$$\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{B}} : \bar{\bar{I}}$$

$$\overline{\overline{A}}\overset{\times}{\times}\overline{\overline{B}} = \overline{\overline{B}}\overset{\times}{\times}\overline{\overline{A}} = \left[(\overline{\overline{A}}:\overline{\overline{I}})(\overline{\overline{B}}:\overline{\overline{I}}) - \overline{\overline{A}}:\overline{\overline{B}}^T\right]\overline{\overline{I}} - (\overline{\overline{A}}:\overline{\overline{I}})\overline{\overline{B}}^T - (\overline{\overline{B}}:\overline{\overline{I}})\overline{\overline{A}}^T + [\overline{\overline{A}}\cdot\overline{\overline{B}} + \overline{\overline{B}}\cdot\overline{\overline{A}}]^T$$

$$: \bar{\bar{I}}$$

$$= 3\left(\mathrm{tr}\bar{\bar{A}}\,\mathrm{tr}\bar{\bar{B}} - \mathrm{tr}\bar{\bar{A}}\cdot\bar{\bar{B}}\right)$$

$$- 2\,\mathrm{tr}\bar{\bar{A}}\,\mathrm{tr}\bar{\bar{B}} + 2\,\mathrm{tr}\bar{\bar{A}}\cdot\bar{\bar{B}}$$

$$= \mathrm{tr}\bar{\bar{A}}\,\mathrm{tr}\bar{\bar{B}} - \mathrm{tr}\bar{\bar{A}}\cdot\bar{\bar{B}}$$

$$= \mathrm{tr}\bar{\bar{A}}\;\bar{\bar{I}}:\bar{\bar{B}} - \bar{\bar{A}}^T:\bar{\bar{B}}$$

$$\bar{\bar{A}}\cdot\bar{\bar{B}} : \bar{\bar{I}}$$

$$\bar{a}_1\bar{a}_2\cdot\bar{b}_1\bar{b}_2 : \bar{\bar{I}}$$

$$= \bar{a}_1\cdot\bar{b}_2\;\bar{a}_2\cdot\bar{b}_1$$

$$= \bar{\bar{A}} : \bar{\bar{B}}^T$$

$$= \bar{\bar{B}} : \bar{\bar{A}}^T$$

$$= (\bar{\bar{B}}\cdot\bar{\bar{A}}) : \bar{\bar{I}}$$

$$\left(\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{I}} - \mathrm{tr}\bar{\bar{A}}\;\bar{\bar{I}} + \bar{\bar{A}}^T\right) : \bar{\bar{B}} = 0$$

$$\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{I}} = \mathrm{tr}\bar{\bar{A}}\;\bar{\bar{I}} - \bar{\bar{A}}^T$$

$$(\bar{b}\times\bar{\bar{I}}) \overset{\times}{\times} \bar{\bar{I}} = \bar{b}\times\bar{\bar{I}}$$

$$\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{I}} = \lambda\bar{\bar{A}}$$

$$\underbrace{\mathrm{tr}\,\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{I}}}_{2\,\mathrm{tr}\bar{\bar{A}}} = \lambda\,\mathrm{tr}\bar{\bar{A}} \quad\Rightarrow\quad (2-\lambda)\,\mathrm{tr}\bar{\bar{A}} = 0$$

1) $\mathrm{tr}\bar{\bar{A}} = 0$ — $\bar{\bar{A}}$ antisymm. $\lambda = +1$

2) $\mathrm{tr}\bar{\bar{A}} = 0$ — $\bar{\bar{A}}$ symmetric $\lambda = -1$

3) $\lambda = +2$

$$\mathrm{tr}\bar{\bar{A}}\;\bar{\bar{I}} = 2\bar{\bar{A}} + \bar{\bar{A}}^T = 3\alpha\bar{\bar{I}}$$

$$\bar{\bar{A}} = \alpha\bar{\bar{I}}$$

$$\bar{\bar{A}} = \underbrace{\frac{1}{2}\left(\bar{\bar{A}} + \bar{\bar{A}}^T\right)}_{tr \to tr\bar{\bar{A}}} + \frac{1}{2}\left(\bar{\bar{A}} - \bar{\bar{A}}^T\right)$$

