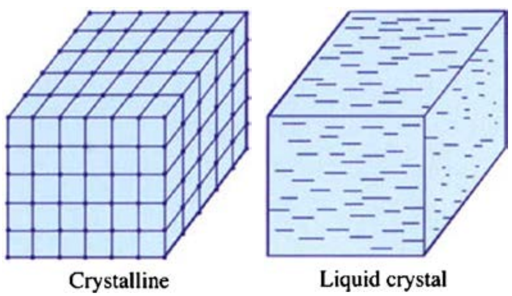
Chapter 6

POLARIZATION OPTICS II

Refractive indices in anisotropic media



D is not in general parallel to **E**

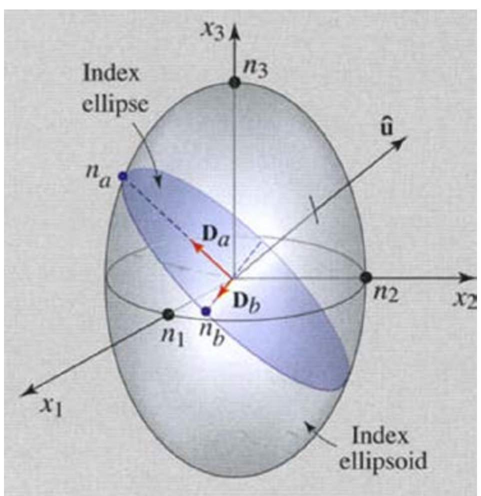
$$D_i = \sum_j \epsilon_{ij} E_j \quad \Leftrightarrow \quad \mathbf{D} = \boldsymbol{\epsilon} \cdot \mathbf{E}$$

The electric *permittivity tensor* is $\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$

$$\mathbf{E} = \boldsymbol{\epsilon}^{-1} \cdot \mathbf{D}$$

$\Rightarrow \ \epsilon_0 \mathbf{E} = \boldsymbol{\eta} \cdot \mathbf{D}$, where $\boldsymbol{\eta} = \epsilon_0 \boldsymbol{\epsilon}^{-1}$ is the *impermeability tensor*.

The *index ellipsoid* $\sum_{ij} \eta_{ij} x_i y_j = 1$ is the geometric representation of tensor $\boldsymbol{\eta}$. If $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ are the principal axes of the material, $\boldsymbol{\eta}$ is diagonal and we have

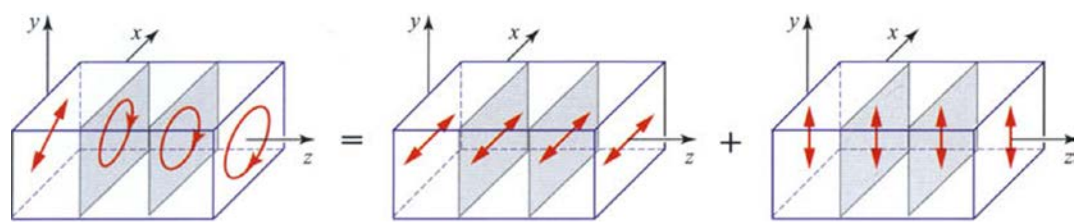


Polarization modes *a* and *b*

$\eta_i = 1/n_i^2$ and

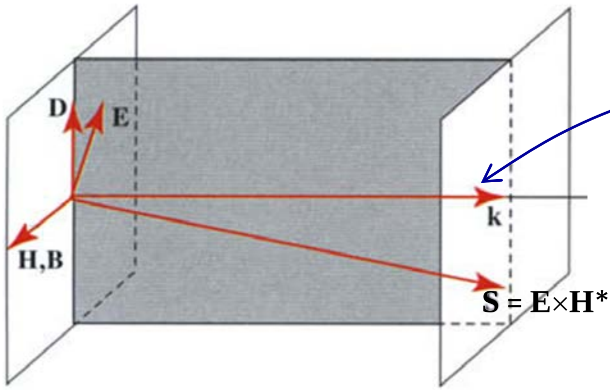
$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1.$$

