prediction error:

$$p(y_1, y_2, \dots, y_T \mid \theta)$$

$$= p(y_T, y_{T-1}, \dots, y_2, y_1 \mid \theta)$$

$$= p(y_T \mid y_{T-1}, \dots, y_1, \theta) \cdot p(y_{T-1}, \dots, y_1 \mid \theta)$$

$$= p(y_T \mid y_{T-1}, \dots, y_1, \theta) \cdot p(y_{T-1} \mid y_{T-2}, \dots, y_1, \theta) \cdot p(y_{T-2}, \dots, y_1 \mid \theta)$$

$$\vdots$$

$$= \prod_{k=1}^{T} p(y_k \mid y_{1:k-1}, \theta)$$

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filter: $p(x_k \mid y_{1:k}, \theta)$

pred: $p(x_k \mid y_{1:k-1}, \theta)$

$$p(y_k, x_k \mid y_{1:k-1}, \theta)$$
$$= p(y_k \mid x_k, y_{1:k-1}, \theta) \cdot p(x_k \mid y_{1:k-1}, \theta)$$
$$= p(y_k \mid x_k, \theta) \cdot p(x_k \mid y_{1:k-1}, \theta)$$

$$p(y_k \mid y_{1:k-1}, \theta)$$
$$= \int p(y_k \mid x_k, \theta) \cdot p(x_k \mid y_{1:k-1}, \theta) \, dx_k$$


likel.
pred.