$$tr\left(\frac{1}{2}\left(\bar{\bar{A}} + \bar{\bar{A}}^T\right) - \frac{1}{3} tr\bar{\bar{A}}\,\bar{\bar{I}}\right) = tr\bar{\bar{A}} - tr\bar{\bar{A}} = 0$$

$$\bar{\bar{A}} = \left(\frac{1}{3} tr\bar{\bar{A}}\right)\bar{\bar{I}} + \left(\frac{1}{2}\bar{\bar{A}} + \frac{1}{2}\bar{\bar{A}}^T - \frac{1}{3} tr\bar{\bar{A}}\,\bar{\bar{I}}\right) + \left(\frac{1}{2}\bar{\bar{A}} - \frac{1}{2}\bar{\bar{A}}^T\right)$$

$$\bar{\bar{X}} \cdot \bar{\bar{X}} = \bar{\bar{I}} \qquad \bar{\bar{X}} = \pm \bar{\bar{I}}$$

$$\bar{\bar{X}} = \pm \left(\bar{\bar{I}} - 2 \frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}} \right) \qquad (\bar{a}\cdot\bar{b} \neq 0)$$

$$\bar{\bar{X}} \cdot \bar{\bar{X}} = \left(\bar{\bar{I}} - 2 \frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}} \right) \cdot \left(\bar{\bar{I}} - 2 \frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}} \right)$$

$$= \bar{\bar{I}} - 2 \frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}} - 2 \frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}} + 4 \frac{\bar{a}\bar{b}\cdot\bar{a}\bar{b}}{(\bar{a}\cdot\bar{b})^2}$$

e.g. $\bar{\bar{I}} - \bar{u}\bar{u}$

EIGENVECTORS / VALUES

$$\bar{\bar{A}} \cdot \bar{a}_i = \alpha_i \bar{a}_i \qquad \text{(right e.v.)}$$

$$\bar{b}_j \cdot \bar{\bar{A}} = \beta_j \bar{b}_j \qquad \text{(left e.v.)}$$

$$(\bar{\bar{A}} - \alpha_i \bar{\bar{I}}) \cdot \bar{a}_i = 0$$

$$\bar{b}_j \cdot (\bar{\bar{A}} - \beta_j \bar{\bar{I}}) = 0$$

$\bar{\bar{A}} - \gamma \bar{\bar{I}}$ is planar (for $\gamma = \alpha_i, \beta_j$)

$$6 \det(\bar{\bar{A}} - \gamma \bar{\bar{I}}) = 0$$

$$(\bar{\bar{A}} - \gamma \bar{\bar{I}}) \overset{\times}{\times} (\bar{\bar{A}} - \gamma \bar{\bar{I}}) : (\bar{\bar{A}} - \gamma \bar{\bar{I}})$$

$$= \underbrace{\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{A}}}_{6 \det \bar{\bar{A}}} - \gamma^3 \underbrace{\bar{\bar{I}} \overset{\times}{\times} \bar{\bar{I}} : \bar{\bar{I}}}_{6} - 3\gamma \underbrace{\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{I}}}_{2 \operatorname{spm} \bar{\bar{A}}} + 3\gamma^2 \underbrace{\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{I}} : \bar{\bar{I}}}_{2 \bar{\bar{A}} : \bar{\bar{I}}}$$

$$\gamma^3 - \gamma^2 \operatorname{tr} \bar{\bar{A}} + \gamma \operatorname{spm} \bar{\bar{A}} - \det \bar{\bar{A}} = 0$$

$$\bar{b}_j \cdot (\bar{\bar{A}} \cdot \bar{a}_i = \alpha_i \bar{a}_i) \Rightarrow \alpha_i \, \bar{b}_j \cdot \bar{a}_i$$

$$(\bar{b}_j \cdot \bar{\bar{A}} = \beta_j \bar{b}_j) \cdot \bar{a}_i \Rightarrow \beta_j \, \bar{b}_j \cdot \bar{a}_i$$

$$\operatorname{tr} \bar{\bar{A}} = \alpha_1 + \alpha_2 + \alpha_3$$

$$\mathrm{tr}\,\bar{\bar{A}} = \alpha_1 + \alpha_2 + \alpha_3$$

$$\det \bar{\bar{A}} = \alpha_1 \alpha_2 \alpha_3$$

$$\mathrm{spn}\,\bar{\bar{A}} = \alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1$$