Change of polarization upon propagation:



mode *a*, n_a **mode *b*,** n_b

The dispersion relation and k surface

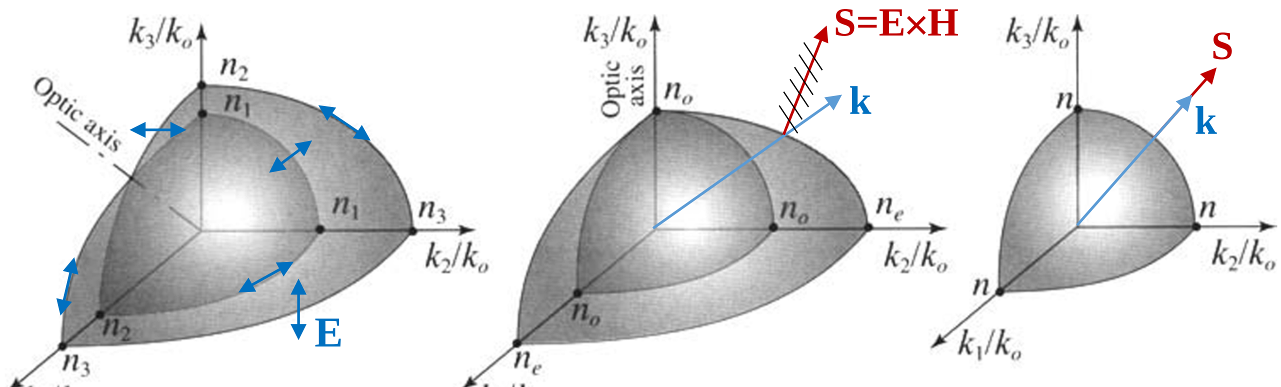


$$\mathbf{k}\times\mathbf{H} = -\omega\mathbf{D}$$
$$\mathbf{k}\times\mathbf{E} = \omega\mu_o\mathbf{H}$$

$$\Rightarrow\ \mathbf{k}\times(\mathbf{k}\times\mathbf{E}) + \omega^2\mu_o\boldsymbol{\epsilon}\mathbf{E} = \mathbf{0}$$

$$\Rightarrow \begin{bmatrix} n_1^2k_o^2 - k_2^2 - k_3^2 & k_1k_2 & k_1k_3 \\ k_2k_1 & n_2^2k_o^2 - k_1^2 - k_3^2 & k_2k_3 \\ k_3k_1 & k_3k_2 & n_3^2k_o^2 - k_1^2 - k_2^2 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Leftrightarrow \mathbf{K}\cdot\mathbf{E} = 0$$

$\Rightarrow$ det{**K**}= 0 - the equation of the **k** surface. The intersection of the normalized $\mathbf{k}/k_0$ surface with the propagation direction gives the *mode refractive indices*.

