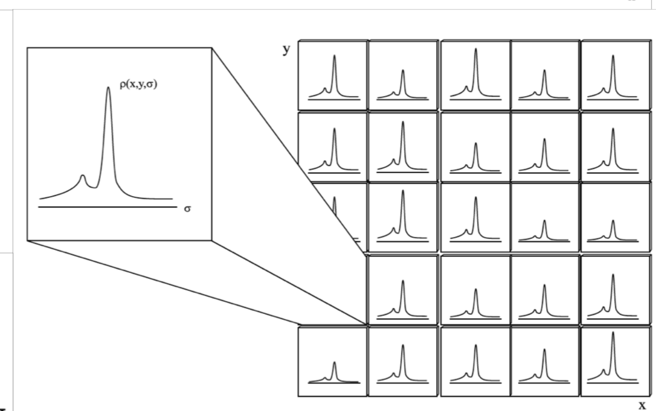
Multi-Slice 2D Imaging

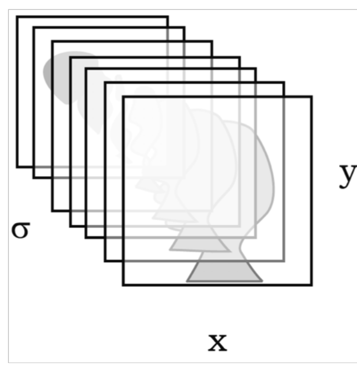


Chemical Shift Imaging









Problem

Problem 8.2

Given the following imaging parameters, determine T_{acq} in each case.

a) Find T_{acq} for a 2D spatial imaging experiment with $N_y = 256$ and $T_R = 1200\,\text{ms}$.

b) Find T_{acq} for a 2D spatial 1D spectral CSI experiment with $N_x = N_y = 256$ and $T_R = 1200\,\text{ms}$.

c) Find T_{acq} for a 3D spatial imaging experiment with $N_x = N_y = 256$, $N_z = 128$ and $T_R = 1200\,\text{ms}$.

d) Find T_{acq} for a 4D imaging experiment with $N_x = N_y = N_z = 16$ and $T_R = 1200\,\text{ms}$.

e) Discuss the implications of your result to the imaging of humans. Assume it is difficult for an average patient to stay in the imaging environment (lying still inside the bore of a magnet) for more than 30 minutes without becoming uncomfortable. Usually 10 minutes is assumed to be an upper limit for a single MRI scan. Assume also that you are imaging the entire human head which requires about 19.2 cm (left-to-right) $\times$ 25.6 cm (head-to-foot with oversampling) $\times$ 22.4 cm (front-to-back) of total spatial coverage. Given that the spatial resolution, as defined later in Ch. 13, is the ratio of FOV (the area over which the image is acquired) to number of encoded points, discuss the trade-off in spatial resolution versus imaging time in going from part (c) to part (d).