$$\bar{a}, \bar{b}, \bar{c} \quad \to \quad \bar{a}' = \frac{\bar{b}\times\bar{c}}{J}, \quad \bar{b}' = \frac{\bar{c}\times\bar{a}}{J}, \quad \bar{c}' = \frac{\bar{a}\times\bar{b}}{J}$$

$$J = \bar{a}\times\bar{b}\cdot\bar{c}$$

$$\bar{\bar{I}} = \bar{a}\bar{a}' + \bar{b}\bar{b}' + \bar{c}\bar{c}'$$

$\bar{\bar{A}}$: r. eigenvectors : $\bar{a}, \bar{b}, \bar{c}$

$$\bar{\bar{A}} = \bar{\bar{A}}\cdot\bar{\bar{I}} = \bar{\bar{A}}\cdot(\bar{a}\bar{a}' + \bar{b}\bar{b}' + \bar{c}\bar{c}')$$

$$= \alpha_a \bar{a}\bar{a}' + \alpha_b \bar{b}\bar{b}' + \alpha_c \bar{c}\bar{c}'$$

$$\left(\bar{\bar{A}} - \alpha_a \bar{\bar{I}}\right)\cdot\left(\bar{\bar{A}} - \alpha_b \bar{\bar{I}}\right)\cdot\left(\bar{\bar{A}} - \alpha_c \bar{\bar{I}}\right) = 0$$

Hermitian dyadic

$$\bar{\bar{H}}^T = \bar{\bar{H}}^*$$

$$\bar{\bar{A}} = \frac{1}{2}\left(\bar{\bar{A}} + \bar{\bar{A}}^{T*}\right) + \frac{1}{2}\left(\bar{\bar{A}} - \bar{\bar{A}}^{T*}\right)$$

$$\bar{\bar{H}} = \bar{\bar{S}} + j\bar{b} \times \bar{\bar{I}}$$

$\bar{\bar{S}}$ symmetric, real

$\bar{b}$ real

GYROTROPIC DYADIC

$$\bar{\bar{G}} \cdot (\bar{g} \times \bar{\bar{I}}) = (\bar{g} \times \bar{\bar{I}}) \cdot \bar{\bar{G}} = \bar{g} \times \bar{\bar{G}}$$

$$= \bar{\bar{G}} \times \bar{g}$$

$$\bar{\bar{G}} = \alpha \bar{\bar{I}} + \bar{g}\bar{g} + \gamma\, \bar{g} \times \bar{\bar{I}}$$

REFLECTION DYADIC

$$\bar{\bar{C}}(\bar{u}) = \bar{\bar{I}} - 2\bar{u}\bar{u}$$

ROTATION DYADIC

$$\bar{\bar{R}}(\bar{u}, \theta) = \bar{u}\bar{u} + \sin\theta \; \bar{u} \times \bar{\bar{I}} + \cos\theta \,(\bar{\bar{I}} - \bar{u}\bar{u})$$

TWO-DIMENSIONAL DYADICS

$$\bar{u} \cdot \bar{\bar{A}} = \bar{\bar{A}} \cdot \bar{u} = 0$$

$$\bar{\bar{I}}_t = \bar{\bar{I}} - \bar{u}\bar{u} = \bar{v}\bar{v} + \bar{w}\bar{w} \, ; \quad \bar{\bar{J}} = \bar{u} \times \bar{\bar{I}} = \bar{w}\bar{v} - \bar{v}\bar{w}$$

$$(= \bar{u} \times \bar{\bar{I}}_t)$$

$$\bar{u}, \bar{v}, \bar{w}$$

$$\bar{\bar{K}} = \bar{v}\bar{v} - \bar{w}\bar{w} \qquad \bar{\bar{L}} = \bar{v}\bar{w} + \bar{w}\bar{v}$$

2D: $$\bar{\bar{A}} = a_1 \bar{\bar{I}}_t + a_2 \bar{\bar{J}} + a_3 \bar{\bar{K}} + a_4 \bar{\bar{L}}$$