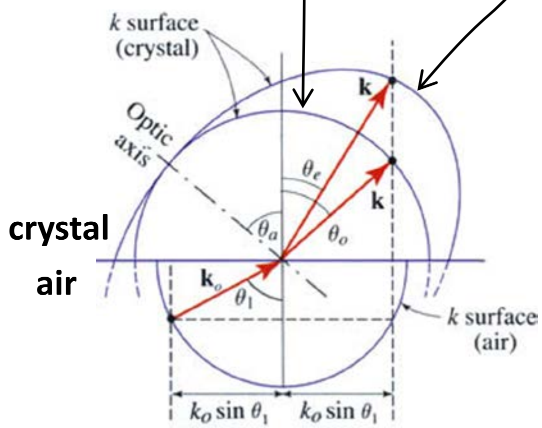


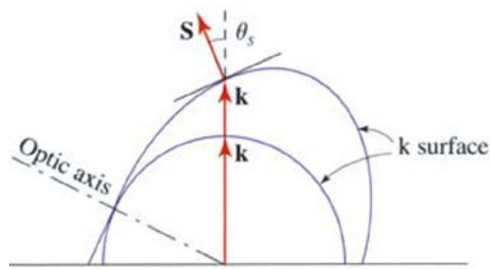
biaxial ***uniaxial*** ***isotropic***

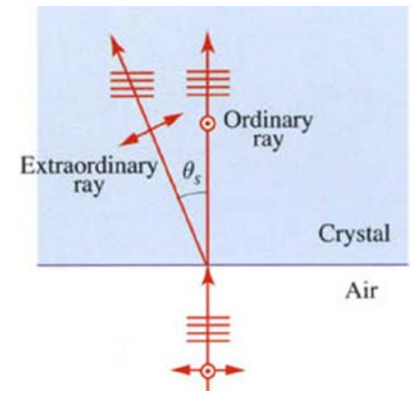
Double refraction

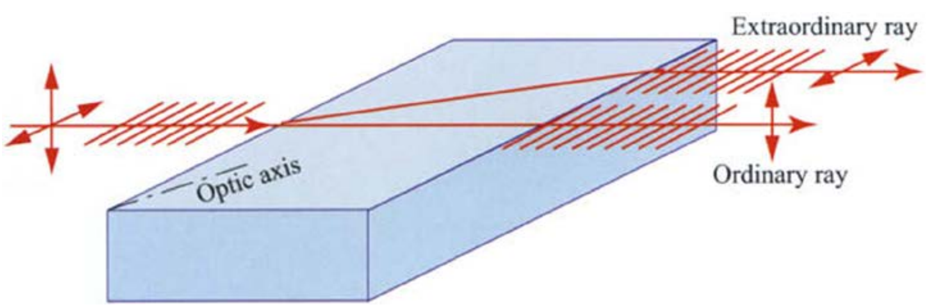
For a uniaxial crystal, $n_1 = n_2 = n_o$ and $n_3 = n_e$, the equation for the **k**-surface is

$$(k^2 - n_o^2 k_o^2)\left(\frac{k_1^2 + k_2^2}{n_e^2} + \frac{k_3^2}{n_o^2} - k_o^2\right) = 0$$



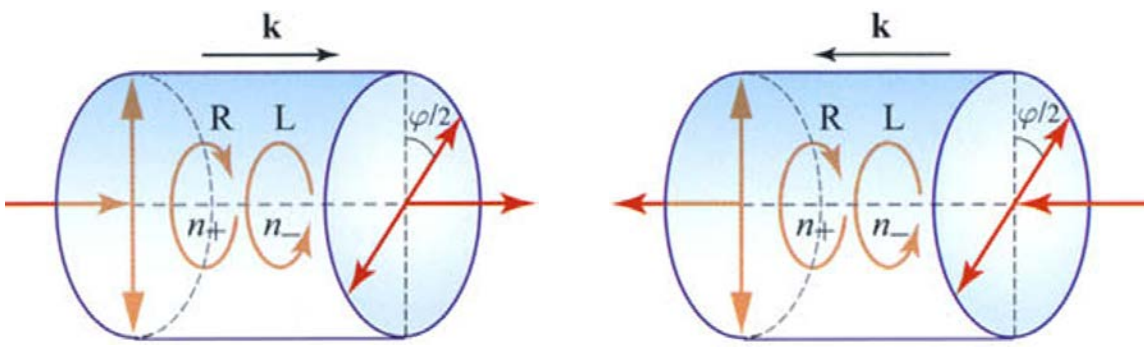






Optical activity

The polarization modes of an optically active medium are RCP and LCP and their refractive indices are n_+ and n_-.



The angle of rotation per unit length is

$$\rho = \frac{\pi}{\lambda_o}(n_- - n_+)$$

Optically active medium is *spatially dispersive*. The medium equation can be written as

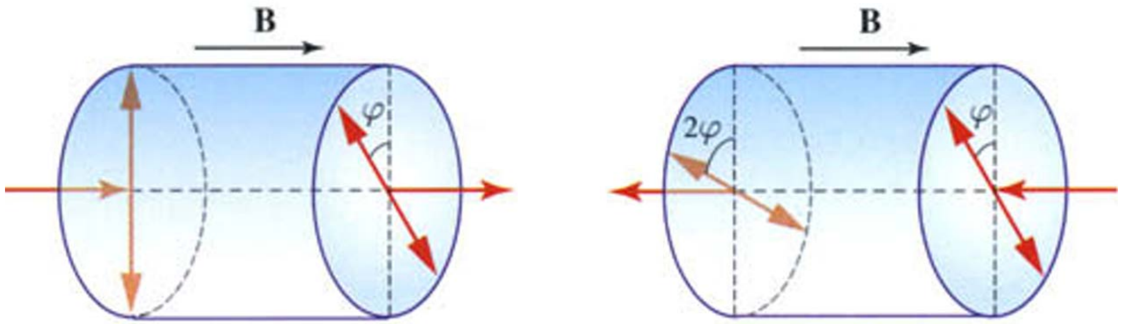
$$\mathbf{D} = \epsilon\mathbf{E} + j\epsilon_o\xi\nabla\times\mathbf{E} = \epsilon\mathbf{E} + j\epsilon_o\mathbf{G}\times\mathbf{E}$$

