

A Course on Open Economy Macroeconomics,
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Problem set ~~4~~, Solutions
3

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INTEREST RATE ARBITRAGE

$$\begin{aligned}r_t &= r_t^f + \Delta s_{t+1}^e \quad \text{UIP} \\r_t &= r_t^f + f_t - s_t \quad \text{CIP}\end{aligned}$$

where r_t is the *domestic* and r_t^f the *foreign* interest rate

Exercise 1 Suppose that the interest rate differential between the Euroarea and US is 0.5% and that the euro is expected to depreciate by 1.5%; what is the risk premium on the underlying Euroarea bond as implied by the covered interest rate parity, CIP?

Solution 2 Given the interest rate arbitrage conditions

$$r_t^{\text{euro}} = r_t^{\text{US}} + f_t - s_t$$

or, since the Euroland is the home country and given the assumptions that a) $r_t^{\text{euro}} - r_t^{\text{US}} = 0.5\%$ and b) $\mathbb{E}_t \Delta s_{t+1} = 1.5\%$ (ie. since S_t is the euro price of the USD, this means that the euro is expected to weaken by 1.5 %)

$$\begin{aligned}r_t^{\text{euro}} - r_t^{\text{US}} &= f_t - s_t = fp_t^{\text{euro}} = rp_t + \mathbb{E}_t \Delta s_{t+1}, \text{ or} \\0.005 &= rp_t + 0.015 \implies rp_t = 0.005 - 0.015 = -0.010 = -1.0\%\end{aligned}$$

Thus the risk premium on the Euroarea bond is negative, ie. that of the dollar bond is positive (comes from the fact the expected depreciation of the euro is pretty large relative to the interest rate differential). We can say that the euro is trading at a discount.

Exercise 3 Suppose the rate of return on a Eurobond is 4.3%, and that of an identical Dollarbond is 4.05 %; assume also that the risk premium on the Euroarea bond is 0.25%. What is the forward premium as implied by the CIP?

Solution 4 Use the same assumptions as in the first problem

$$\begin{aligned} r_t^{US} &= 4.05\% \\ r_t^{euro} &= 4.30\% \\ rp_t^{US} &= 0.25\% \end{aligned}$$

Euroland = home. The notation in the last expression indicates the risk premium on the dollar bond. Now, given CIP (and that ; hence S_t is the euro price of the USD)

$$\begin{aligned} r_t^{euro} - r_t^{US} &= f_t - s_t = fp_t^{euro} = -fp_t^{US}, \text{ or} \\ 0.0430 - 0.0405 &= 0.0025 = fp_t^{euro} = -fp_t^{US} \end{aligned}$$

Exercise 5 Use the interest rate parity equation to calculate the spot exchange rate (S_t) when $r = 0.045$, $r^f = 0.043$, and $S_{t+1}^e = \mathbb{E}_t S_{t+1} = 1.094$. Is the resulting S_t greater or less than 1.094?

Solution 6 We have (*Euroland = home*, S_t is the euro price of the USD)

$$\begin{aligned} r_t^{US} &= 0.043 \\ r_t^{euro} &= 0.045 \\ S_{t+1}^e &= \mathbb{E}_t S_{t+1} = 1.094 \implies \ln \mathbb{E}_t S_{t+1} = \ln(1.094) = 0.089841 \end{aligned}$$

where S_t denotes the euro price of the USD (so that $1/S_t$ denotes the USD price of the euro). This time use the UIP ($r_t^{euro} - r_t^{US} = \Delta s_{t+1}^e$) gives

$$\begin{aligned} r_t^{euro} &= r_t^{US} + \mathbb{E}_t \Delta s_{t+1} \text{ or} \\ r_t^{euro} - r_t^{US} &= (\ln \mathbb{E}_t S_{t+1} - \ln S_t) \implies \\ 0.045 - 0.043 &= 0.002 = 0.089841 - \ln S_t \iff \\ 0.002 - 0.089841 &= -0.087841 = -\ln S_t \iff S_t = e^{0.087841} = 1 - 1.91815 < 1.094 \end{aligned}$$

So $S_t = 1.091815$ should be the current spot euro price of the USD that is consistent with the interest rates and expected spot rate. Note that the approximation $\mathbb{E}_t \ln S_{t+1}$ cannot be used, since we do not know the realized value of the future spot euro price of the USD. Anyway, to be consistent with the interest rate differential and expected euro price of the dollar, the euro has to be relatively strong currently.

Exercise 7 Suppose r_t , r_t^f , and S_{t+1}^e are at the values specified in Ex3, but suppose that the spot exchange rate S_t is initially equal to 1.094. Describe the market forces that would drive the spot rate to the answer you found in Ex3.

Solution 8 Now, if the the current spot euro price of the USD were 1.094 instead of the 1.091815 found in the previous problem, the euro would appear to be slightly undervalued relative to the observed interest rate differential and expected spot rate. In the eyes of the risk neutral investors (Note! We are under

the UIP!), investing in the euro bond seems more attractive than investing the corresponding dollar bond. So, that's what they do resulting in an increase in the demand for euros. This puts pressure on the euro to strengthen, ie. dollar to weaken. This is the mechanism that brings the euro price of the USD back in line with the observed interest rate differential between the US and euroarea economies as well as with the expected future spot euro price of the the USD.