where ξ is a constant and $\mathbf{G} = \xi\mathbf{k}$ is the gyration vector. The refractive indices are then given by

$$n_\pm = \sqrt{n^2 \pm G} \;\Rightarrow\; \rho \approx -\frac{\pi G}{\lambda_o n}$$

Faraday effect

The modes are circularly polarized as well, and n_+ and n_- are different. However, the rotation is not reciprocal.



The material equation can be written in the same form:

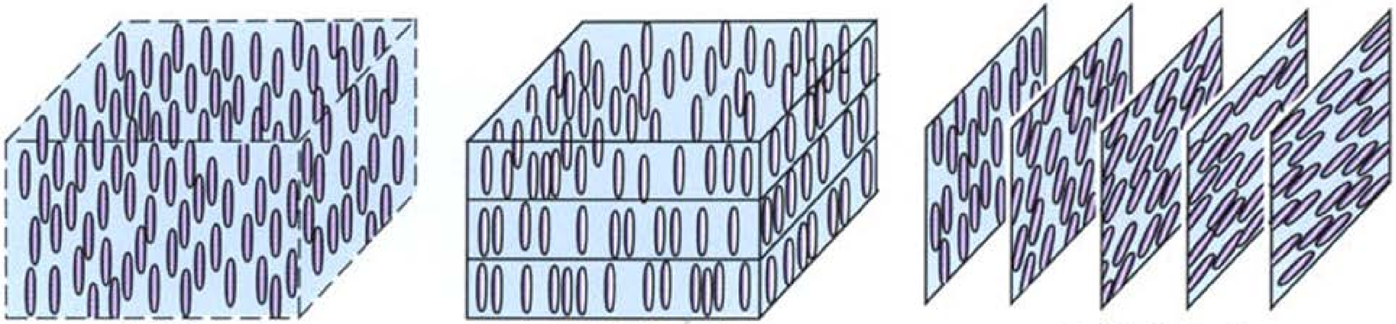
$$\mathbf{D} = \epsilon \mathbf{E} + j\epsilon_o \mathbf{G} \times \mathbf{E},$$

where $\mathbf{G} = \gamma \mathbf{B}$ and γ is the magnetogyration coefficient. The rotatory power is

$$\rho = \mathfrak{V} B \approx -\frac{\pi G}{\lambda_o n}$$

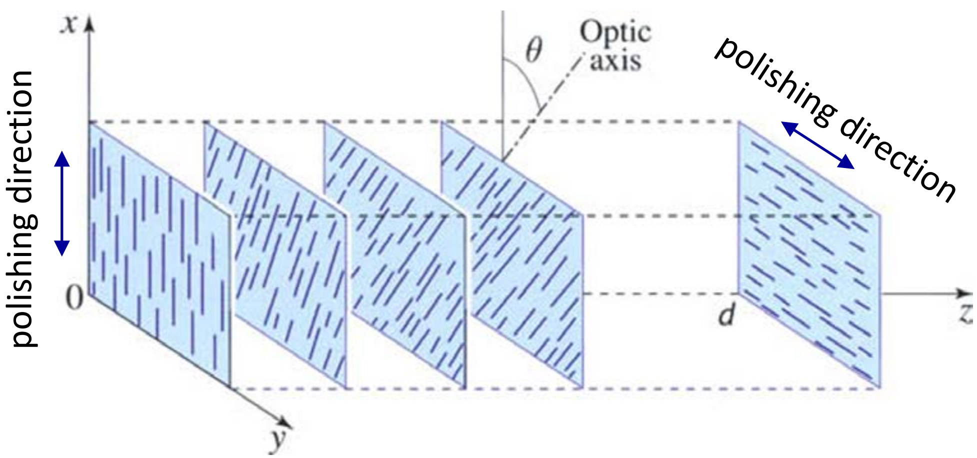
$$\Rightarrow \mathfrak{V} \approx -\frac{\pi \gamma}{\lambda_o n}$$ (Verdet constant)

Optics of liquid crystals



(a) Nematic (b) Smectic (c) Cholesteric

Twisted nematic liquid crystals:



Each slice acts as a uniaxial crystal with the optic axis at

$$\theta = \alpha z$$

and the phase retardation coefficient

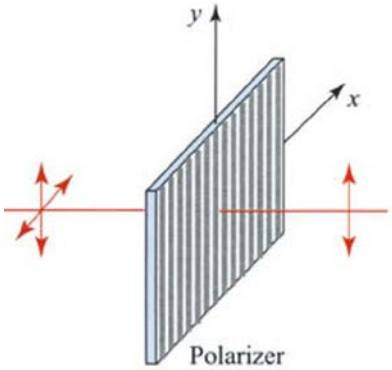
$$\beta = (n_e - n_o)k_0 >> \alpha.$$

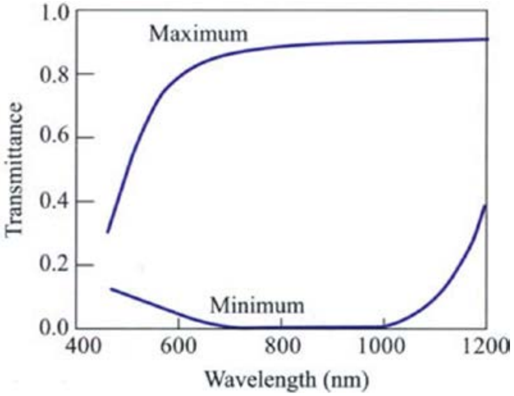
The modes are *linearly polarized* along and perpendicular to the optic axis at each coordinate z and having the refractive indices n_o and n_e. The orientation of the molecules can be changed with an electric field, which is used in modulators.

Polarization devices

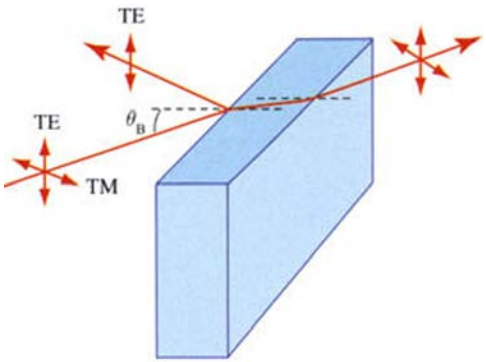
Polarizers

Anisotropic absorption (dichroic material) or reflection:

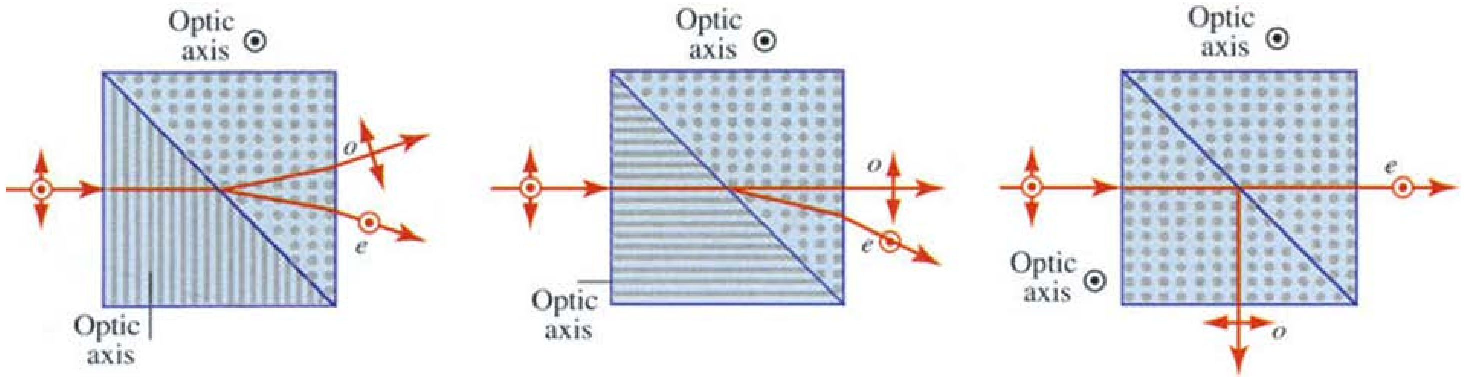




Reflection at the Brewster angle:



Refraction and reflection in a polarizing beam splitter:



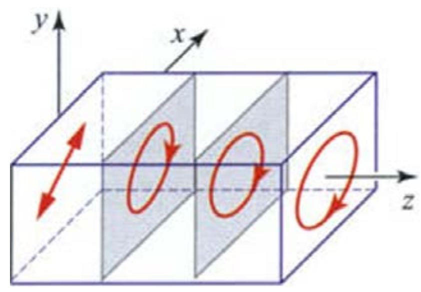
(*a*) Wollaston prism (*b*) Rochon prism (*c*) Glan–Thompson prism

Polarization devices

Wave retarders

Anisotropic crystals, $n_1 < n_2$

(*including tunable Pockels-type crystals and liquid crystals*)

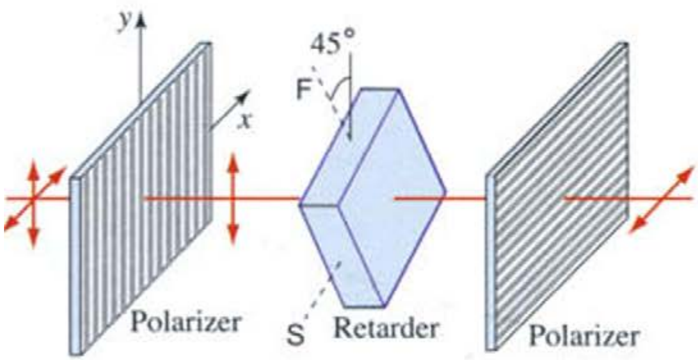


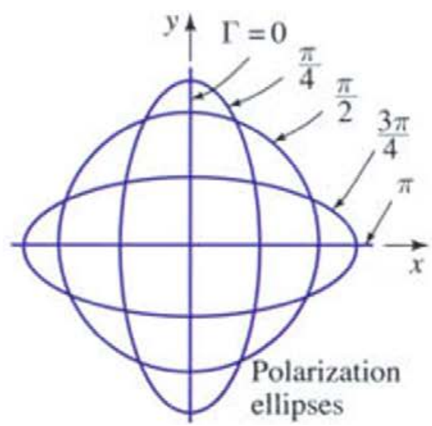
Phase retardation is

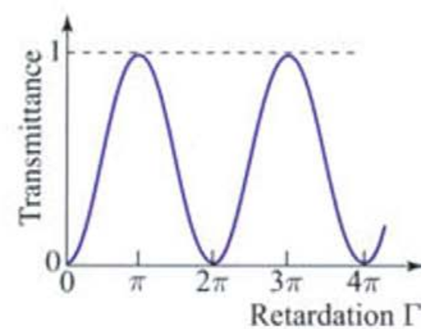
$$\Gamma = (n_2 - n_1) k_0 d.$$

In mica, $\Gamma = \pi$ for d = 60 μm at λ = 600 nm.

A tunable wave retarder between crossed polarizers = optical modulator







$T = \sin^2(\Gamma/2)$

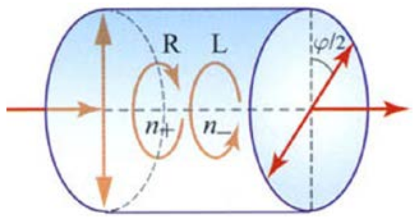
Electrically tunable liquid crystal or Pockels cell

Polarization devices

Polarization rotators

Optically active and Faraday rotators

$n_{\mathrm{RCP}} < n_{\mathrm{LCP}}$



Optical isolators